

Portfolio Management

Financial Markets, Day 4, Class 1

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Outline for Day 4

- Class 1: Portfolio Management.
- Class 2: Risk Management.
- Class 3: Chinese Stock Market.
- Class 4: Chinese Bond Market.
- Class 5: Financial Institutions in China.
- Class 6: Review and quiz.

Outline for Class 1

- The process of portfolio management.
- Optimal portfolio selection with one risky asset.
- The Optimal risky portfolio.
- Limitations of portfolio theory.
- The Black-Litterman asset allocation model.

Policy Portfolio, Harvard Management Company, 2002

	Min	Policy	Max	Benchmark
Domestic equities	10	15	25	80% S&P 500, 8% S&P 400, 12% Russell 2000
Foreign equities	5	10	15	93% EAFE, 7% MSCI Small Cap ex US ex EAFE
Emerging markets	2	5	8	80% MSCI EM Investable, 20% MSCI EM Inv + 5%
Private equities	8	13	18	Cambridge Associates Weighted Composite
Total	30	43	60	
Absolute return	8	12	16	20% equity composite, 20% LIBOR+5%, 60% funds of funds
High-yield	2	5	8	60% Sal. High-Yield/Bankrupt Weighted Composite, 40% EMBI+
Commodities	8	13	18	23% GSCI and 77% NCREIF Timberland Index
Real estate	6	10	14	50% CPI+6, 25% NCREIF, 25% REIT. Leverage adjusted
Total	25	40	50	
Domestic bonds	6	11	21	Lehman 5+ year Treasury Index
Foreign bonds	0	5	10	J.P. Morgan Non U.S.
Inflation-indexed	0	6	15	Salomon 5+ year TIPS
Cash	-10	-5	10	One-month LIBOR
Total	0	17	30	

The Process of Portfolio Management

Objectives of the Portfolio:

- Client Risk Tolerance
- Universe of Assets
- Passive vs. Active
- Stock Selections vs. Asset Allocation
- Tactical vs. Strategic Asset Allocation
- Selection of Benchmark

Tactical Asset Allocation

- Switching between asset classes
- Enhanced Indexing
- Market Timing

Security Selection within Asset Class

- Selection of Individual Stocks
- Top-Down vs. Bottom-up
- Growth vs. Value (“Style”)
- Fundamental vs. Technical
- Macro vs. Micro
- Quantitative vs. Traditional
- Long/Short Plays

- What is your value-added?
- What can you promise?
- How do you deal with poor performance?
- Client services

Performance Evaluation

- Comparison with benchmark
- Risk adjustments
- Performance Attribution

Compensation

- Base + pay for performance
- Objective: incentive alignment
- High-water mark
- Clawback

The Investment Opportunity

Monthly Returns from 198706-201009

	mean	std	Sharpe ratio
riskfree r_f	0.33%	0	NA
risky asset R_p			
CRSP VW	0.80%	4.65%	0.1011
Magellan	0.79%	5.15%	0.0893
PIMCO	0.70%	1.25%	0.2960
Hedge Fund Index*	0.77%	2.23%	0.1973

*Hedge fund data starts in 199401.

A Mean-Variance Investor

$$\mathbf{Utility} = \text{mean} - \frac{1}{2} \times \text{risk aversion} \times \text{variance}$$

asset class	mean	variance	risk aversion	utility
riskfree	0.33%	0	any	0.33%
CRSP	0.80%	$(4.65\%)^2$	1	0.69%
	0.80%	$(4.65\%)^2$	4	0.37%
	0.80%	$(4.65\%)^2$	10	-0.28%
PIMCO	0.70%	$(1.25\%)^2$	1	0.69%
	0.70%	$(1.25\%)^2$	4	0.67%
	0.70%	$(1.25\%)^2$	10	0.62%

Portfolio Construction with One Risky and One Riskfree

Assume the investor's risk aversion = 4. He invests a fraction y of his wealth in the risky asset (CRSP), and leaves $1 - y$ in the riskfree.

$$\tilde{R}_y = (1 - y) r_f + y \tilde{R}_p$$

strategy	y	mean	variance	utility	Sharpe ratio
pure riskfree	0	0.33%	0	0.3300%	N/A
conservative	20%	0.42%	$(0.93\%)^2$	0.4067%	0.1011
half & half	50%	0.57%	$(2.33\%)^2$	0.4569%	0.1011
aggressive	80%	0.71%	$(3.72\%)^2$	0.4292%	0.1011
pure risky	100%	0.80%	$(4.65\%)^2$	0.3675%	0.1011
leveraged	200%	1.27%	$(9.30\%)^2$	-0.4598%	0.1011
short	-50%	0.095%	$(2.33\%)^2$	-0.0131%	-0.1011
optimal	54.34%	0.59%	$(2.53\%)^2$	0.4577%	0.1011

The Tradeoff of Risk and Return

- For any investor with a known risk aversion, we can actually solve the optimal portfolio weight y^* for him.
- Adding more risky asset increases the mean of the portfolio:

$$\text{mean} = (1 - y) 0.33\% + y0.80\% = 0.33\% + (0.80\% - 0.33\%)y$$

- But it also increases the volatility of the portfolio:

$$\text{variance} = (4.65\%)^2 y^2$$

- Recall

$$\text{Utility} = \text{mean} - \frac{1}{2} \times \text{risk aversion} \times \text{variance}$$

- The optimal portfolio weight y^* is the portfolio mix that maximizes the investor's utility.

The Optimal Risk and Return Tradeoff

The optimal risk and return tradeoff can be achieved by:

$$y^* = \frac{\text{risk premium}}{\text{variance} \times \text{risk aversion}}$$

- All else equal, a more risk averse investor invests less in the risky asset.
- All else equal, a higher risk premium induces investor to hold more of the risky asset.
- All else equal, a lower volatility induces investor to hold more of the risky asset.

Portfolio Construction with Two Risky and One Riskfree

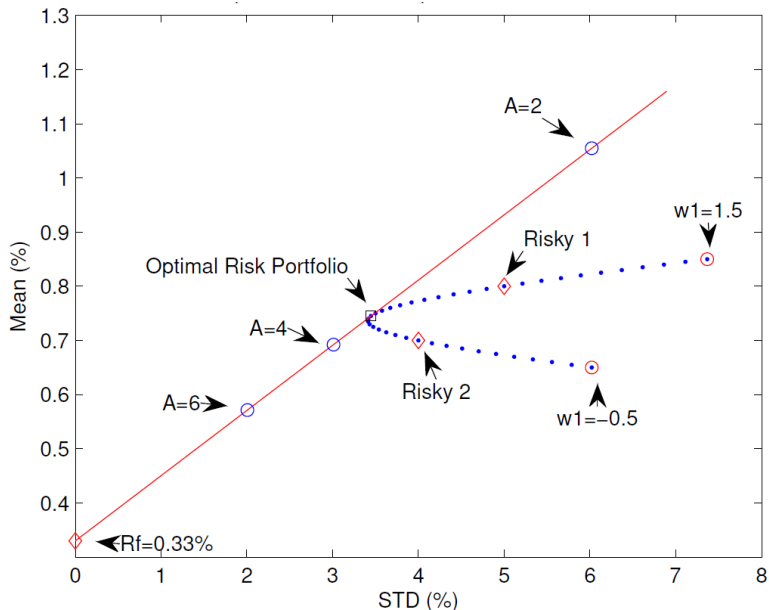
Invest a fraction w_1 in risky asset 1, w_2 in risky asset 2, and leave $w_0 = 1 - w_1 - w_2$ in the riskfree account:

$$\tilde{R}_w = w_0 r_f + w_1 \tilde{R}_1 + w_2 \tilde{R}_2 .$$

	mean	std
riskfree r_f	0.33%	
risky asset 1	0.80%	5.00%
risky asset 2	0.70%	4.00%

- mean = $w_0 \times 0.33\% + w_1 \times 0.80\% + w_2 \times 0.70\%$
- variance = $w_1^2 \times (5.00\%)^2 + w_2^2 \times (4.00\%)^2 + 2 \times w_1 \times w_2 \times \text{cov.}$
- cov = $\text{corr} \times (5.00\%) \times (4.00\%)$; $\text{corr} = 20\%$.

Mean-Variance Spanned by Two Risky Assets



Portfolio Strategies and Investor Utility

Fix risk aversion at 4:

strategy	w_0	w_1	w_2	mean	variance	utility	Sharpe ratio
riskfree	100%	0	0	0.33%	0	0.3300%	N/A
all risky 1	0	100%	0	0.80%	$(5.00\%)^2$	0.3000%	0.0940
all risky 2	0	0	100%	0.70%	$(4.00\%)^2$	0.3800%	0.0925
risky 1 & 2	60%	20%	20%	0.50%	$(1.40\%)^2$	0.4588%	0.1200
equal weight	1/3	1/3	1/3	0.61%	$(2.33\%)^2$	0.5011%	0.1200
optimal	12.70%	39.32%	47.98%	0.69%	$(3.01\%)^2$	0.5112%	0.1204

The Optimal Portfolio Solution

risk aversion	w_0^*	w_1^*	w_2^*	$w_1^*/(w_1^* + w_2^*)$	Sharpe ratio
1	-249%	157%	192%	45.04%	0.1204
2	-74.61%	78.65%	95.96%	45.04%	0.1204
4	12.70%	39.32%	47.98%	45.04%	0.1204
6	41.80%	26.22%	31.99%	45.04%	0.1204
10	65.08%	15.73%	19.19%	45.04%	0.1204

- Regardless of their risk aversion, investors hold *the same* optimal risky portfolio: $w_1^*/(w_1^* + w_2^*)$ is the same for all investors.
- This optimal risky portfolio, also known as the tangent portfolio, has the highest Sharpe ratio attainable.
- What separates a more risk averse investor from his more risk tolerant counterpart is the optimal weight w_0 invested in the riskfree asset.

The Optimal Portfolio Solution

- When there is only one risky asset, the optimal portfolio weight

$$y^* = \frac{\text{risk premium}}{\text{variance} \times \text{risk aversion}}$$

- Now we have two risky assets, so the portfolio weight has two elements,

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

- The risk premium also has two elements,

$$\text{risk premium} = \begin{pmatrix} 0.80\% - 0.33\% \\ 0.70\% - 0.33\% \end{pmatrix} = \begin{pmatrix} 0.47\% \\ 0.37\% \end{pmatrix}$$

- The variance now has variance as well as covariance,

$$\Sigma = \begin{pmatrix} \text{variance 1} & \text{covariance} \\ \text{covariance} & \text{variance 2} \end{pmatrix} = \begin{pmatrix} 5\%^2 & 5\% \times 4\% \times 0.2 \\ 5\% \times 4\% \times 0.2 & 4\%^2 \end{pmatrix}$$

and it is now called variance-covariance matrix.

- The optimal portfolio weight:

$$\frac{\text{risk premium}}{\text{variance} \times \text{risk aversion}}$$

still applies, except that we need to use matrix notation:

$$w^* = \frac{1}{\text{risk aversion}} \times \Sigma^{-1} \times \text{risk premium}$$

- For example, an investor with risk aversion = 4:

$$w^* = \frac{1}{4} \times \begin{pmatrix} 5\%^2 & 5\% \times 4\% \times 0.2 \\ 5\% \times 4\% \times 0.2 & 4\%^2 \end{pmatrix}^{-1} \begin{pmatrix} 0.47\% \\ 0.37\% \end{pmatrix}$$

Matrix Operations

Some useful tips for matrix operation in *Excel*:

- the command for summation is still “+”
- the command for multiplication is “mmult”
- the command for inverse, say Σ^{-1} , is “minverse”

Some useful tips for matrix operation in *Matlab*:

- the command for summation is still “+”
- the command for multiplication is still “*”
- the command for inverse, say Σ^{-1} , is “inv(Σ)”

Magellan, PIMCO, and Hedge Fund

Monthly Returns from 199401 to 201009

	mean	std	corr(.,P)	corr(.,H)
riskfree r_f	0.28%	0		
risky asset				
Magellan	0.60%	5.18%	13.51%	58.70%
PIMCO	0.61%	1.21%	100%	22.98%
Hedge Fund Index*	0.77%	2.23%	22.98%	100%

The optimal risky portfolio: -7.14%, 70.88%, and 36.26%.

On Modern Portfolio Theory

- The Modern Portfolio Theory is about *optimal diversification* and *optimal risk and return tradeoff*.
- This intellectual foundation should be central to any portfolio management.
- The actual math, however, needs human supervision.
- Otherwise, it could be harmful instead of useful.

Limitations of Mean-Variance Optimization

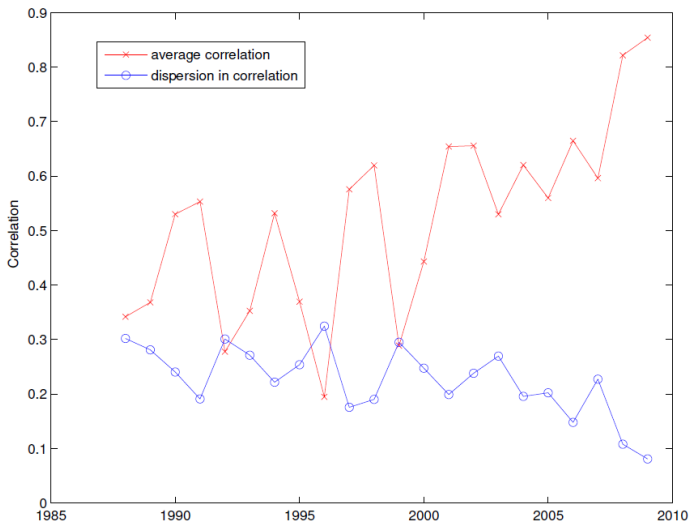
- Despite mean-variance optimization's potential for positive contribution to portfolio structuring, dangerous conclusions may be reached if poorly considered forecasts enter the modeling process.
- Some of the most egregious errors committed with mean-variance analysis involve inappropriate use of the historical data. As a result, unconstrained mean-variance runs usually provide solutions unrecognizable as reasonable portfolios.
- One critic of mean-variance analysis writes: "The unintuitive character of many optimized portfolios can be traced to the fact that mean-variance optimizers are 'estimation error' maximizers."

Other Limitations of the Mean-Variance Analysis

- Evidence suggests that distributions of security returns might not be normal.
- The way in which asset classes relate to each other may not be stable.
- Even more disturbing, market crises tend to cause otherwise distinct markets to behave in a similar fashion.
- Mean-variance optimization assumes that expected return and risk completely define asset class characteristics. The framework fails to consider other important attributes, such as liquidity and marketability.

International Diversification

Correlations Averaged Across 23 Developed Countries



The Traditional Portfolio Theory

- So far, the portfolio optimizer asks the user to input a complete set of expected returns (or risk premiums) and the variance-covariance matrix, and generates the optimal portfolio weights.
- Due to the complex mapping between expected returns and portfolio weights, users of the standard portfolio optimizers often find that their specification of expected returns produces output portfolio weights which may not make sense.
- These unreasonable results stem from two well-recognized problems:
 - 1 Expected returns are very difficult to estimate. Investors typically have knowledgeable views about absolute or relative returns in only a few markets. A standard optimization model, however, requires them to provide expected returns for all assets.
 - 2 The optimal portfolio weights of standard asset allocation models are extremely sensitive to the return assumptions used.

The Traditional Portfolio Theory

- These two problems compound each other; the standard model has no way to distinguish strongly held views from auxiliary assumptions, and the optimal portfolio it generates, given its sensitivity to the expected returns, often appears to bear little or no relation to the views the investor wishes to express.
- In practice, therefore, despite the obvious attractions of a quantitative approach, few global investment managers regularly allow quantitative models to play a major role in their asset allocation decision.

The Black-Litterman model

Mix beliefs with portfolio theory:

- The Black-Litterman asset allocation model, developed when both authors were working for Goldman Sachs, is a significant modification of the traditional mean-variance approach.
- In the Black-Litterman model, the user inputs any number of views or statements about the expected returns of arbitrary portfolios, and the model combines the views with equilibrium, producing both the set of expected returns of assets as well as the optimal portfolio weights.
- Since publication in 1990, the Black-Litterman asset allocation model has gained wide application in many financial institutions.

An illustrative example

Asset allocation among G7 countries:

Annualized volatilities and market-capitalization weights:

Country	Volatility (%)	Portfolio Weight (%)
Australia	16.0	1.6
Canada	20.3	2.2
France	24.8	5.2
Germany	27.1	5.5
Japan	21.0	11.6
UK	20.0	12.4
USA	18.7	61.5

Correlations among the Equity Index Returns

	Australia	Canada	France	Germany	Japan	UK
Canada	0.488					
France	0.478	0.664				
Germany	0.515	0.655	0.861			
Japan	0.439	0.310	0.355	0.354		
UK	0.512	0.608	0.783	0.777	0.405	
USA	0.491	0.779	0.668	0.653	0.306	0.652

The variance-covariance matrix Σ

	AUS	CAN	FRA	GER	JAP	UK	USA
AUS	0.0256	0.0159	0.0190	0.0223	0.0148	0.0164	0.0147
CAN	0.0159	0.0412	0.0334	0.0360	0.0132	0.0247	0.0296
FRA	0.0190	0.0334	0.0615	0.0579	0.0185	0.0388	0.0310
GER	0.0223	0.0360	0.0579	0.0734	0.0201	0.0421	0.0331
JAP	0.0148	0.0132	0.0185	0.0201	0.0441	0.0170	0.0120
UK	0.0164	0.0247	0.0388	0.0421	0.0170	0.0400	0.0244
USA	0.0147	0.0296	0.0310	0.0331	0.0120	0.0244	0.0350

details:

- volatility of CAN=20.3%, volatility of USA=18.7%, and correlation=77.9%.
- $\text{cov}(\text{CAN}, \text{CAN}) = \text{var}(\text{CAN}) = 0.203^2 = 0.0412$.
- $\text{cov}(\text{USA}, \text{USA}) = \text{var}(\text{USA}) = 0.187^2 = 0.0350$.
- $\text{cov}(\text{CAN}, \text{USA}) = 0.203 \times 0.187 \times 0.779 = 0.0296$.

The equilibrium portfolio weights w^{EQ} :

- Let's think of the market as a whole. Lending and borrowing sum up to zero (assuming no government borrowing). We have $w_0 = 0$.
- The U.S. stock market accounts for 61.5% of the total wealth invested in the seven stock market, while Canada accounts for 2.2%. We have $w^{USA} = 61.5\%$ and $w^{CAN} = 2.2\%$.
- More generally,

$$w^{EQ} = \begin{pmatrix} \text{AUS} & 1.6\% \\ \text{CAN} & 2.2\% \\ \text{FRA} & 5.2\% \\ \text{GER} & 5.5\% \\ \text{JAP} & 11.6\% \\ \text{UK} & 12.4\% \\ \text{USA} & 61.5\% \end{pmatrix}$$

The Equilibrium Risk Premium

- If investors are fully optimized, then the equilibrium portfolio weight must be their optimal portfolio weight.
- We can use this information and backout the equilibrium risk premiums that give rise to the equilibrium portfolio holdings.
- Recall that,

$$w^* = \frac{1}{\text{risk aversion}} \times \Sigma^{-1} \times \text{risk premium}$$

- Reverse the direction gives

$$\text{risk premium} = \text{risk aversion} \times \Sigma \times w^* .$$

The Neutral View Risk Premium

- Assuming that the average risk aversion of all investors is 2.5:

$$\text{risk premium}^{\text{EQ}} = 2.5 \times \Sigma \times w^{\text{EQ}} = \begin{pmatrix} \text{AUS} & 3.9\% \\ \text{CAN} & 6.9\% \\ \text{FRA} & 8.4\% \\ \text{GER} & 9.0\% \\ \text{JAP} & 4.3\% \\ \text{UK} & 6.8\% \\ \text{USA} & 7.6\% \end{pmatrix}$$

- Black and Litterman call this the *neutral view risk premium*.

Historical Returns

Table I Historical Excess Returns, January 1975–August 1991*

	<i>Germany</i>	<i>France</i>	<i>Japan</i>	<i>U.K.</i>	<i>U.S.</i>	<i>Canada</i>	<i>Australia</i>
Total Mean Excess Return							
Currencies	-20.8	3.2	23.3	13.4		12.6	3.0
Bonds	15.3	-2.3	42.3	21.4	-4.9	-22.8	-13.1
Equities	112.9	117.0	223.0	291.3	130.1	16.7	107.8
Annualized Mean Excess Return							
Currencies	-1.4	0.2	1.3	0.8		0.7	0.2
Bonds	0.9	-0.1	2.1	1.2	-0.3	-1.5	-0.8
Equities	4.7	4.8	7.3	8.6	5.2	0.9	4.5
Annualized Standard Deviation							
Currencies	12.1	11.7	12.3	11.9		4.7	10.3
Bonds	4.5	4.5	6.5	9.9	6.8	7.8	5.5
Equities	18.3	22.2	17.8	24.7	16.1	18.3	21.9

* Bond and equity returns in U.S. dollars, currency hedged and in excess of the London interbank offered rate (LIBOR); returns on currencies are in excess of the one-month forward rates.

Source: Black and Litterman (1992)

Neutral View

- The neutral view risk premiums are the risk premiums that give rise to the market portfolio weights.
- Put it differently, these are the risk premiums expressed by the market as a whole through the portfolio holdings aggregate over all investors.
- As an individual investor, he or she will be following the crowd's opinion by holding a portfolio with weights proportional to w^{EQ} .
- What if you have an opinion of your own?

A Naïve Treatment of a View

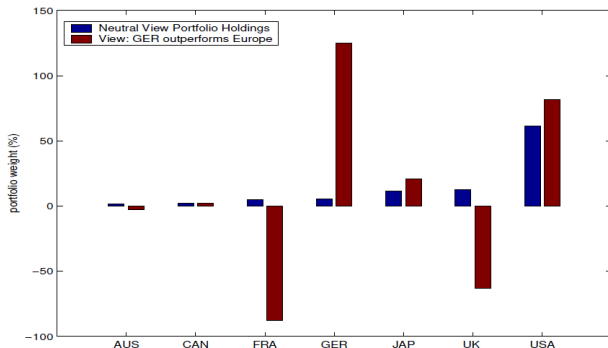
- Your view: Germany will outperform the rest of Europe by 5%.
- Add 2.5% to Germany and subtract 2.5% each from French and UK:

$$\text{risk premium}^{\text{view}} = \begin{pmatrix} \text{AUS} & 3.9\% \\ \text{CAN} & 6.9\% \\ \text{FRA} & 8.4\% - 2.5\% \\ \text{GER} & 9.0\% + 2.5\% \\ \text{JAP} & 4.3\% \\ \text{UK} & 6.8\% - 2.5\% \\ \text{USA} & 7.6\% \end{pmatrix} = \begin{pmatrix} 3.9\% \\ 6.9\% \\ 5.9\% \\ 11.5\% \\ 4.3\% \\ 4.3\% \\ 7.6\% \end{pmatrix}$$

A Naïve Treatment of a View

The portfolio optimizer gives (risk aversion = 2.5):

$$w^{\text{view}} = \frac{1}{2.5} \times \Sigma^{-1} \times \text{risk premium}^{\text{view}}$$



Problem with this naïve treatment: small changes in risk premiums result in wild changes in the optimal portfolio weights.

The Black-Litterman Treatment of a View

In Black-Litterman, a view is expressed in terms of three elements

- The view portfolio:

	AUS	CAN	FRA	GER	JAP	UK	USA
$P =$	0	0	-30%	100%	0	-70%	0

The view is expressed here in terms of *view portfolio weights*: overweight GER and underweight other European countries. Notice that the France and UK view portfolio weights are proportional to their market portfolio weights.

- The view premium:

$$Q = 5\%$$

If *view portfolio* is to express direction, then view premium is to express magnitude. In this case, it is an overperformance of 5% in risk premium.

The Black-Litterman Treatment of a View

- The view confidence:

$$\Omega = 0.20^2$$

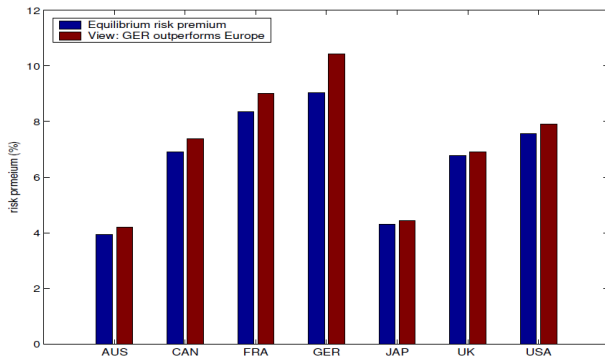
Black and Litterman also allow investor to express their confidence in their view.

- ▶ If they are 100% sure about their view, they would assign $\Omega = 0\%$.
- ▶ The larger the Ω , the less confident they are about their view.
- ▶ Assigning a very large number to Ω , you might as well not have a view.

The Black-Litterman Risk Premium

The Black-Litterman Risk Premium $\text{risk premium}^{\text{BL}}$

$$= (\Sigma^{-1} + P' \times \Omega^{-1} \times P)^{-1} \times \left(\Sigma^{-1} \times \text{risk premium}^{\text{EQ}} + P' \times \Omega^{-1} \times Q \right)$$



Calculation details:

- Market (neutral view) inputs:

$$\text{risk premium}^{\text{EQ}} = \begin{pmatrix} 3.9\% \\ 6.9\% \\ 8.4\% \\ 9.0\% \\ 4.3\% \\ 6.8\% \\ 7.6\% \end{pmatrix}; \quad \Sigma = \begin{pmatrix} 0.0256 & 0.0159 & 0.0190 & 0.0223 & 0.0148 & 0.0164 & 0.0147 \\ 0.0159 & 0.0412 & 0.0334 & 0.0360 & 0.0132 & 0.0247 & 0.0296 \\ 0.0190 & 0.0334 & 0.0615 & 0.0579 & 0.0185 & 0.0388 & 0.0310 \\ 0.0223 & 0.0360 & 0.0579 & 0.0734 & 0.0201 & 0.0421 & 0.0331 \\ 0.0148 & 0.0132 & 0.0185 & 0.0201 & 0.0441 & 0.0170 & 0.0120 \\ 0.0164 & 0.0247 & 0.0388 & 0.0421 & 0.0170 & 0.0400 & 0.0244 \\ 0.0147 & 0.0296 & 0.0310 & 0.0331 & 0.0120 & 0.0244 & 0.0350 \end{pmatrix}$$

- View inputs:

$$P = (0 \quad 0 \quad -0.3 \quad 1 \quad 0 \quad -0.7 \quad 0); \quad Q = 5\%; \quad \Omega = 0.20^2$$

- The Black-Litterman risk premium:

$$\text{risk premium}^{\text{BL}} = \left(\Sigma^{-1} + P' \times \Omega^{-1} \times P \right)^{-1} \times \left(\Sigma^{-1} \times \text{risk premium}^{\text{EQ}} + P' \times \Omega^{-1} \times Q \right) = \begin{pmatrix} 4.2\% \\ 7.4\% \\ 9.0\% \\ 10.4\% \\ 4.4\% \\ 6.9\% \\ 7.9\% \end{pmatrix}$$

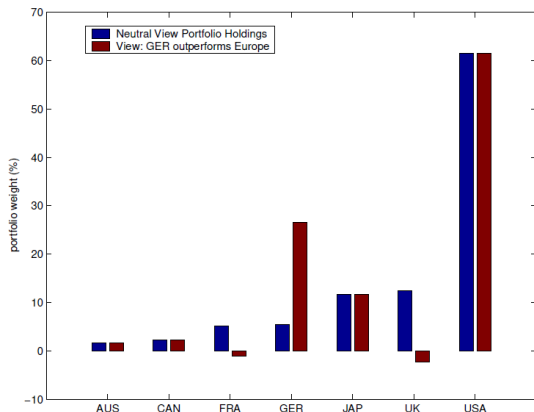
Additional Tips

- *Excel* command for matrix operation:
 - ▶ the command for summation is still “+”
 - ▶ the command for multiplication is “mmult”
 - ▶ the command for inverse, say Σ^{-1} , is “minverse”
 - ▶ P' is the transpose of P , Excel command: “transpose”
- *Matlab* command for matrix operation:
 - ▶ the command for summation is still “+”
 - ▶ the command for multiplication is still “*”
 - ▶ the command for inverse, say Σ^{-1} , is “inv(Σ)”
 - ▶ the command for transpose, say P' is P'

The Black-Litterman portfolio weights:

Plug in the Black-Litterman risk premium, we have

$$w^{BL} = \frac{1}{\text{risk aversion}} \times \Sigma^{-1} \times \text{risk premium}^{BL}$$



Black-Litterman with Multiple Views

Suppose that in addition to the Germany-outperforming-Europe view, you also believe that Canada will outperform the U.S. by 3%.

- The portfolio view P :

AUS	CAN	FRA	GER	JAP	UK	USA
0	0	-30%	100%	0	-70%	0
0	1	0	0	0	0	-1

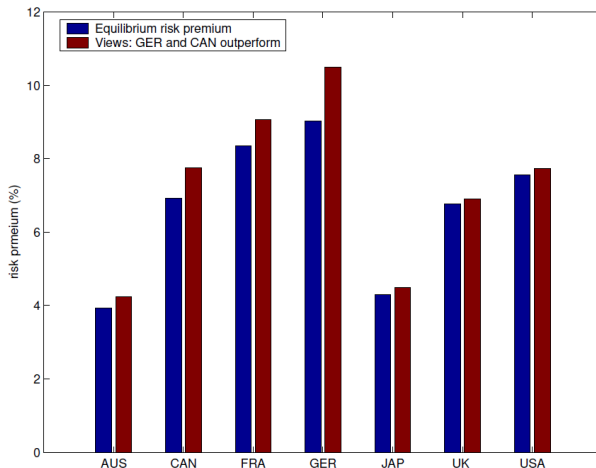
- The view premium:

$$Q = \begin{pmatrix} 5\% \\ 3\% \end{pmatrix}$$

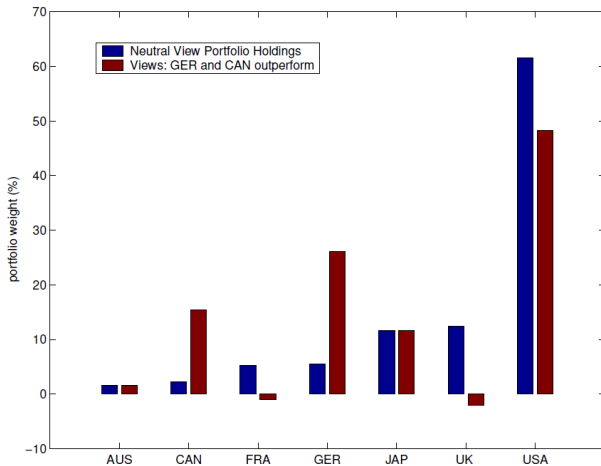
- The view confidence:

$$\Omega = \begin{pmatrix} 0.20^2 & 0 \\ 0 & 0.30^2 \end{pmatrix}$$

The Black-Litterman Risk Premium with 2 Views



The Black-Litterman Optimal Portfolio Weights with 2 Views



The Black-Litterman Method with Portfolio Constraints

- Arriving at the optimal portfolio is somewhat more complex in the presence of constraints.
- In general, when there are constraints, the easiest way to find the optimal portfolio is to use the Black-Litterman model to generate the expected returns for the assets, and then use a mean-variance optimizer to solve the constrained optimization problem.
- In these situations, the intuition of the Black-Litterman model is more difficult to see.
- Again, the portfolios about which the investor has views play a critical role in the optimal portfolio construction, even in the constrained case.

The Practical Application of the Black-Litterman Model

Excerpts from “The Intuition Behind Black-Litterman Model Portfolios,” Goldman-Sachs working paper, He and Litterman (1999):

- In the Quantitative Strategies group at Goldman Sachs Asset Management, we develop quantitative models and use these models to manage portfolios.
- The Black-Litterman model is the central framework for our modeling process. Our process starts with finding a set of views that are profitable.
- For example, it is well known that portfolios based on certain value factors and portfolios based on momentum factors are consistently profitable.
- We forecast the expected returns on portfolios which incorporate these factors and construct a set of views.

The Practical Application of the Black-Litterman Model

- The Black-Litterman model takes these views and constructs a set of expected returns on each asset.
- Although we manage many portfolios for many clients, using different benchmarks, different targeted risk levels, and different constraints on the portfolios, the same set of expected returns from the Black-Litterman model is used throughout.
- Even though the final portfolios may look different due to the differences in benchmarks, targeted risk levels and constraints, all portfolios are constructed to be consistent with the same set of views, and all will have exposures to the same set of historically profitable return-generating factors.