Modeling the Yield Curve

Financial Markets, Day 3, Class 3

Jun Pan

Shanghai Advanced Institute of Finance (SAIF)
Shanghai Jiao Tong University

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Outline

- Term structure models.
- How to calibrate the model to the data?
- Use term structure models to identify trading opportunities.

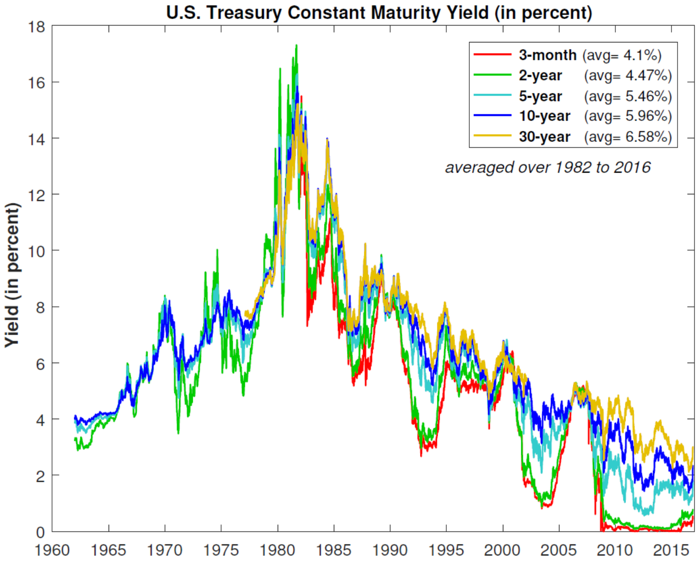
Trading the Yield Curve

- Most fixed-income trading strategies involve buying and/or selling various parts of the yield curve.
- Trading in the fixed-income markets as compared with that in the equity market:
  - The risk and return tradeoffs.
  - The role of a pricing model.
  - The major risk factors.

large Term Structure Modeling

- Term-structure modeling is one of the major success stories in the application of financial models to everyday business problems.
- It ranges from managing the risk of a bond portfolio to the design, pricing and hedging of interest-rate derivatives and collateralized mortgage obligations.
- Each major investment bank has its own proprietary term-structure model, and it is claimed that the industry has the most sophisticated term-structure models.
- The main object of concern is the random fluctuation or the dynamic evolution of interest rates – not just one rate, but the entire term structure of interest rate.

Treasury Yield Curve



Some Well-Known Term-Structure Models

- One-factor short-rate models: the entire term structure at any given time depends on only one factor — the short rate. This includes the Merton (Ho-Lee) model, the Black-Karasinski (Black-Derman-Toy) model, the Vasicek model, and the Cox-Ingersoll-Ross model.
- Multi-factor short-rate models: use multiple stochastic factors.
  - Multi-factor versions of the Vasicek and CIR models
  - The affine models (Duffie and Kan)
- The HJM (Heath-Jarrow-Morton) model: use stochastic factors to model the instantaneous forward rates.
- In general, a good term-structure model is flexible enough to capture the rich dynamics, but also tractable enough to allow for fast pricing and hedging calculations.

The Vasicek Model

- The Vasicek model is a continuous-time term-structure model:

$$dr_t = \kappa\,(\bar{r} - r_t)\;dt + \sigma\,dB_t$$

- For this class, let's use the discrete-time version of the model. Let r_t be the three-month T-bill rate at time t, and $r_{t+\Delta}$ be the three-month T-bill rate at the next Δ instant:

$$r_{t+\Delta} - r_t = (\bar{r} - r_t)\,\kappa\,\Delta + \sigma\,\sqrt{\Delta}\,\epsilon_{t+\Delta}$$

- At any time t, the short rate is subject to a new shock $\epsilon_{t+\Delta}$, which is standard normally distributed. Shocks are independent across time.

The Parameters for the Model

- $\bar{r}$ controls the normal level of the interest rates, or the long-run mean of the interest rates:

$$E(r_t) = \bar{r}$$

- σ controls the conditional variance:

$$\text{var}\left(r_{t+\Delta} \middle| r_t\right) = \sigma^2 \, \frac{1 - \exp(-2\kappa\Delta)}{2\kappa} \approx \sigma^2 \Delta$$

- κ controls the rate at which the interest rate reverts to its long-run mean $\bar{r}$.
  - When κ is big, any deviation from the long-run mean will be pulled back to its normal level $\bar{r}$ pretty quickly.
  - When κ is small, it takes a long time for the interest rate to come back to its normal level. Interest rates are very persistent in this situation.

Bond Pricing:

Suppose that today's three-month T-bill rate is r and suppose that we know κ, σ, and $\bar{r}$. According to the Vasicek model, the price of T-year zero-coupon bond with face value of \$1 is determined by

$$P = e^{A+B\,r},$$

where

$$B = \frac{e^{-\kappa T} - 1}{\kappa}$$

$$A = \bar{r}\left(\frac{1 - e^{-\kappa T}}{\kappa} - T\right) + \frac{\sigma^2}{2\kappa^2}\left(\frac{1 - e^{-2\kappa T}}{2\kappa} - 2\frac{1 - e^{-\kappa T}}{\kappa} + T\right)$$

Calibrating the Model, using Time-Series Data

- To be useful, the model parameters need to be calibrated to the data.
- Suppose you are given a time-series of three-month T-bill rates, observed with monthly frequency.
- The Vasicek model is equivalent to

$$r_{t+1} = a + b\, r_t + c\, \epsilon_{t+1}\,,$$

  where $a = \kappa \bar{r} \Delta$, $b = 1 - \kappa \Delta$, and $c = \sigma \sqrt{\Delta}$.
- If the interest rates are observed in monthly frequency, then $\Delta = 1/12$.
- If we know that $\kappa = 0.1$, $\sigma = 0.01$, and $\bar{r} = 5\%$, then $a = \kappa \bar{r}/12 = 0.005$, $b = 1 - \kappa/12 = 0.9917$, and $c = \sigma\sqrt{1/12} = 0.007071$.
- Conversely, if you know how to estimate a, b, c, you can back out κ, σ, and $\bar{r}$.

Calibrating the model, using the Yield Curve

- Instead of using the historical time-series data to estimate the model, the industry practice is to calibrate the model to the yield curve.
- Given r, κ, $\bar{r}$, and σ, the model can price bonds of any maturities.
- On any given day, we observe prices and yields of various maturities. We can take advantage of these market-traded prices by forcing the model to price such bonds as precisely as possible.
- In other words, κ, $\bar{r}$, and σ are calibrated to today's yield curve. Tomorrow, we repeat the same exercise and end up with a different set of model parameters.

What Have We Learned?

- The economic drivers of the term structure of interest rates.
- The statistical analysis of the term structure of interest rates.
- The analytical modeling of the term structure of interest rates:
  - The model itself.
  - The bond pricing formula.
  - How to calibrate the model to the data?

Use Term Structure Models to Identify Trading Opportunity

- First, pick a term structure model: a two-factor model at the minimum. Most often, a three-factor model is used to capture level, slope, and volatility (or the short-end, long-end and volatility).
- Second, calibrate the model to the data.
  - From an academic point of few, the best way is to use the time-series of historical data (Treasury or Swap) to calibrate the model.
  - If it is a three-factor model, then the 6M, 5Y, 10Y Treasury/Swap yields can be used to back out the three factors, day by day.
  - One can even think about using Swaptions to help identify the volatility factor. (Important if the model is to be used for derivatives pricing.)
  - Econometrics tools as such Maximum-Likelihood Estimation (MLE) can be used to estimate the model parameters from the time-series data.

Use Term Structure Models to Identify Trading Opportunity

- Now you have everything for the model: parameters and the factors.
- Use the model to price the entire yield curve. The deviation of the actual market-traded yield curve from the model-produced yield curve gives rise to potential trading opportunities.
- The model can also price fixed-income derivatives for you. Again, any deviation from the model gives rise to trading opportunities.
- As usual, following your model with blind faith is not a good idea because models are always limited compared with the richness of the reality.
- As always, know the economic forces and the institutional reasons behind your trading opportunity.

Relative Value Investing

Excerpts from Chifu Huang's Guest Lecture at MIT Sloan in March 2011

- Relative value investing is such an example.
- It takes the view that deviations from any reasonable/good model is created by transitory supply/demand imbalances originated from
  - Clientele effects and institutional rigidity;
  - Derivatives hedging;
  - Accounting/tax rules;
- These imbalances dissipate over time as
  - Economics of substitution takes hold.
  - Imbalances reverse themselves as market conditions change.

Relative Value Investing

Excerpts from Chifu Huang's Guest Lecture at MIT Sloan in March 2011

- Do not make judgment on level of interest rates or slope of the curve.
- Assume that a few points on the yield curve are always fair. For example:
  - 10-year rate: capturing the level of long-term interest rates.
  - 2-year rate: together with 10-yr rate, capturing the slope of the curve.
  - 1-month rate: capturing short term interest rate/expectation on monetary policy in the near term.
- Predicting level of interest rates of other maturities or their cheapness/richness "relative to" the presumed fair maturities.
- Buying/selling cheap/rich maturities hedged with fair maturities to make the portfolio insensitive to changes of the level and the slope of the yield curve and to changes of monetary policy.

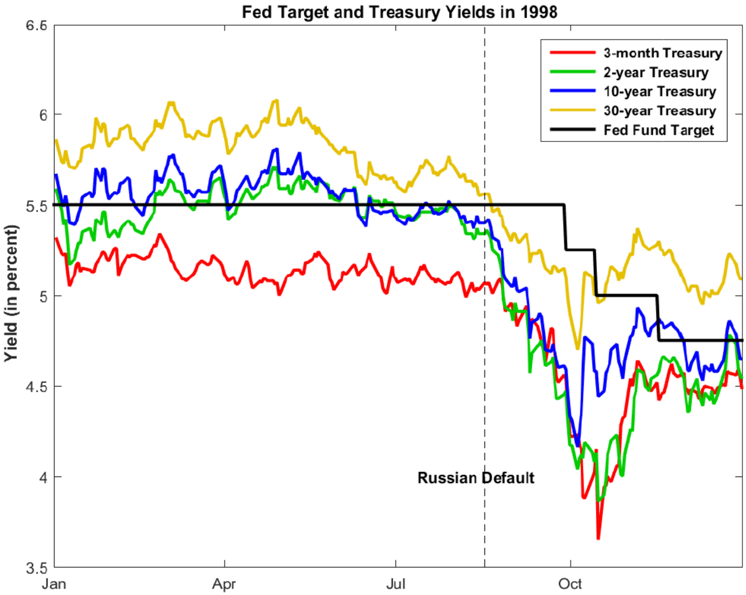
Market Price vs. Model Price

Crisis Behavior of 30-yr Cheapness/Richness

Aug-Sep 1998 Russia default:

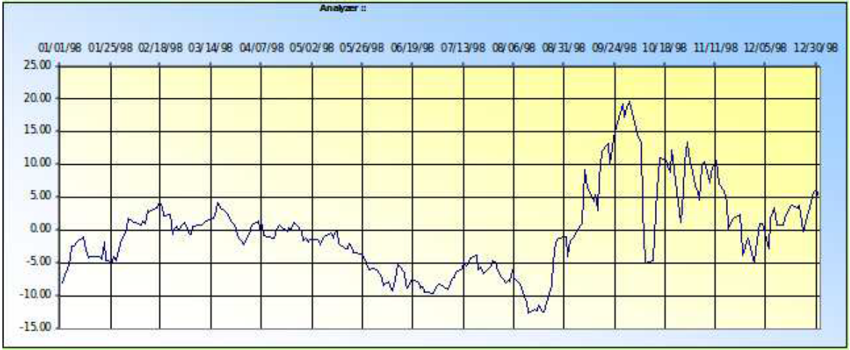
- Bond markets rallied anticipating Fed to cut rate.
- 2-10 steepened: rate cut would have more impact on 2-yr rate.
- Macro traders active in 2-yr and mortgages hedges active in 10-yr – clientele effect.
- Pension funds are natural players in 30-yr but they are not "traders" but are "portfolio rebalancers" who rebalance their portfolio periodically – clientele effect/institutional rigidity.
- Life Insurance companies active in the 30-yr as well. They typically are rate-targeted buyers and as market rallied, they back away from buying – clientele effect/institutional rigidity.

The LTCM Crisis



Model Performance in LTCM

All the above led to “cheapening” of the 30-yr sector – market rates did not rally as much as the model said they should.



Then there is Lehman in 2008: