Yield and Duration

Financial Markets, Day 3, Class 1

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Outline for Day 3

- Class 1: Yield and duration.
- Class 2: Factors influencing the yield curve.
- Class 3: Modeling the yield curve.
- Class 4: Interest rate swaps.
- Class 5: Corporate bonds and credit risk.
- Class 6: Review and quiz.

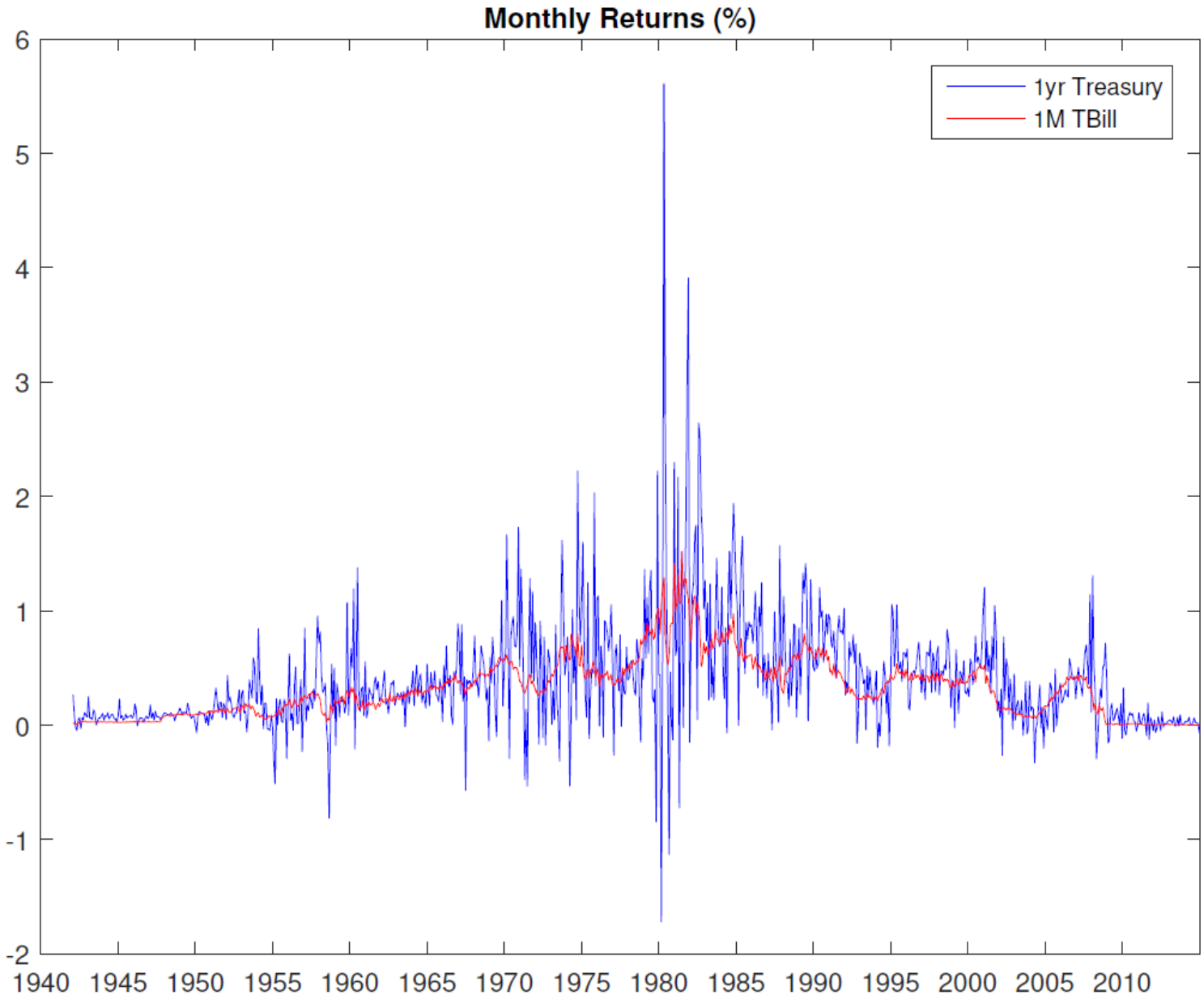
Outline for Class 1

- From equity to fixed income.
- Bond price and yield: duration and convexity.
- The universe of fixed income securities.

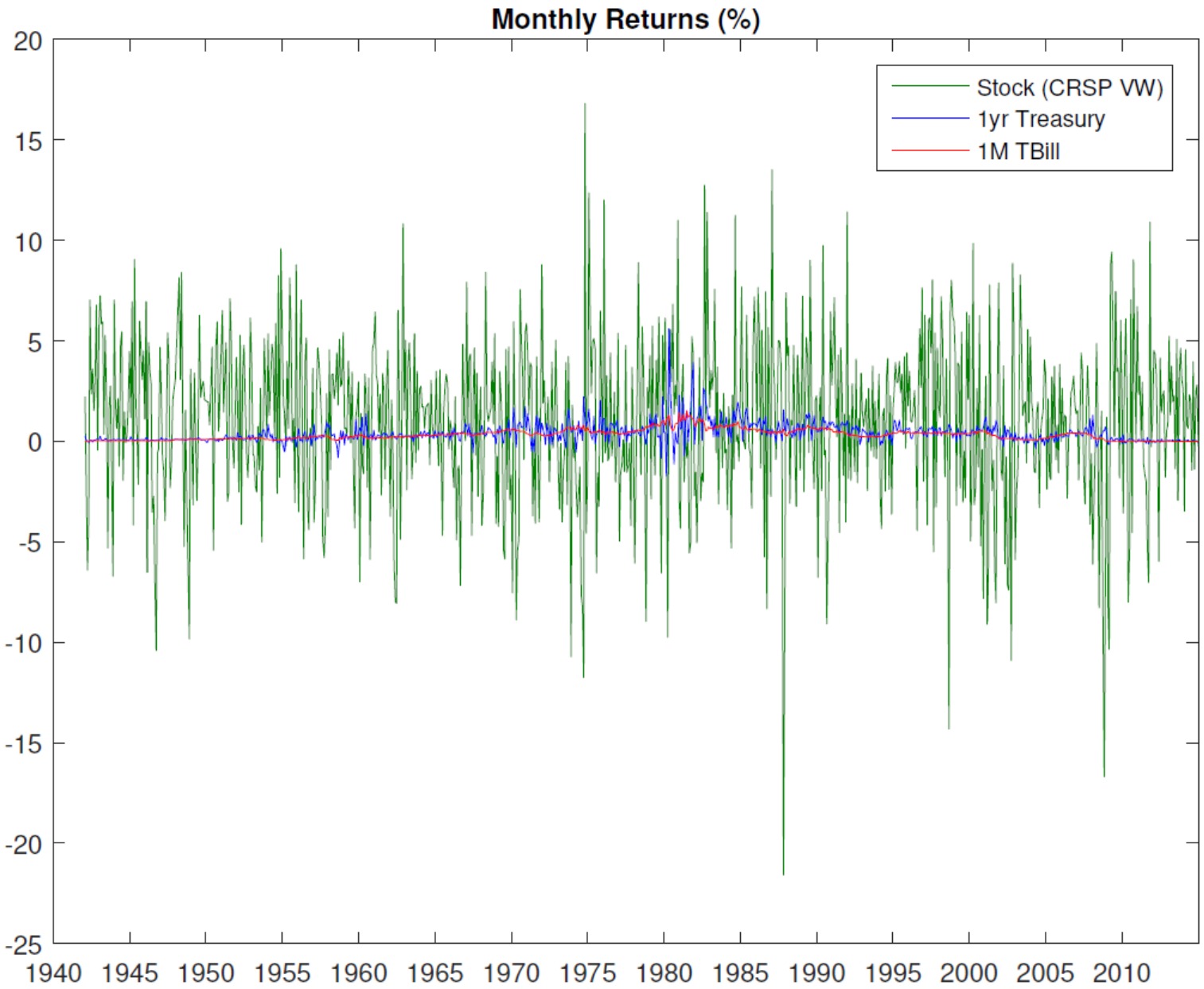
From Equity to Fixed Income

- So how are things different? (Michael Lewis: Equities in Dallas.)
- A bond matures. At maturity, the bond pays back the principal.
- Before maturity, it has scheduled coupon payments.
- Its key risk factor: interest rate exposure, which is measured by *duration*.
- Very often, we will refer to buying bonds as buying duration.
- This becomes quite useful when moving from bonds to interest-rate swaps: different in structure but same as vehicles for *duration*.
- So **beta** in equity, and **duration** in fixed income.
- The cheapness and richness of a bond is often measured in the space of yields to maturity.
- So Black-Scholes **implied vol** in options and **yield** in fixed income.

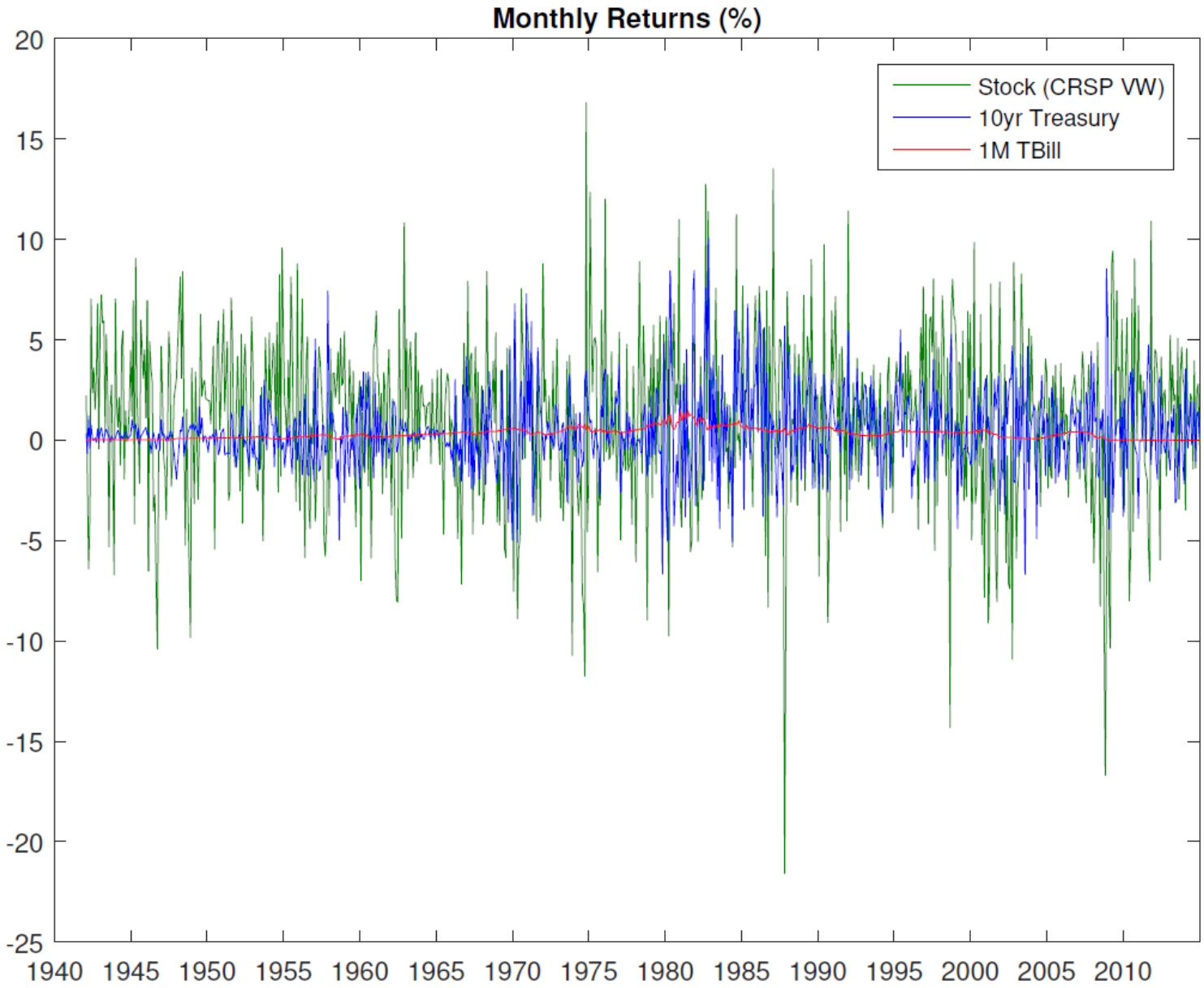
Monthly Returns



Monthly Returns



Monthly Returns

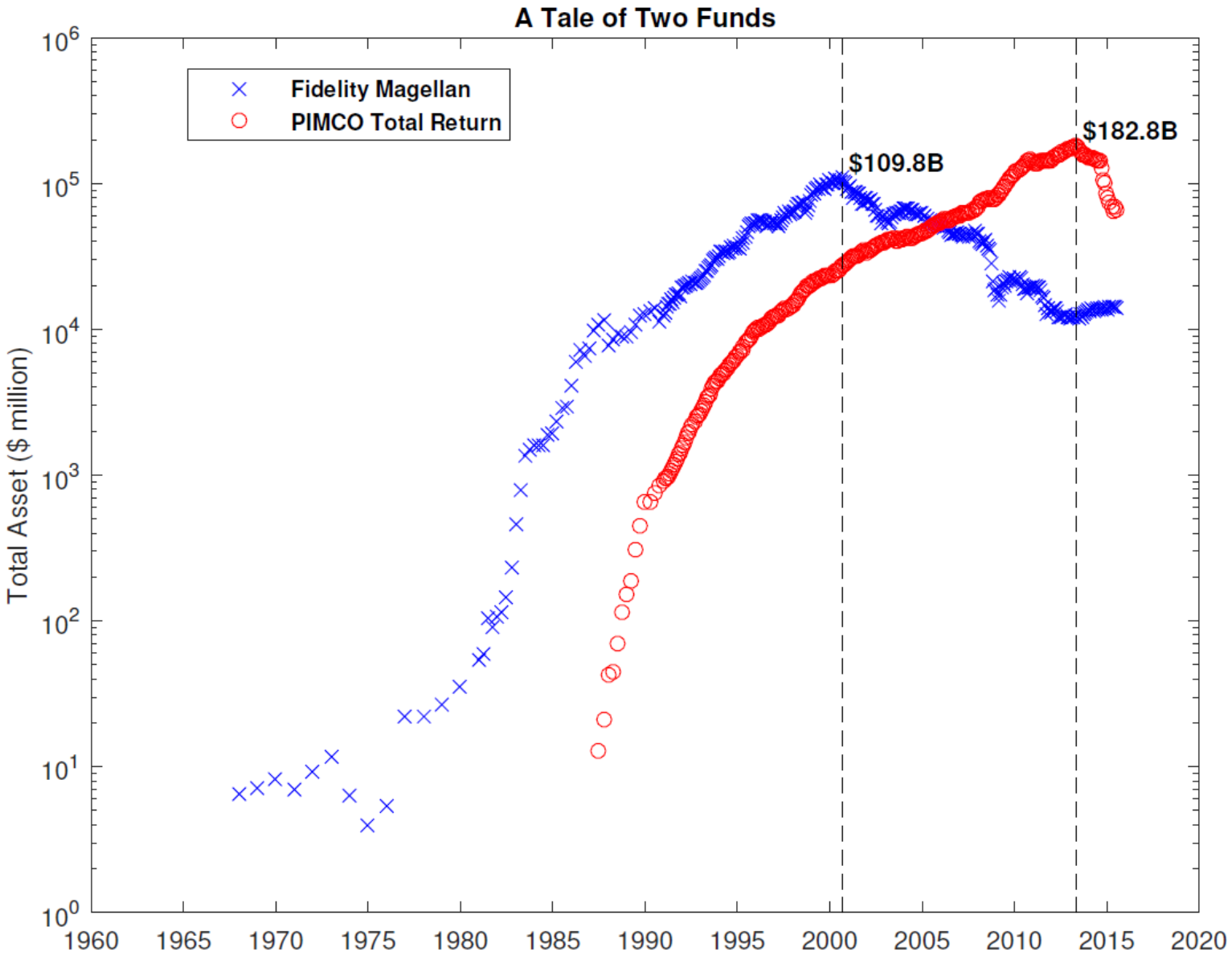


Stock and Bond Returns

Returns of Stock and Bond and Inflation

Monthly Returns 1942-2014	mean (%)	std (%)	Sharpe ratio	min (%)	max (%)	correlation with Stock	TBill	10Y
Stock (CRSP VW)	1.03	4.16	0.17	-21.58	16.81	1.00	-0.05	0.10
10Y Bond	0.47	2.00	0.08	-6.68	10.00	0.10	0.12	1.00
5Y Bond	0.46	1.38	0.10	-5.80	10.61	0.07	0.19	0.90
2Y Bond	0.42	0.77	0.13	-3.69	8.42	0.08	0.37	0.76
1Y Bond	0.40	0.50	0.16	-1.72	5.61	0.08	0.59	0.62
1M TBill	0.32	0.26		-0.00	1.52	-0.05	1.00	0.12
CPI	0.31	0.45		-1.92	5.88	-0.07	0.26	-0.07
Monthly Returns 1990-2014	mean (%)	std (%)	Sharpe ratio	min (%)	max (%)	correlation with Stock	TBill	10Y
Stock (CRSP VW)	0.87	4.22	0.15	-16.70	11.41	1.00	0.01	-0.06
10Y Bond	0.57	1.99	0.16	-6.68	8.54	-0.06	0.07	1.00
5Y Bond	0.50	1.24	0.20	-3.38	4.52	-0.10	0.15	0.93
2Y Bond	0.39	0.54	0.26	-1.30	2.07	-0.11	0.41	0.74
1Y Bond	0.33	0.31	0.26	-0.33	1.31	-0.03	0.72	0.51
1M TBill	0.25	0.19		-0.00	0.68	0.01	1.00	0.07
CPI	0.21	0.34		-1.92	1.22	-0.04	0.18	-0.16

Bond and Equity Funds



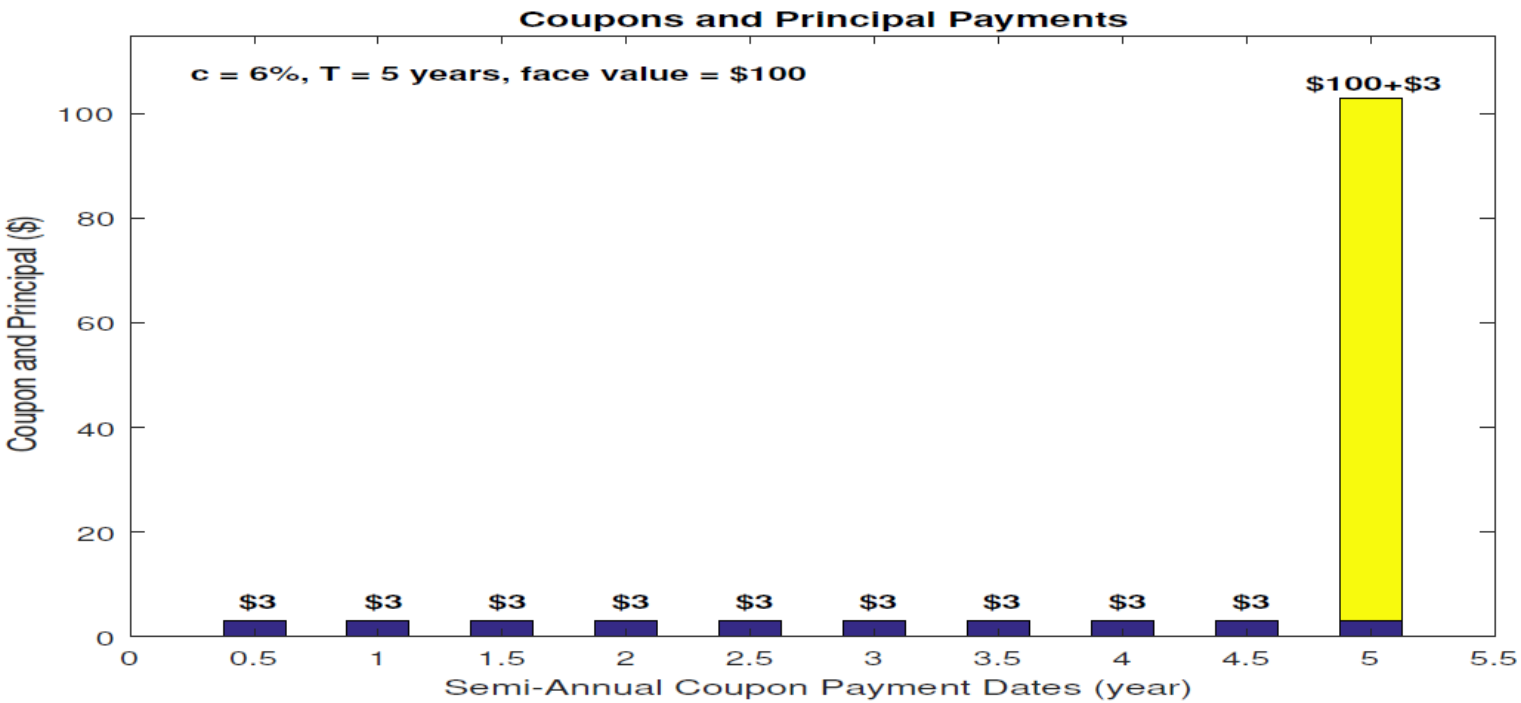
Yield to Maturity y and Bond Price P

- At issuance, a Treasury bond has the following terms fixed: **face value** $= \$100$; **coupon rate** $= c$; **maturity** $= \mathrm{T}$ years.
- Treasury bonds pay coupon semi-annually, and, at issuance, the coupon rate c is chosen so that the bond is priced at par: $P = \$100$ and $c = y$.
- Later, with interest rate fluctuations, both P and y change and there is a *deterministic*, inverse relationship between the two:

$$P = \sum_{n=1}^{2T} \frac{\frac{c}{2} \times 100}{\left(1 + \frac{y}{2}\right)^n} + \frac{100}{\left(1 + \frac{y}{2}\right)^{2T}}.$$

- Increasing interest rate is bad news for bonds and decreasing interest rate is good news for bonds.
- Decreasing interest rate after issuance turns the bond into premium $P > \$100$, and increasing interest rate turns it into discount $P < \$100$.

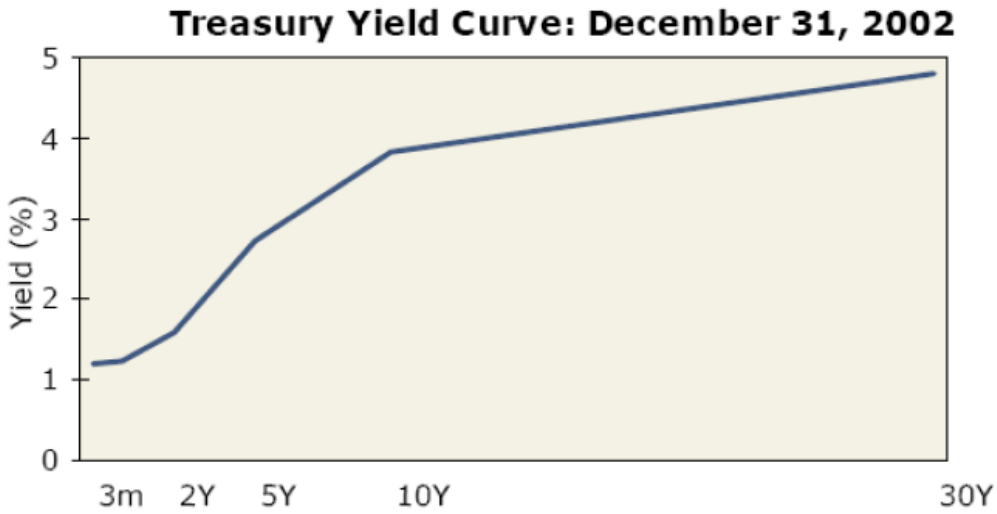
Fixed-Rate Coupon Bonds



$$P = \sum_{n=1}^{2T} \frac{\frac{c}{2} \times 100}{\left(1 + \frac{y}{2}\right)^n} + \frac{100}{\left(1 + \frac{y}{2}\right)^{2T}}.$$

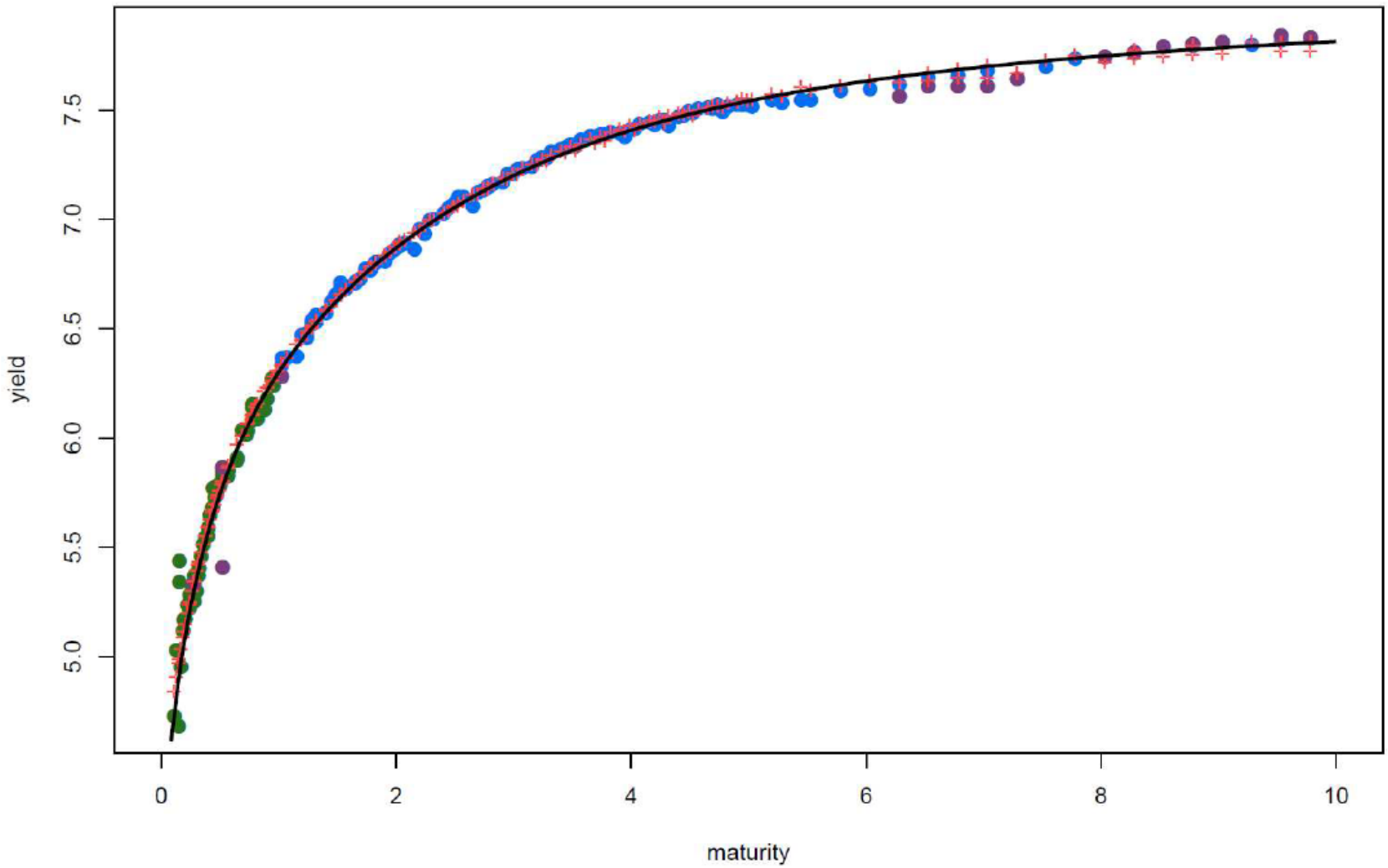
Treasury Yield Curve

- A typical yield curve (also called the term structure of interest rate):

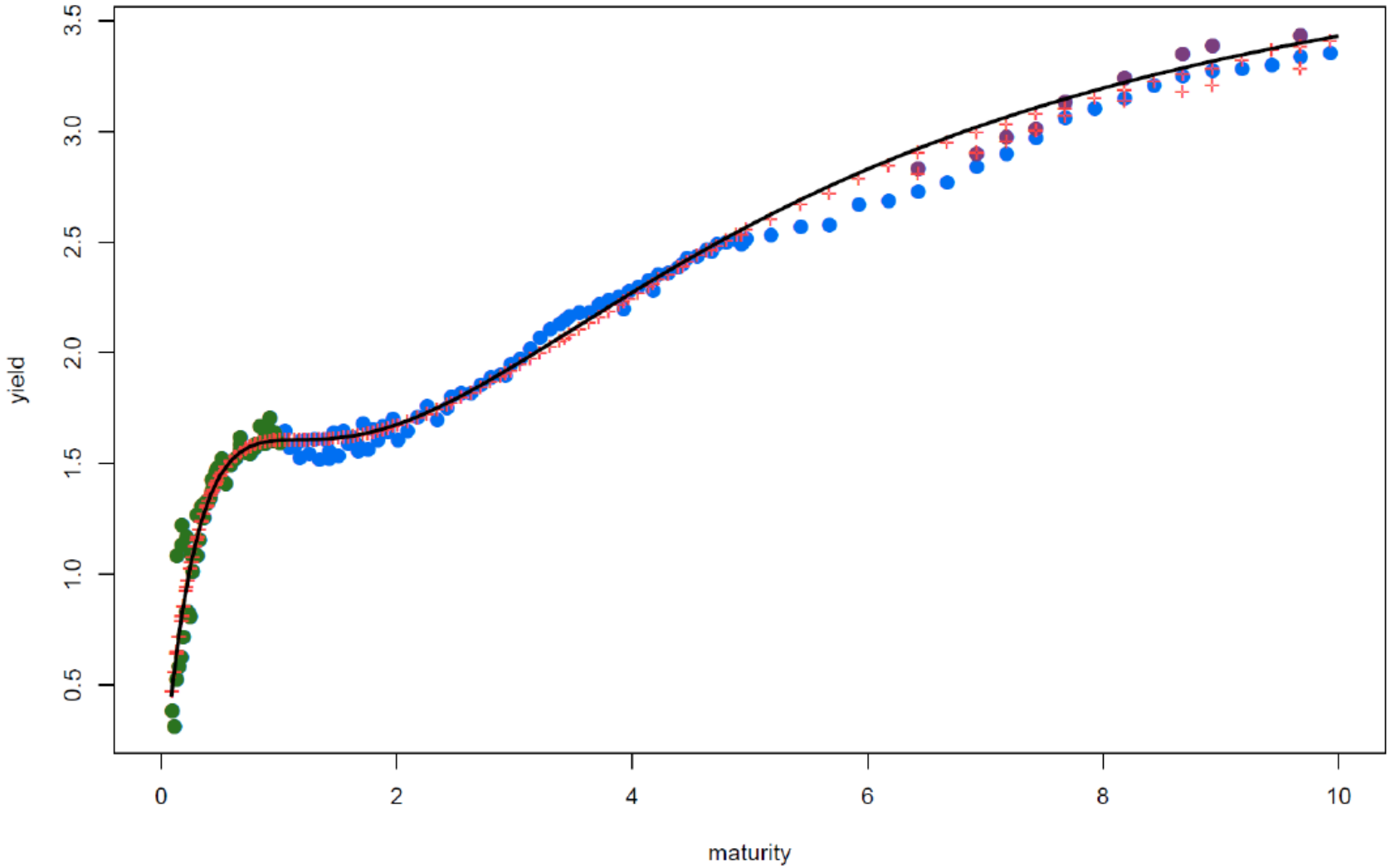


- A yield curve can be created for any specific segment, from triple-A rated mortgage-backed securities to single-B rated corporate bonds.
- The Treasury bond yield curve is the most widely used. The normal shape of the yield curve is upward, but, occasionally, it slopes downward, or inverts.

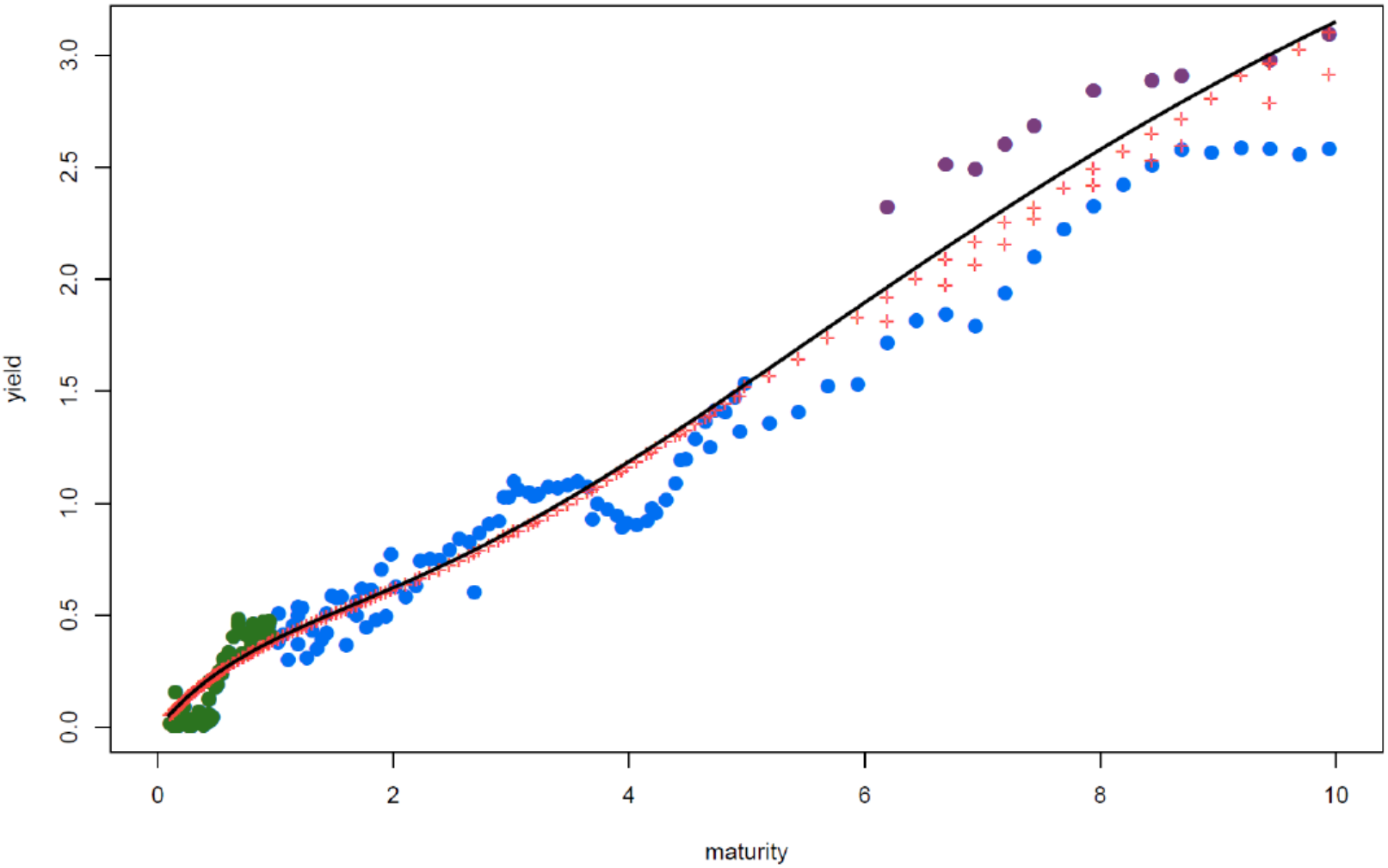
Treasury Yield Curve on November 8, 1994 (Noise=2.60)



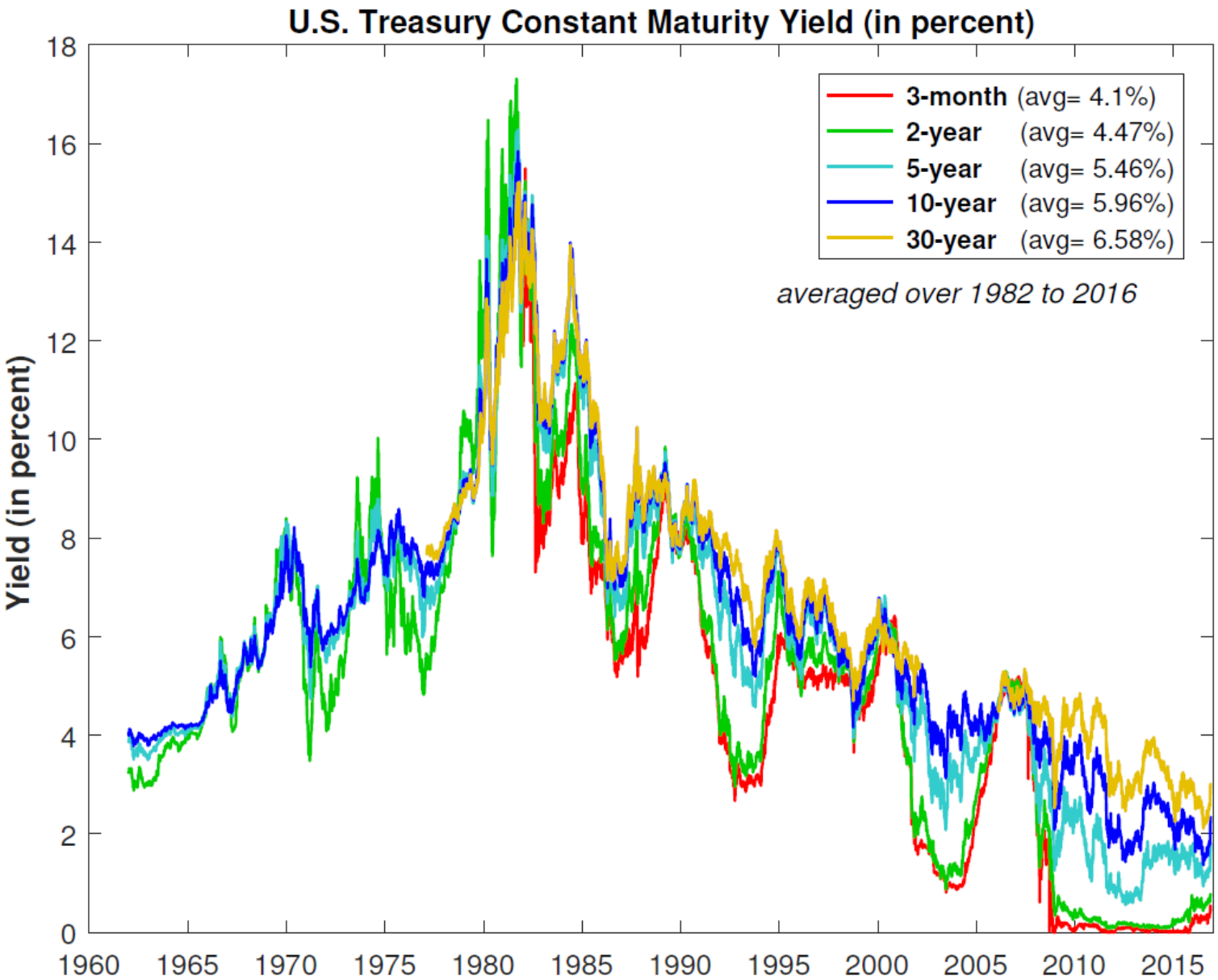
Treasury Yield Curve on September 15, 2008 (Noise=6.64)



Treasury Yield Curve on December 11, 2008 (Noise=20.4)



Treasury Constant Maturity Yields



Daily Changes in Treasury Yields

Daily Changes in Treasury Yields

sample	maturity	std (bp)	min (bp)		max (bp)	
1982-2015	3M	7.63	-104	19820222	169	19820201
	2Y	6.86	-84	19871020	80	19820201
	10Y	6.80	-75	19871020	44	19820201
	30Y	6.30	-76	19871020	42	19820201
1990-2008	3M	5.18	-64	20070820	58	20001226
	2Y	6.05	-54	20010913	36	19940404
	10Y	5.78	-23	19950613	39	19940404
	30Y	4.99	-33	20011031	32	19940404
2008-2015	3M	4.94	-81	20080917	76	20080919
	2Y	4.86	-45	20080915	38	20080919
	10Y	6.42	-51	20090318	24	20080930
	30Y	6.12	-32	20081120	28	20110811

Dollar Duration (DV01) and Modified Duration

- Dollar Duration:

$$-\frac{\partial P}{\partial y} = \frac{1}{1+\frac{y}{2}}\left[\sum_{n=1}^{2T} \frac{n}{2} \times \frac{\frac{c}{2}\times 100}{\left(1+\frac{y}{2}\right)^n} + T \times \frac{100}{\left(1+\frac{y}{2}\right)^{2T}}\right],$$

  which is the negative of \$ change in bond price per unit change in yield.
- DV01 = Dollar Duration/10000 (\$ per 1 basis point change in yield):
- Modified Duration:

$$-\frac{1}{P}\frac{\partial P}{\partial y} = \frac{1}{1+\frac{y}{2}} \frac{\sum_{n=1}^{2T} \frac{n}{2} \times \frac{\frac{c}{2}\times 100}{\left(1+\frac{y}{2}\right)^n} + T \times \frac{100}{\left(1+\frac{y}{2}\right)^{2T}}}{\sum_{n=1}^{2T} \frac{\frac{c}{2}\times 100}{\left(1+\frac{y}{2}\right)^n} + \frac{100}{\left(1+\frac{y}{2}\right)^{2T}}},$$

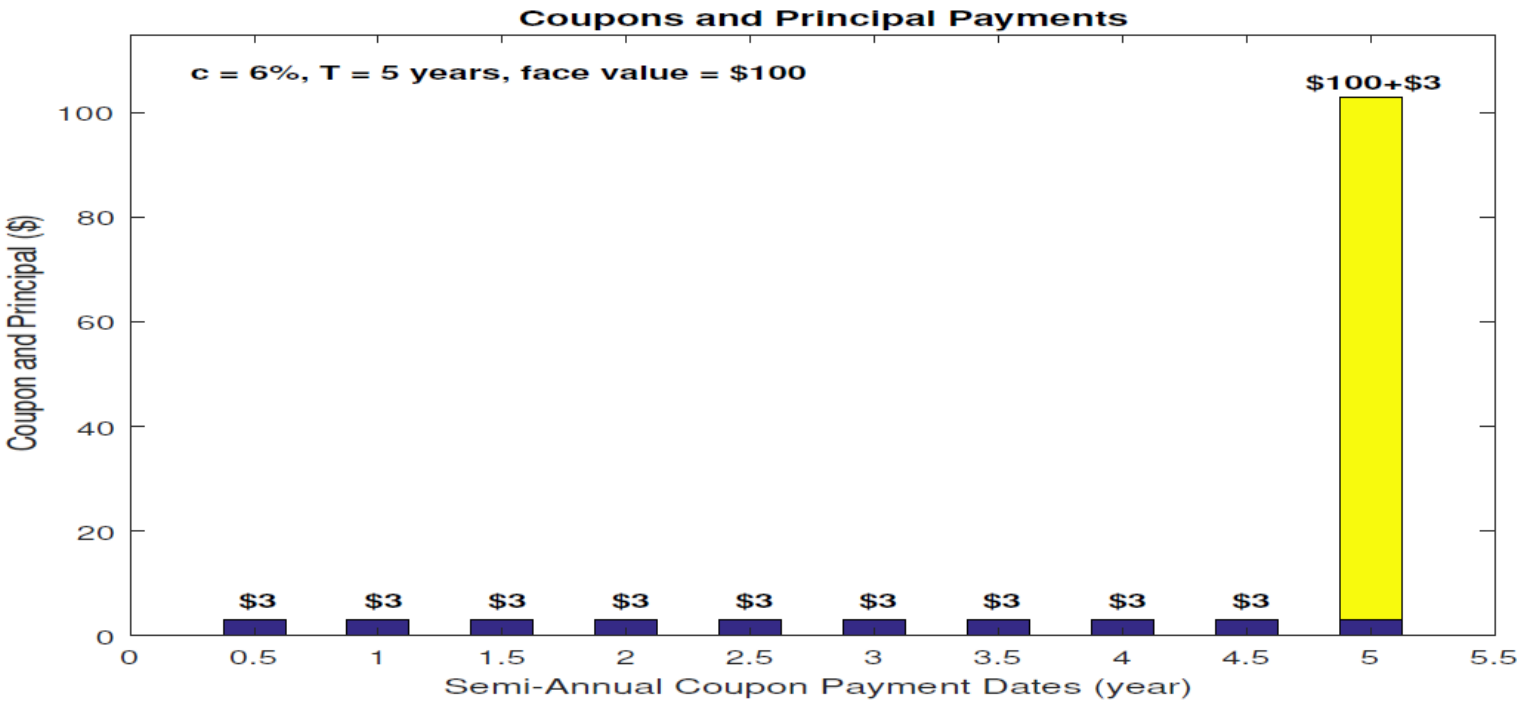
  which is effectively a weighted sum of semi-annual coupon payment dates: $6m$, $1y$, $1.5y$, ..., and T years. It captures the percentage change in bond price (i.e., bond return) per unit change in yield.

Modified Duration

Modified Duration

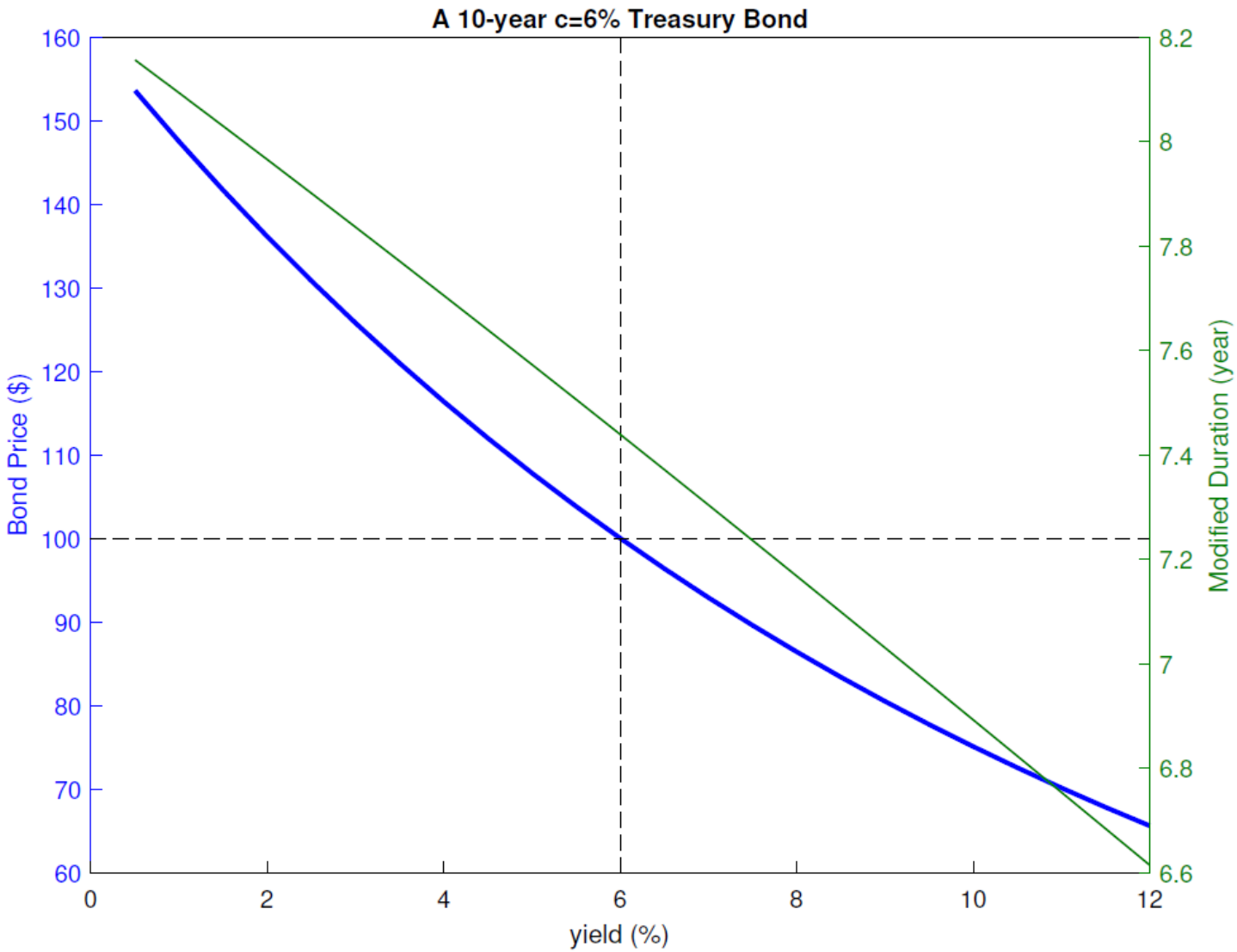
yield y	2%	5%	6%	6%	6%	7%	10%
coupon c	2%	5%	4.8%	6%	7.2%	7%	10%
$T = 1$	0.99	0.96	0.96	0.96	0.95	0.95	0.93
$T = 2$	1.95	1.88	1.87	1.86	1.84	1.84	1.77
$T = 3$	2.90	2.75	2.74	2.71	2.68	2.66	2.54
$T = 5$	4.74	4.38	4.36	4.27	4.18	4.16	3.86
$T = 7$	6.50	5.85	5.81	5.65	5.51	5.46	4.95
$T = 10$	9.02	7.79	7.71	7.44	7.21	7.11	6.23
$T = 20$	16.42	12.55	12.12	11.56	11.13	10.68	8.58
$T = 30$	22.48	15.45	14.46	13.84	13.39	12.47	9.46

Calculating Modified Duration



$$D^{\text{mod}} = \frac{1}{1+\frac{y}{2}} \frac{\sum_{n=1}^{2T} \frac{n}{2} \times \frac{\frac{c}{2}\times 100}{\left(1+\frac{y}{2}\right)^n} + T \times \frac{100}{\left(1+\frac{y}{2}\right)^{2T}}}{\sum_{n=1}^{2T} \frac{\frac{c}{2}\times 100}{\left(1+\frac{y}{2}\right)^n} + \frac{100}{\left(1+\frac{y}{2}\right)^{2T}}}$$

Bond Price, Yield, and Duration



Duration and Convexity

- Duration and convexity are meaningful only because we work in the yield space (for convenience), and the profit/loss is in the dollar space.
- Duration is a bridge that connects the two:
  - Dollar Duration:

$$\Delta P_t = P_t - P_{t-1} \approx -\mathrm{D}^{\$} \times (y_t - y_{t-1}) = -\mathrm{D}^{\$} \times \Delta y_t$$

  - Modified Duration:

$$R_t = \frac{\Delta P_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} \approx -\mathrm{D}^{\mathrm{mod}} \times (y_t - y_{t-1}) = -\mathrm{D}^{\mathrm{mod}} \times \Delta y_t$$

- The relation between price and yield is not linear, but convex:
  - With decreasing y, duration increases: profits amplified.
  - With increasing y, duration decreases: losses dampened.
- Bonus from positive convexity, not offered by a security linear in y.

The Universe of Fixed Income Securities

- US treasuries: bills, notes, bonds.
- Treasury inflation protected securities (TIPS).
- Muni's
- Agencies, government sponsored enterprises (GSE)
- Mortgage-backed
- Corporate bonds
- Emerging market bonds
- LIBOR and swaps
- Fixed income derivatives
- Credit derivatives

Key Risk Factors in Fixed Income

- Yield curve uncertainties:
  - **level** of interest rates.
  - the **slope** of the yield curve (long-term yield minus short-term yield).
  - interest rate **volatility** (e.g., swaption implied vol).
- **Credit** risk (e.g., yield spread between US investment grade and US Treasury bond of similar maturity).
- Counterparty risk and other interesting spreads: LIBOR-OIS, swap spread, old bond and new bond spread, CDS-bond basis.

Outstanding US Bond Market Debt in $ Billions

	Municipal	Treasury	Mortgage Related	Corporate Debt	Federal Agency Securities	Money Markets	Asset-Backed	Total
1991	1,272.1	2,471.6	1,577.1	1,463.6	421.5	565.9	92.6	7,864.4
1992	1,295.4	2,754.1	1,774.3	1,563.3	462.4	579.0	117.3	8,545.9
1993	1,361.7	2,989.5	2,209.0	1,790.2	550.8	580.0	133.3	9,614.6
1994	1,325.8	3,126.0	2,352.9	1,941.7	727.7	623.5	162.8	10,260.3
1995	1,268.2	3,307.2	2,432.1	2,096.9	924.0	700.4	215.7	10,944.4
1996	1,261.6	3,444.7	2,606.4	2,268.0	925.8	803.0	298.2	11,607.7
1997	1,318.5	3,441.7	2,871.8	2,478.1	1,021.8	979.4	394.9	12,506.2
1998	1,402.7	3,340.5	3,243.4	2,809.8	1,302.1	1,172.6	478.0	13,749.1
1999	1,457.1	3,266.0	3,832.2	3,163.5	1,620.0	1,402.4	583.5	15,324.7
2000	1,480.7	2,951.9	4,119.3	3,461.3	1,853.7	1,614.0	701.9	16,182.8
2001	1,603.4	2,968.0	4,711.0	3,877.1	2,157.4	1,474.0	812.0	17,602.9
2002	1,762.8	3,205.3	5,289.4	4,075.5	2,377.7	1,374.9	904.8	18,990.4
2003	1,900.4	3,575.2	5,714.5	4,370.1	2,626.2	1,292.9	995.3	20,474.6
2004	2,877.7	3,945.8	6,301.7	4,595.6	2,700.6	1,399.1	1,100.2	22,920.7
2005	3,101.4	4,170.0	7,218.1	4,664.6	2,616.0	1,644.2	1,281.4	24,695.7
2006	3,287.5	4,328.0	8,389.9	4,904.7	2,634.0	1,958.4	1,656.9	27,159.4
2007	3,551.7	4,522.6	9,386.0	5,337.7	2,906.2	1,788.9	1,963.5	29,456.6
2008	3,667.6	5,783.6	9,467.4	5,514.0	3,210.6	1,599.8	1,829.5	31,072.5
2009	3,850.8	7,260.6	9,352.5	6,100.1	2,727.5	1,138.0	1,712.1	32,141.5
2010	3,961.4	8,853.0	9,258.4	6,743.9	2,538.8	1,057.6	1,507.8	33,921.0
2011	3,923.7	9,928.4	9,075.5	6,862.5	2,326.9	969.3	1,359.0	34,445.3
2012	3,920.9	11,046.1	8,838.1	7,266.2	2,095.8	952.4	1,280.3	35,399.8
2013	3,852.8	11,854.4	8,742.6	7,682.2	2,056.9	951.6	1,285.7	36,426.3
2014	3,806.9	12,504.8	8,842.0	8,045.0	2,028.7	930.4	1,349.4	37,507.2
2015	3,821.7	13,191.6	8,894.8	8,284.7	1,995.4	941.5	1,383.7	38,513.3
2016	3,866.4	13,908.2	9,023.4	8,688.5	1,971.7	884.9	1,397.1	39,740.2
2017	3,879.3	14,468.8	9,304.5	8,994.0	1,934.7	965.9	1,468.9	41,016.1
2018	3,821.1	15,608.0	9,732.1	9,200.7	1,841.6	1,076.1	1,677.1	42,956.7

Issuance in the US Bond Markets (USD Billions)

Year	M	Municipal	Treasury	Mortgage-Related	Corporate Debt	Federal Agency Securities	Asset-Backed	Total
1996		182.9	612.4	551.8	337.4	277.9	121.2	2,083.5
1997		218.6	540.0	725.2	455.3	323.1	143.1	2,405.3
1998		284.1	438.4	1,260.6	588.5	596.4	184.2	3,352.1
1999		224.4	364.6	1,121.6	602.1	548.0	196.3	3,057.0
2000		198.2	312.4	779.9	575.1	446.6	240.3	2,552.5
2001		286.2	380.7	1,816.7	770.6	941.0	261.4	4,456.7
2002		355.7	571.6	2,514.9	636.7	1,041.5	268.6	5,389.0
2003		380.1	745.2	3,537.1	774.3	1,219.5	287.6	6,943.8
2004		358.1	853.3	2,428.3	775.8	877.8	330.6	5,623.8
2005		407.1	746.2	2,764.1	750.1	635.0	473.7	5,776.2
2006		385.9	788.5	2,691.1	1,058.4	691.8	658.2	6,273.9
2007		429.2	752.3	2,434.2	1,141.0	831.2	827.6	6,415.5
2008		389.3	1,037.3	1,394.0	717.0	924.8	215.2	4,677.5
2009		409.5	2,074.9	2,172.1	945.4	1,244.4	177.9	7,024.3
2010		433.1	2,319.8	2,012.6	1,055.4	1,362.1	125.9	7,309.0
2011		295.1	2,103.3	1,724.8	1,025.1	1,025.3	151.0	6,324.5
2012		382.6	2,304.5	2,195.1	1,371.2	925.5	259.0	7,438.0
2013		335.3	2,140.0	2,120.2	1,379.9	652.9	304.1	6,932.5
2014		339.1	2,215.4	1,439.6	1,438.4	558.7	393.4	6,384.6
2015		405.1	2,122.5	1,800.7	1,494.8	645.5	333.4	6,801.9
2016		445.8	2,169.4	2,044.2	1,527.6	927.9	325.4	7,440.3
2017		448.0	2,224.3	1,934.8	1,652.4	731.3	550.3	7,541.2
2018		338.3	2,684.6	1,898.6	1,336.7	649.2	516.9	7,424.3

Average Daily Trading Volume (USD Billions)

	Municipal	Treasury	Agency MBS	Non-Agency MBS	ABS	Corporate Debt	Federal Agency Securities
1996	1.1	203.7	38.1	-	-	-	31.1
1997	1.1	212.1	47.1	-	-	-	40.2
1998	3.3	226.6	70.9	-	-	-	47.6
1999	8.3	186.5	67.1	-	-	-	54.5
2000	8.8	206.5	69.5	-	-	-	72.8
2001	8.8	297.9	112.0	-	-	-	90.2
2002	10.7	366.4	154.5	-	-	17.8	81.8
2003	12.6	433.5	206.0	-	-	18.0	81.7
2004	14.8	499.0	207.4	-	-	17.3	78.8
2005	16.9	554.5	251.8	-	-	16.6	78.8
2006	23.1	524.7	254.6	-	-	16.9	74.4
2007	25.1	570.2	320.1	-	-	16.4	83.0
2008	19.4	553.1	344.9	-	-	14.3	104.5
2009	12.5	407.9	299.9	-	-	19.9	77.7
2010	13.3	528.2	320.6	-	-	20.5	11.2
2011	11.3	567.8	243.3	4.4	1.5	20.6	9.6
2012	11.3	518.9	280.4	4.5	1.5	22.6	9.7
2013	11.2	545.4	222.8	4.1	1.3	24.7	6.6
2014	9.9	504.2	178.0	3.7	1.5	26.7	5.3
2015	8.6	490.1	193.0	3.1	1.4	27.9	4.5
2016	10.6	519.1	206.6	2.9	1.3	29.6	5.4
2017	10.8	505.2	209.1	2.5	1.4	30.6	4.1
2018	11.6	547.8	219.0	2.5	1.4	31.2	3.4

Average daily dollar trading volume in September 2015:

Equity **$321bn**, Treasury **$499bn**, and Corporate Bonds **$25bn**.

The Benchmark Role of Treasury Yield Curve

- Market participants rely on the Treasury curve to assess the cost of funds at different borrowing horizons.
- Price discovery about inflation prospects and other macroeconomic fundamentals occurred mainly in the Treasury market.
- This benchmark status derives from features unique to Treasuries:
  - The most credit-worthy, essentially free of default risk.
  - Large amount outstanding, highly liquid.
  - A wide range of maturities, facilitating the construction of yield curves.
  - Well developed repo and derivatives markets for Treasuries, enabling long and short positions to reflect views of future interest rates.
- This benchmark role of Treasuries is facing increasing competition from private sector debt instruments (e.g., interest rate swaps).

The Determinants of the Yield Curve

Some often used explanations (not mutually exclusive):

- Investor's expectations of future interest rates.
- Premiums required by investors to hold long-term bonds (e.g., risk premium and liquidity preference).
- Monetary policy.
- Expectations of future macroeconomic conditions (e.g., economic growth and inflation).
- Fiscal policy.
- Market segmentation; temporary imbalance of supply and demand.