Options and Market Crashes

Financial Markets, Day 2, Class 4

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April 19, 2019

Outline

- Bring the Black-Scholes model to the data:
  - Time-varying volatility.
  - Over-pricing of ATM options.
  - Volatility smirks/smiles.
- When crash happens:
  - Bank of volatility.
  - The 2008 crisis.
- Beyond the Black-Scholes model:
  - Market prices and financial models.
  - A model with market crash.
  - A model with stochastic volatility.

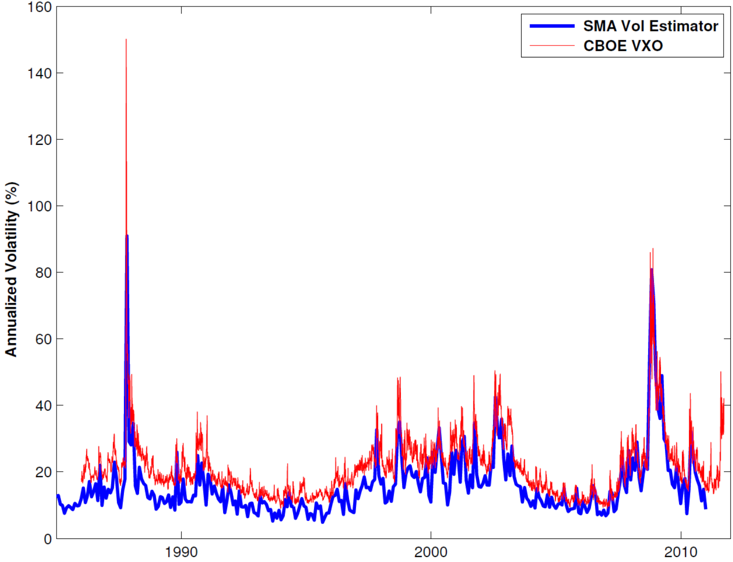
Bring the Black-Scholes Model to the Data

- The key assumptions of the model: constant volatility, continuous price movements, and log-normal distribution.
- The data: S&P 500 index options of different levels of moneyness and time to expiration.
- The basic tool: the Black-Scholes Option Implied Volatility.

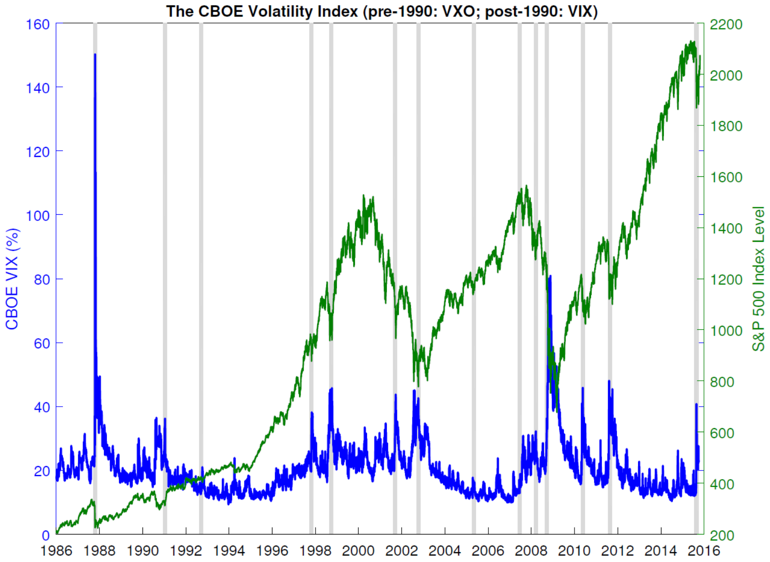
Disagreements between the Model and Data

1. Volatility is not a constant.
2. The volatility implied by the options market is on average higher than that observed directly from the underlying stock market.
3. On any given day, options (both puts and calls) with different strike prices exhibit a pattern of "smile" or "smirk":
   - OTM puts have higher implied-vol than ATM options and OTM calls.
   - This "smile" pattern is more pronounced in short-dated options.
4. Moreover, the volatility implied by long-dated options differs from that implied by short-dated options.

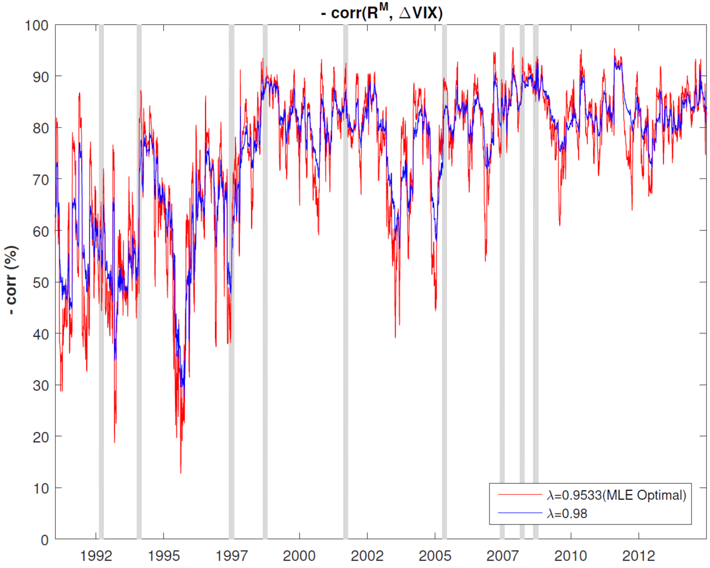
SMA vs. Option-Implied



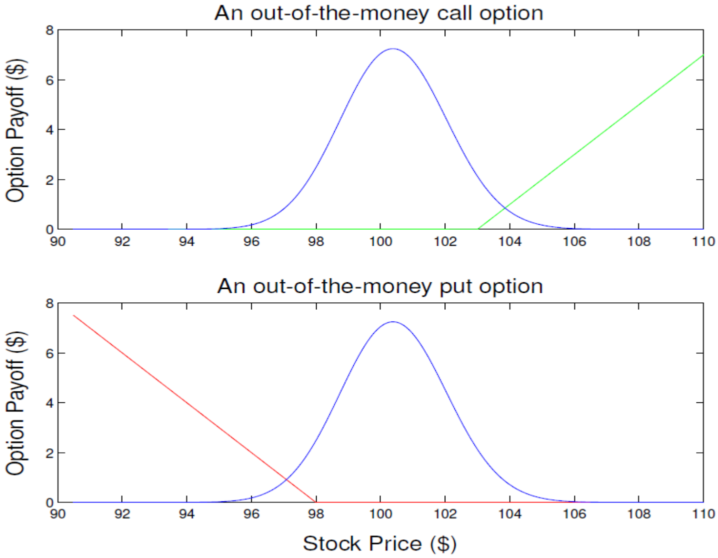
Stock Price and VIX



Correlation between Returns and Changes in VIX



Out-of-the-Money Options: Sampling the Tails



Option Pricing and Tail Events

- The pricing of a call option is linked to the right tail, $\mathbf{1}_{S_T>K}$

$$
\begin{aligned}
C_0 &= E^Q\left(e^{-rT}(S_T-K)\mathbf{1}_{S_T>K}\right) \\
&= \boxed{e^{-rT}\,E^Q\left(S_T\mathbf{1}_{S_T>K}\right)} - \boxed{e^{-rT}\,K\,E^Q\left(\mathbf{1}_{S_T>K}\right)}
\end{aligned}
$$

- The pricing of a put option is linked to the left tail, $\mathbf{1}_{S_T<K}$

$$
\begin{aligned}
P_0 &= E^Q\left(e^{-rT}(K-S_T)\mathbf{1}_{S_T<K}\right) \\
&= \boxed{e^{-rT}\,K\,E^Q\left(\mathbf{1}_{S_T<K}\right)} - \boxed{e^{-rT}\,E^Q\left(S_T\mathbf{1}_{S_T<K}\right)}
\end{aligned}
$$

OTM Put Option under the Black-Scholes Model

- Under the Black-Scholes model:

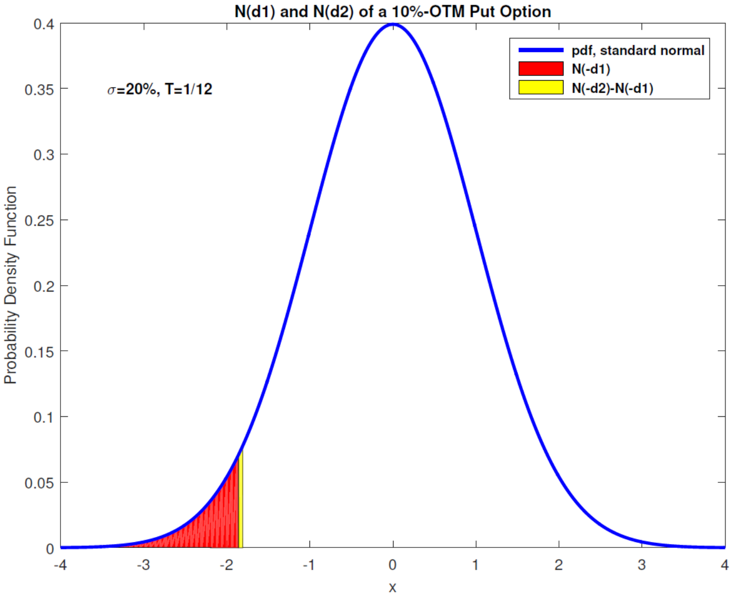
$$
\begin{aligned}
P_0 &= E^Q\left(e^{-rT}(K - S_T)\mathbf{1}_{S_T<K}\right) \\
&= \boxed{e^{-rT}\,K\,E^Q\left(\mathbf{1}_{S_T<K}\right)} - \boxed{e^{-rT}\,E^Q\left(S_T\mathbf{1}_{S_T<K}\right)} \\
&= \boxed{e^{-rT}\,K\,N(-d_2)} - \boxed{S_0\,N(-d_1)}
\end{aligned}
$$

- A 10% out-of-the-money put option striking at $K = S_0\,e^{r\,T} \times 90\%$:

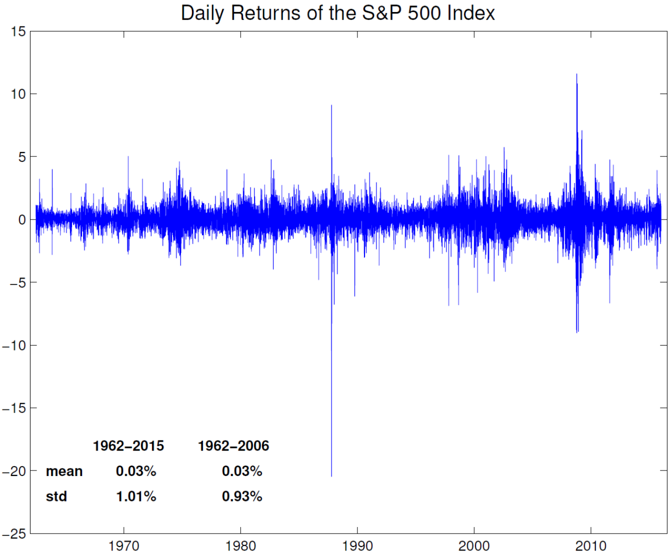
$$
\begin{aligned}
\frac{P_0}{S_0} &= \frac{e^{-rT}K}{S_0}\boxed{N(-d_2)} - \boxed{N(-d_1)} \\
&= 0.90 \times \boxed{N(-d_2)} - \boxed{N(-d_1)}
\end{aligned}
$$

- For $T = 1/12$ and $\sigma = 20\%$, $d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)\,T}{\sigma\sqrt{T}} = 1.8574$ and $d_2 = d_1 - \sigma\sqrt{T} = 1.7996$.

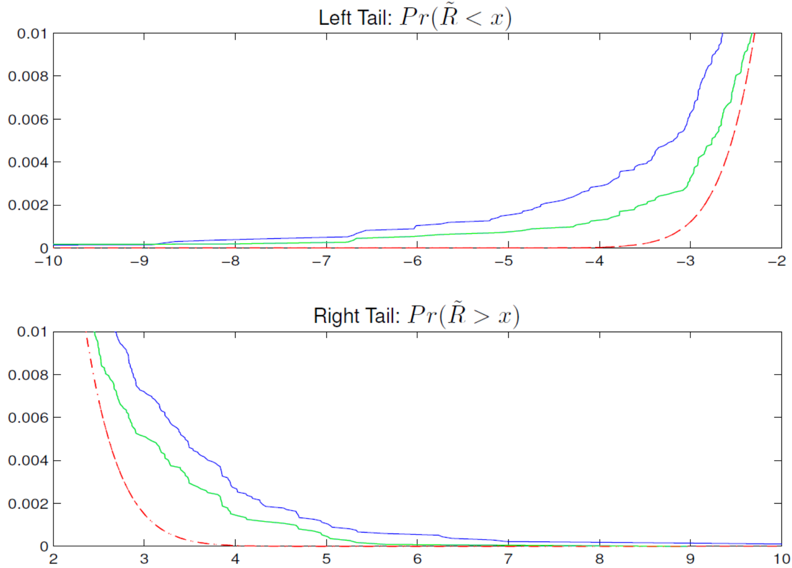
OTM Put Options



Daily Stock Returns



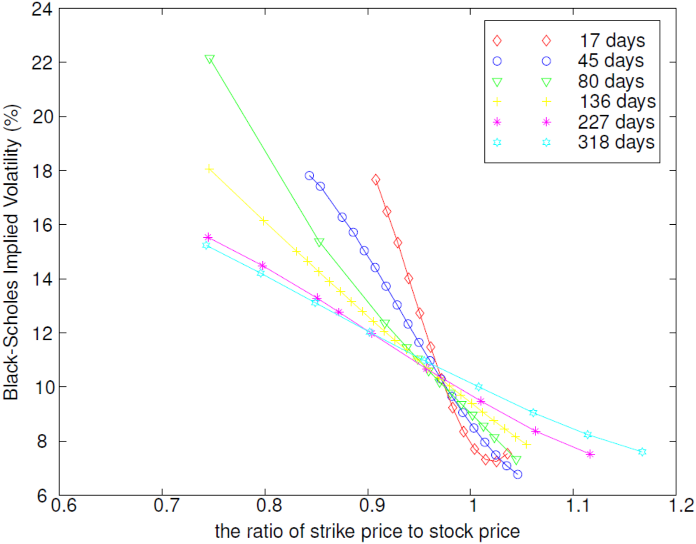
Tail Events



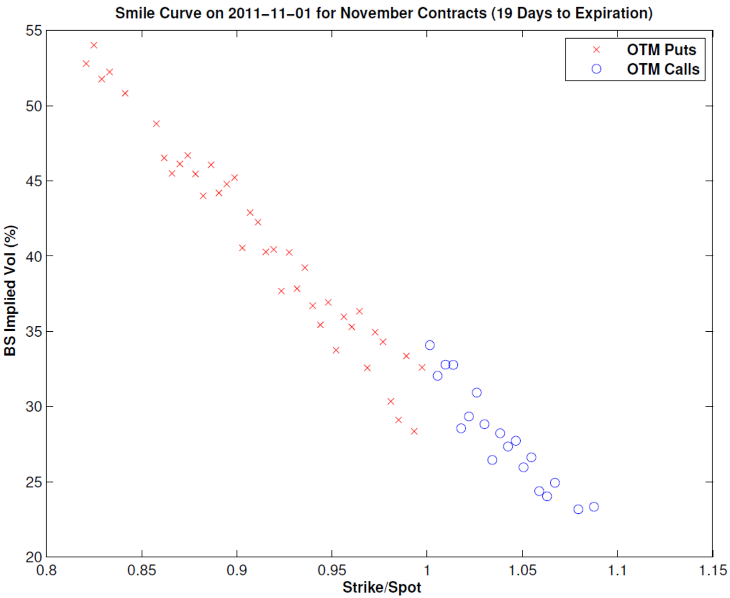
Left Tail: $Pr(\tilde{R} < x)$
0.01
0.008
0.006
0.004
0.002
0
−10
−9
−8
−7
−6
−5
−4
−3
−2
Right Tail: $Pr(\tilde{R} > x)$
0.01
0.008
0.006
0.004
0.002
0
2
3
4
5
6
7
8
9
10

Option Implied Smile

S&P 500 Index Options on Nov. 2, 1993



Option Implied Smile



Index Options with Varying Moneyness:

On March 2, 2006, the following put options are traded on CBOE:

P_0	S_0	K	T	r	q	σ^I
9.30	1287	1285	16/365	0.04	0.02	10.06%
6.00	1287	1275	16/365	0.04	0.02	10.64%
2.20	1287	1250	16/365	0.04	0.02	12.74%
1.20	1287	1225	16/365	0.04	0.02	15.91%
1.00	1287	1215	16/365	0.04	0.02	17.24%
0.40	1287	1170	16/365	0.04	0.02	22.19%

Index Options with Varying Moneyness:

On March 2, 2006, the following put options are traded on CBOE:

P_0	S_0	K	OTM-ness	T	σ^I	P_0^{BS}
9.30	1287	1285	0.15%	16/365	10.06%	?
6.00	1287	1275	0.93%	16/365	10.64%	5.44
2.20	1287	1250	2.87%	16/365	12.74%	0.92
1.20	1287	1225	4.82%	16/365	15.91%	0.075
1.00	1287	1215	5.59%	16/365	17.24%	0.022
0.40	1287	1170	9.09%	16/365	22.19%	0.000013

P_0^{BS} is the Black-Scholes price assuming $\sigma = 10.06\%$.

Expected Option Returns

Coval and Shumway, *Journal of Finance*, 2000

Strike - Spot	-15 to -10	-10 to -5	-5 to 0	0 to 5	5 to 10
Weekly SPX Put Option Returns (in %)					
mean return	-14.56	-12.78	-9.50	-7.71	-6.16
max return	475.88	359.18	307.88	228.57	174.70
min return	-84.03	-84.72	-87.72	-88.90	-85.98
mean BS β	-36.85	-37.53	-35.23	-31.11	-26.53
corrected return	-10.31	-8.45	-5.44	-4.12	-3.10

Data sample period from Jan. 1990 through Oct. 1995

Crash and Crash Premium

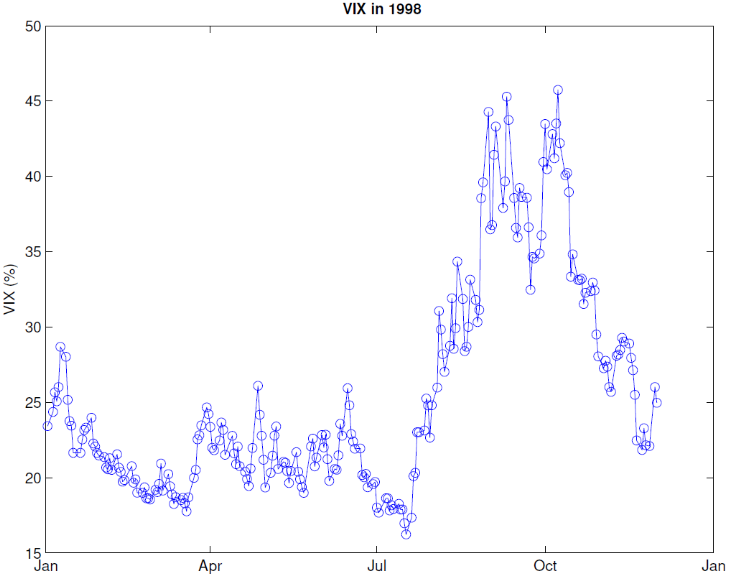
- The empirical evidence we've seen so far indicates that strategies involving selling volatility and selling crash insurance are profitable, and their risk profile differs significantly from that of stock portfolios.
- In the presence of tail risk, options are no longer redundant and cannot be dynamically replicated, and their pricing has two components:
  - the likelihood and magnitude of the tail risk.
  - aversion or preference toward such tail events.
- As such, the "over-pricing" of put options on the S&P 500 index reflects not only the probability and severity of market crashes, but also investors' aversion to such crashes — crash premium.
- In fact, the crash premium accounts for most of the "over-pricing" in short-dated OTM puts and ATM options. This "over-pricing" is not severe for OTM calls because they are less sensitive to the left tail.

The Bank of Volatility

Excerpts from *"When Genius Failed"* by Roger Lowenstein

- Early in 1998, LTCM began to short large amounts of equity volatility.
- Betting that implied volatility would eventually revert to its long-run mean of 15%, they shorted options at prices with an implied volatility of 19%.
- Their position is such that each percentage change in implied vol will make or lose $40 million in their option portfolio.
- Morgan Stanley coined a nickname for the fund: the Central Bank of Volatility.

VIX in 1998



Implications for the 2008 Crisis

- The OTM put options on the S&P 500 index is a very good example for us to remember what an insurance on the market looks like.
- So next time when you see one, you will recognize it for what it is.
- As we learned from the recent crisis, some supposedly sophisticated investors wrote insurance on the market without knowing, the willingness to know, or the integrity to acknowledge the consequences.
- $0 \times$ \$100 billion $= 0$, but only if the zero is really zero.
- Small probability events have a close to zero probability, but not zero!
- So $10^{-9} \times$ \$100 billion $\neq 0$! And the math is in fact more complicated.
- And if this small probability event has a market-wide impact, then you need to be very careful.

Excerpts from *Fool's Gold* by Gillian Tett

- By 2006, Merrill topped the league table in terms of underwriting CDO's, selling a total of \$52 billion that year, up from \$2 billion in 2001.
- Behind the scenes, Merrill was facing the same problem that worried Winters at J.P.Morgan: what to do with the super-senior debt?
- Initially, Merrill solved the problem by buying insurance for its super-senior debt from AIG.
- In late 2005, AIG told Merrill it would no longer offer that service.
- The CDO team decided to start keeping the risk on Merrill's books.
- In 2006, sales of the various CDO notes produced some \$700 million worth of fees. Meanwhile, the retained super-senior rose by more than \$5 billion each quarter.

Excerpts from *Fool's Gold* by Gillian Tett

- As the CDS team posted more and more profits, it became increasingly difficult for other departments, or even risk controllers, to interfere.
- O'Neal himself could have weighted in, but he was in no position to discuss the finer details of super-senior risk.
- The risk department did not even report directly to the board.
- O'Neal faces absolutely no regulatory pressure to manage the risk any better.
- Far from it. The main regulator of the brokerages was the SEC, which had recently removed some of the old constraints.

Excerpts from *Fool's Gold* by Gillian Tett

- Citigroup was also keen to ramp up the output of its CDO machine.
- Unlike the brokerages, though, Citi could not park unlimited quantities of super-senior on its balance sheet, since the US regulatory system did still impose a leverage limit on commercial banks.
- Citi decided to circumvent that rule by placing large volumes of its super-senior in an extensive network of SIVs and other off balance sheet vehicles that it created.
- The SIVs were not always eager to buy the risk, so Citi began throwing in a type of "buyback" sweetener: it promised that if the SIVs ever ran into problems with the super-senior notes, Citi itself would buy them back.
- By 2007, it had extended such "liquidity puts" on $25 billion of super-senior notes. It also held more than $10 billion of the notes on its own books.

Market Prices and Financial Models

- Students often ask, *"If not Black-Scholes, then which model is used in pricing options?"* One implicit belief in that question is that market prices are determined by models.
- In practice, prices are determined through trading: buying and selling by a wide spectrum of investors with a wide spectrum of motives. Some trade because of (legal) private information; some trade for hedging or portfolio rebalancing.
- So the market prices we observe arise from a rather "organic" process. It is clearly not the result of one or several models.
- In a less liquid market, this "organic" process is less effective, and there is more reliance on models.

How do We Think about Models?

- Some people are highly skeptical about models: *"Investors should be skeptical of history-based models. Constructed by a nerdy-sounding priesthood using esoteric terms such as beta, gamma, sigma and the like, these models tend to look impressive. Too often, though, investors forget to examine the assumptions behind the symbols. Our advice: Beware of geeks bearing formulas."*
- Regardless of the negative sentiment, Finance models such as the CAPM, the Black-Scholes model, term-structure models, and credit-risk models play important roles in the day-to-day practice of Finance.
- This is especially true for the areas involving options, fixed-incomes, and credit instruments, where people rely quite heavily on models.

Use Models Wisely

- It is important to remember that models are just a means, not an end to itself. The final decision making lies with the investors.
- Chi-Fu Huang: *"Models are important (for the purpose of identifying trading opportunities). But what is more important is for the investor to understand the* **economic force** *and the* **institutional reasons** *that generate the trading opportunity."*
- In other words, use the model wisely; use it as a tool, not as a machine for self-delusion. Your model should not have the final say.
- It is also advisable to know your model well. Don't be content with a black box. Look under the hood. Locate the existing components of the model. Identify the flaky parts. And, most important of all, be aware of the missing parts.

Fear of Crash and Crash Premium

- Options, especially OTM puts, provide a unique opportunity to gauge investors' *assessment* and *attitudes* toward market crashes.
- As such, the market prices of OTM puts reflect not only the probability and magnitude of market crashes, but also investors' aversion to such crashes.
- How much are people willing to pay to have the crash hedged out? Is this willingness consistent with the magnitude and frequency of historical crashes? Are investors willing to pay a crash premium because of their fear of crash?
- To answer such questions, we need a model to help us back out the *probability* and *magnitude* of market crashes that have been priced into the OTM puts.

Diffusion vs. Market Crashes

- The Black-Scholes model is a pure diffusion model with Brownian motions as the driving force. Brownian motions have continuous paths.
- The increment $(B_T - B_0)$ has a variance of T. So as the time horizon T shrinks to zero, the risk goes to zero as well. In fact, this is what makes the dynamic hedging feasible.
- These features of Brownian motion are counter to the *discontinuous* and *sudden* nature of market crashes. So we need to go beyond the Black-Scholes model. This was done by Merton (1976) and Cox and Ross (1976).
- Their work gave rise to jump-diffusion models in Finance: a diffusion component to capture the day-to-day random fluctuations and a jump component to capture the sudden and adverse events such as market crashes.

A Model with Market Crash

- In Group Project 3, we work with a simplified version of Merton (1976). In that model, we have two additional parameters for the crash component: the one-month probability of "jump" ($p = 2\%$) and the "jump size" given its arrival (jump size = -20%).
- In Merton (1976), the jump arrival is dictated by a Poisson process with a jump arrival intensity of λ. Over a one-month horizon, the jump probability is $p = 1 - e^{-\lambda T}$, where $T = 1/12$. So $p = 2\%$ implies a jump intensity of $\lambda = 24.24\%$ per year.
- In Merton (1976), the jump size is normally distributed. So given jump arrival, there is uncertainty in jump amplitude. In our simplified model, we work with a constant jump size of -20%.
- In Merton (1976), the option pricing formula builds on the Black-Scholes model. For convenience, we use the cumbersome method of simulation.

What We Learned from the Crash Model?

- We find that in order to generate realistic volatility smirk to match the options data, we need the market to crash much more often than what has been historically observed.
- Conversely, if we plug into the model more realistic jump parameters (moderate p and jump size), then the model cannot generate the steep option-implied smirk as observed in the options data.
- In other words, investors are willing to pay a very high premium to have the crash risk hedged out of their portfolio. Conversely, selling OTM put options on the market can be a "good" investment strategy if you believe that such people suffers from "paranoia."
- Then rare events such as 2008 happens, and you realize that such "paranoia" is in fact rational: the "over-pricing" or the extra premium is due to a high level of risk aversion towards market crashes.

The Assumption of Constant Volatility

- The fact that volatility is not constant can be seen from both the underlying stock market and the options market.
- Some empirical "regularities":
  1. volatility is stochastic (random)
  2. volatility is persistent
  3. volatility is mean-reverting
  4. the volatility shocks are negatively correlated with the underlying price shocks

Index Options with Varying Time to Expiration:

On March 2, 2006, the following put options are traded on CBOE:

P_0	S_0	K	T	r	q	σ^I
17.70	1287	1300	16/365	0.04	0.02	9.96%
51.00	1287	1300	288/365	0.04	0.02	12.19%
63.00	1287	1300	470/365	0.04	0.02	12.85%

Index Options with Varying Time to Expiration:

On March 2, 2006, the following put options are traded on CBOE:

P_0	S_0	K	T	σ^I	P_0^{BS}
17.70	1287	1300	16/365	9.96%	?
51.00	1287	1300	288/365	12.19%	24.17
63.00	1287	1300	470/365	12.85%	46.85

P_0^{BS} is the Black-Scholes price assuming $\sigma = 9.96\%$.

Term Structure of Option Implied Volatility

- Let's focus on options with similar degrees of money and vary the time to expiration from one month to one year.
- Plotting the option-implied volatility over the expiration horizon, we find that the implied volatility exhibit some interesting patterns.
- First, it is usually not flat (contrary to the Black-Scholes model).
- Second, it is sometimes upward sloping (longer maturity options having higher implied volatility); sometimes downward sloping.
- These patterns are a result of a time-varying volatility that tends to mean revert over a relative short horizon (as compared with the mean reversion in interest rate).

Stochastic Volatility Model

- The Black-Scholes model (under the risk-neutral measure):

$$\frac{dS_t}{S_t} = r\,dt + \sigma\,dB_t$$

  where σ is a constant.

- A stochastic volatility model relaxes the assumption that σ is a constant and makes it time varying (σ_t). There are three well-known models of stochastic volatility: Hull and White (1987), Stein and Stein (1991), Heston (1993).

- Among these, the Heston model has been used quite widely by practitioners and academics. Its tractability in option pricing is a key reason for its success.

The Heston Model

- The Heston model rewrites the Black-Scholes model by ($\sigma_t = \sqrt{V_t}$),

$$\frac{dS_t}{S_t} = r\,dt + \sqrt{V_t}\,dB_t^s$$

- It then models the variance process V_t as a square-root process:

$$dV_t = \kappa\,(\theta - V_t)\ dt + \sigma_v \sqrt{V_t}\,dB_t^v$$

  Here κ is a coefficient that controls the mean-reversion of the variance process, θ is the long-run mean of the variance process, and σ_v is the volatility coefficient for the variance process.

- Notice that there are two Brownian motions: one B^s for the stock price and one B^v for the variance. These two Brownian motions are allowed to be correlated to capture the fact the stock returns and volatility are negatively correlated (ranges from -50% to -90%).