Alpha, Beta and the CAPM

Financial Markets, Day 1, Class 3

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Outline

- The risk that matters.
- Running regressions to estimate the CAPM alpha and beta.

The Risk that Matters

- So far, we've focused on one time series.
- This time series turns out to be a very important risk factor.
- According to the CAPM, investors are only rewarded for bearing *systematic risk*, the type of risk that *cannot* be diversified away.
- They should not be rewarded for bearing *idiosyncratic risk*, since this uncertainty can be mitigated through appropriate diversification.
- The U.S. aggregate stock market has been commonly adopted as a proxy for the systematic risk.
- With this time series serving as an anchor, we can now talk about the pricing of individual stocks or other portfolios.
- Let's start by referring to it as R^M.

The CAPM

- The CAPM identifies one single portfolio, the market portfolio R^M, to be the only source of risk that matters.
- The market risk premium $= E(R^M) - r_f$, where r_f is the riskfree rate.
- So far, our estimate for $E(R^M)$ is around 12% per year (and very noisy). The riskfree rate is on average 4% per year. So a good estimate for the market risk premium is around 8%.
- The risk of each individual stock, say GE, is measured *not by its own volatility* but by its *exposure* to the market risk:

$$\beta^{GE} = \frac{\text{covariance}\left(R^{GE}, R^M\right)}{\text{variance}\left(R^M\right)}$$

- The reward is proportional to the risk:

$$E(R^{GE}) - r_f = \beta^{GE} \times \left(E(R^M) - r_f\right)$$

Running Regression to Estimate the CAPM β:

- Identify an index as the market portfolio. Typical choice: the CRSP value-weighted index (e.g., the academics), the S&P 500 index (e.g., Merrill Lynch's beta book), and the NYSE index (e.g., Value-line).
- Identify the stock or portfolio of interest.
- Collect time-series of returns:
  - For the market portfolio: $R_t^M, \quad t = 1, 2, 3, \ldots T$.
  - For the test portfolio: $R_t^{GE}, \quad t = 1, 2, 3, \ldots T$.
- Run the following regression (typically monthly data over a five-year rolling window):

$$R_t^{GE} - r_f = \alpha + \beta \left(R_t^{\mathrm{M}} - r_f \right) + \epsilon_t$$

Two Sources of Uncertainty in a Stock

- By running this regression, we break the total uncertainty in a stock into two components:

$$R_t^{GE} - r_f = \alpha + \beta \left(R_t^M - r_f \right) + \epsilon_t$$

  - One is due to its exposure to the market portfolio: $\beta \left(R_t^M - r_f \right)$.
  - The other is idiosyncratic, as captured by the regression residual ϵ_t .
- By construction, the residual of a regression is uncorrelated with the explanatory variable: $\text{cov}(R_t^M, \epsilon_t) = 0$.
- The R-squared tells us how much of GE's variance can be explained by the variance in the market portfolio:

$$\text{R-squared} = \frac{\beta^2 \, \text{var}(R^M)}{\text{var}(R^{GE})} = \frac{\beta^2 \text{var}(R^M)}{\beta^2 \text{var}(R^M) + \text{var}(\epsilon)}$$

The CAPM α

- If you re-arrange that regression equation, you get

$$\alpha = R_t^{GE} - r_f - \beta\left(R_t^M - r_f\right) - \epsilon_t \, .$$

- Taking expectations on both sides, we have

$$\alpha = E\left(R_t^{GE} - r_f\right) - \beta E\left(R_t^M - r_f\right) \, ,$$

  α is the *expected* excess stock return, after taking out the reward associated with the systematic component.

- So testing the CAPM pricing formula is the same as testing whether or not α is zero.
- Conversely, if we can construct many portfolios with positive and statistically significant α's, then the CAPM pricing formula is under a severe challenge.

Using t-stat

$$\text{t-stat} = \frac{\text{estimate}}{\text{s.e.}}$$

- In finance, we often use historical data to estimate financial models. The model parameters (e.g., α and β) are always estimated with noise.
- The standard errors and t-stat inform us on the precision. We can then decide whether or not to take the estimates seriously.
- As a rule of thumb, we take an estimate seriously if the absolute value of its t-stat is larger than 1.96:

$$|\text{t-stat}| \geq 1.96$$

- If it is less than 1.96, then we don't take it as seriously: statistically insignificant from zero.

Alpha, Beta, and R-Squared

Ticker	Start	mean (%)	std (%)	Alpha (%)	Beta	R2 (%)
GE	192701	1.14 [4.58]	7.98	0.13 [0.86]	1.18 [43.62]	65.12
AAPL	198101	2.28 [3.10]	14.19	1.13 [1.73]	1.45 [10.25]	22.08
BRK	197611	2.04 [5.79]	7.21	1.23 [3.87]	0.68 [9.80]	18.69
ONXX	199606	3.10 [1.71]	24.72	2.26 [1.31]	1.52 [4.38]	9.36
GOOG	200409	2.72 [2.23]	11.46	2.15 [2.00]	1.10 [5.04]	22.63

All time series end on 201112

Alpha, Beta, and R-Squared

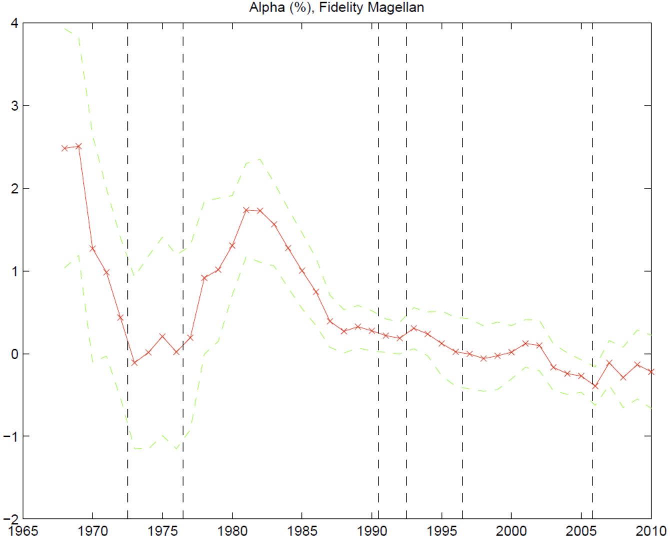
Ticker	Start	mean (%)	std (%)	Alpha (%)	Beta	R2 (%)
GE	200409	0.03 [0.03]	9.01	-0.64 [-1.02]	1.38 [10.88]	57.63
AAPL	200409	4.30 [3.54]	11.39	3.68 [3.60]	1.25 [6.06]	29.70
BRK	200409	0.45 [0.80]	5.27	0.12 [0.24]	0.44 [4.22]	16.99
ONXX	200409	1.76 [0.85]	19.43	1.24 [0.62]	0.97 [2.37]	6.08
GOOG	200409	2.72 [2.23]	11.46	2.15 [2.00]	1.10 [5.04]	22.63

All time series start on 200409 and end on 201112

Wall Street's Search for Alpha

- Searching for investment opportunities with positive alpha is the goal of every active fund manager.
- One thing for sure, without taking on idiosyncratic risk ϵ, a portfolio manager's alpha is always zero. So effectively, he is hoping to get α as a reward for holding ϵ_t.
- In the world of the CAPM, this is impossible.
- So do active fund managers actually provide positive α? We need to look into the data to find out.

Alpha of a Mutual Fund



Alpha of Hedge Funds