Estimation Using Financial Data

Financial Markets, Day 1, Class 2

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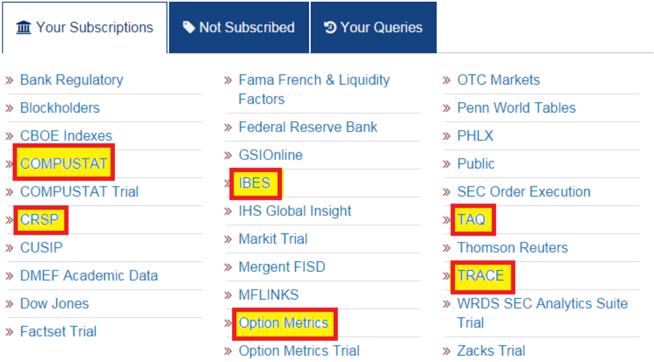
Outline

- Where to get financial data?
- Modelng random events in financial markets.
- Test financial models using financial data.
- Estimating the expected return.

Where to Get Data

- Bloomberg
- Datastream
- WRDS: $\rightarrow$



- CRSP:
  - Stock
  - Treasury Bonds
  - Mutual Funds
- Prof. Ken French's Website.

Computing Realized Stock Returns

- For a publicly traded firm, we can get
  - its stock price P_t at the end of year t.
  - its cash dividend D_t paid during year t.
- At the end of year t, we calculate the **realized** return on the stock:

$$R_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}}$$

- Returns = capital gains yield + dividend yield.
- For the US markets, the best place to get reliable and clean holding-period returns is CRSP.
- I've applied a CRSP class account for us.

The Expected Return

- For any financial instrument, the single most important number is its **expected** return.
- Suppose right now we are in year t, let R_{t+1} denote the stock return to be realized next year. Our investment decision relies on the **expectation**:

$$\mu = E(R_{t+1}) \; .$$

- Just to emphasize, μ is a number, while R_{t+1} is a random variable, drawn from a distribution with mean μ and standard deviation σ.
- To estimate this number μ with precision is the biggest headache in Finance.

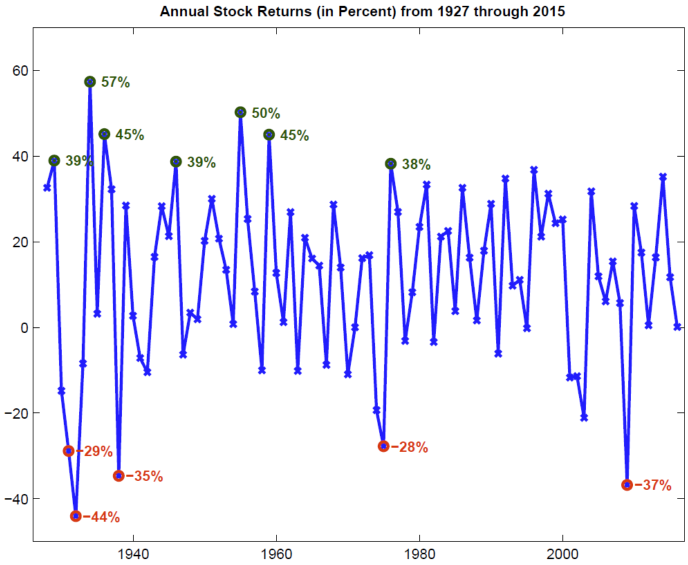
Estimating the Expected Return μ

- We estimate μ by using historical data:

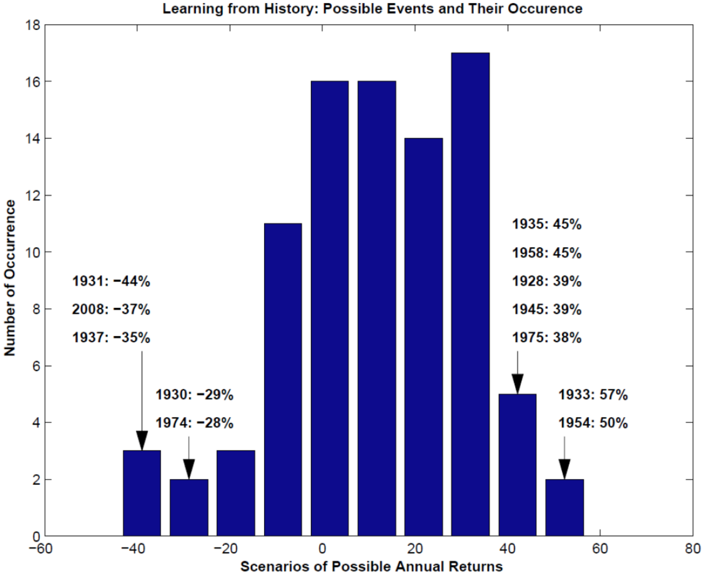
$$\hat{\mu} = \frac{1}{N} \sum_{t=1}^{N} R_t \,.$$

- It is as simple as taking a sample average.
- Why can this sample average of *past* realized returns help us form an expectation of the *future*?
- Because our assumption that history repeats itself. Each R_t in the past was drawn from an identical distribution with mean μ and standard deviation σ.

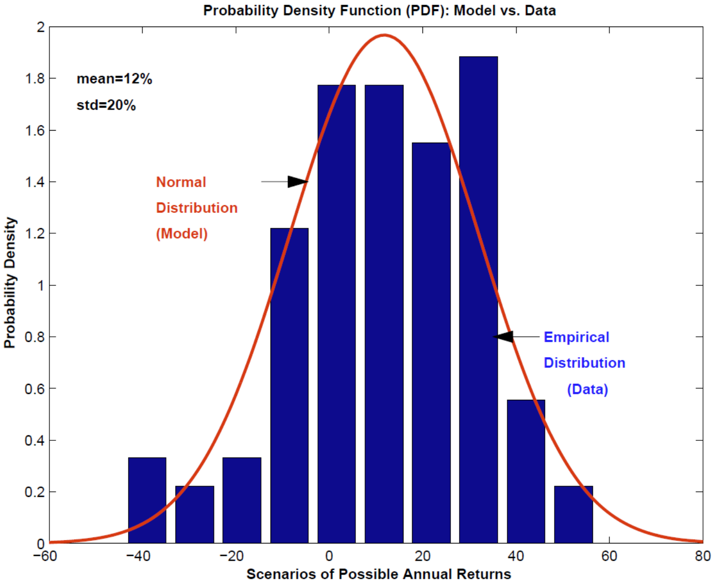
Time Series of Annual Stock Returns



Scenarios and Their Likelihood



Probability Distribution of a Random Event



The Estimator Has Noise

- We use historical returns to estimate the number μ:

$$\hat{\mu} = \frac{1}{N} \sum_{t=1}^{N} R_t$$

- Recall that R_t is a random variable, drawn every year from a distribution with mean μ and standard deviation σ.
- As a result, $\hat{\mu}$ inherits the randomness from R_t. In other word, it is not really a number: $\text{var}(\hat{\mu})$ is not zero.
- If this variance $\text{var}(\hat{\mu})$ is large, then the estimator is noisy.

The Standard Error of $\hat{\mu}$

- Let's first calculate $\text{var}(\hat{\mu})$:

$$\text{var}\left(\frac{1}{N}\sum_{t=1}^{N} R_t\right) = \frac{1}{N^2}\sum_{t=1}^{N}\text{var}(R_t) = \frac{1}{N^2}\times N \times \sigma^2 = \frac{1}{N}\sigma^2$$

- The **standard error** of $\hat{\mu}$ is the same as $\text{std}(\hat{\mu})$:

$$\text{standard error} = \frac{\text{std}(R_t)}{\sqrt{N}} = \frac{\sigma}{\sqrt{N}}$$

Estimating μ for the US Aggregate Stock Market

- Using annual data from 1927 to 2014, we have 88 data points.
- The sample average is $\text{avg}(R) = 12\%$. The sample standard deviation is $\text{std}(R) = 20\%$.
- The **standard error** of $\hat{\mu}$:

$$\text{s.e.} = \text{std}(R)/\sqrt{N} = 20\%/\sqrt{88} = 2.13\%$$

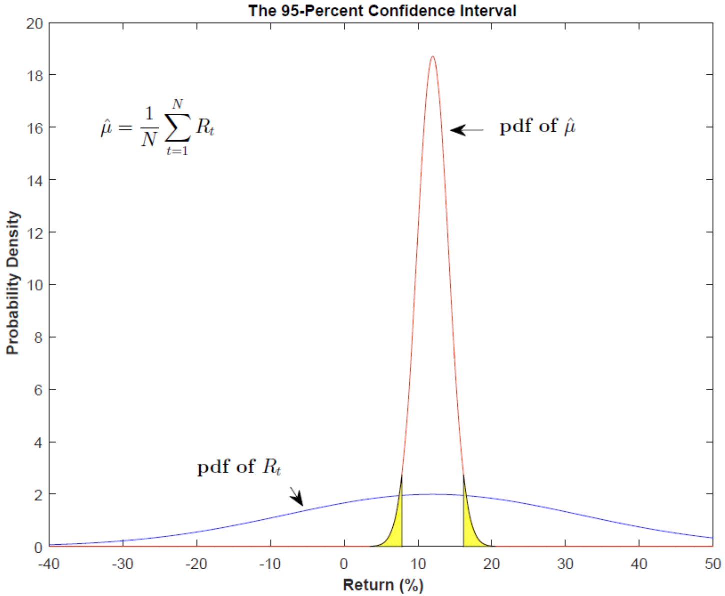
- The 95% confidence interval of our estimator:

$$[12\% - 1.96 \times 2.13\%, 12\% + 1.96 \times 2.13\%] = [7.8\%, 16.2\%]$$

- The **t-stat** of this estimator is (signal-to-noise ratio),

$$\text{t-stat} = \frac{\text{avg}(R)}{\text{std}(R)/\sqrt{N}} = \frac{12\%}{2.13\%} = 5.63\,.$$

The Distributions of R_t and $\hat{\mu}$



How to Improve the Precision?

- Not much, really!
- We got a t-stat of 5.63 for $\hat{\mu}$ using 88 years of data!
- Usually, the time series we are dealing with are much shorter. For example, the average life span of a hedge fund is around 5 years.
- Also, the volatility of individual stocks is much higher than that of the aggregate market. For example, the annual volatility for Apple is 49.16%. For smaller stocks, the number is even higher: around 100%.
- What about designing a derivatives product whose value would depend on μ? (No)
- What about polling investors for their individual assessments of μ and then aggregate the information? (Not very useful)

Estimating μ Using Monthly Returns

- Since the standard error of $\hat{\mu}$ depends on the number of observations, why don't we use monthly returns to improve on our precision?
- Using monthly aggregate stock returns from January 1927 through December 2011, we have 1020 months. So N=1020!
- The mean of the time series is 0.91%, and std is 5.46%.
- So the standard error of $\hat{\mu}$ is:

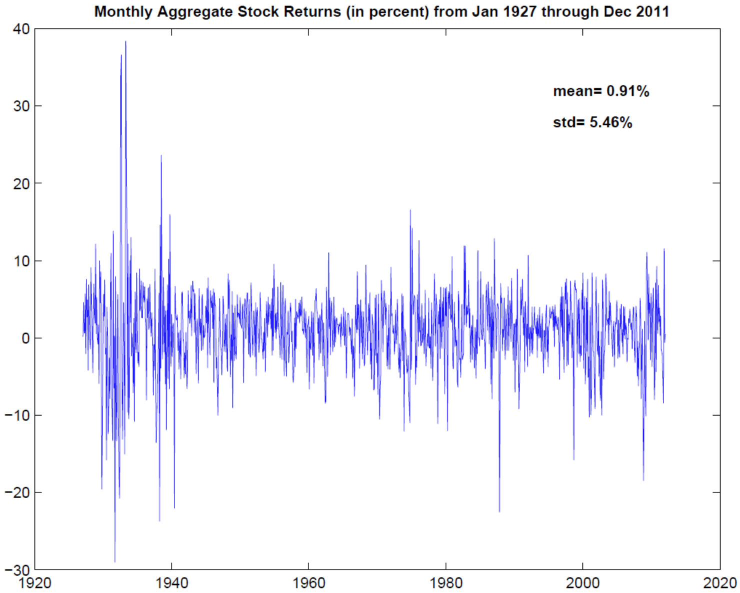
$$\text{s.e.} = 5.46\%/\sqrt{1020} = 0.1718\%$$

- The signal-to-noise ratio:

$$\text{t-stat} = \frac{0.91\%}{0.1718\%} = 5.30$$

- We increased N by a factor of 12. Yet, the t-stat remains more or less the same as before. What is going on?

Time Series of Monthly Stock Returns



Chopping the Time Series into Finer Intervals?

- It is actually a very straightforward calculation (give it a try) to show that when it comes to the precision of $\hat{\mu}$, it is the length of the time series that matters. Chopping the time series into finer intervals does not help.
- Professor Merton has written a paper on that. See "On Estimating the Expected Return on the Market," *Journal of Financial Economics*, 1980.
- But when it comes to estimating the volatility of stock returns, this approach of chopping does help and is widely used. We will come back to this.