Fixed Income, Credit Risk

15.433 Financial Markets

December 5, 2017

Outline

- Corporate Bonds
  - Default Intensity
  - Loss Given Default
- Modeling Default:
  - Structural Approach
  - Reduced-Form Approach
- Credit Default Swaps

Fixed Income

- Key Risk Factors
  - Yield curve uncertainties: **Level, Slope**, and interest rate **Volatility**.
  - **Counterparty risk** in OTC derivatives.
  - **Credit risk** in corporate bonds, CDS, bank loans, mortgages, muni's, commercial paper, CDO/CLO.
  - **Liquidity risk**, often coupled with credit events.
  - **Optionalities:** callable and puttable bonds, prepayment in MBS, etc.
- Measures of Risk:
  - **Term Spreads:** long-term yield minus short-term yield.
  - **Volatility:** swaption implied vol.
  - **Credit/Liquidity Spreads:** LIBOR-Treasury, LIBOR-OIS, Swap-Treasury, Old Bond/New Bond, Corp Spread, CDS, etc.

Outstanding US Bond Market Debt in $ Billions

	Muni	Treasury	Mortgage Related	Corp Debt	Agency Securities	Money Markets	Asset Backed	Total
1980	399.4	623.2	111.4	458.6	164.3	480.7		2,237.7
1985	859.5	1,437.7	399.9	776.6	261.0	950.9	1.2	4,686.7
1990	1,178.6	2,195.8	1,340.1	1,350.3	421.5	1,328.9	66.2	7,881.5
1995	1,268.2	3,307.2	2,432.1	2,087.5	924.0	1,367.6	214.9	11,601.4
2000	1,480.7	2,951.9	4,119.3	3,400.5	1,853.7	2,815.8	699.5	17,321.5
2005	3,019.3	4,165.9	7,206.4	4,604.0	2,616.0	3,536.6	1,275.0	26,423.2
2006	3,189.3	4,322.9	8,376.0	4,842.5	2,634.0	4,140.0	1,642.7	29,147.3
2007	3,424.8	4,516.7	9,372.6	5,254.3	2,906.2	4,310.8	1,938.8	31,724.2
2008	3,517.2	5,783.6	9,457.6	5,417.5	3,210.6	3,939.3	1,799.3	33,125.2
2009	3,672.5	7,260.6	9,341.6	5,934.5	2,727.5	3,243.9	1,682.1	33,862.7
2010	3,772.1	8,853.0	9,221.4	6,543.4	2,538.8	2,980.8	1,476.3	35,385.9
2011	3,719.4	9,928.4	9,043.8	6,618.1	2,326.9	2,719.3	1,330.0	35,685.9
2012	3,714.4	11,046.1	8,814.9	7,049.6	2,095.8	2,612.3	1,253.6	36,586.7
2013	3,671.2	11,854.4	8,720.1	7,458.6	2,056.9	2,713.7	1,252.5	37,727.3
2014	3,652.4	12,504.8	8,729.4	7,846.2	2,028.7	2,903.3	1,336.5	39,001.3
2015	3,777.0	13,191.6	8,759.0	8,172.3	1,995.4	2,806.9	1,358.8	40,061.0
2016Q2	3,826.1	13,417.8	8,829.8	8,433.4	2,005.1	2,849.2	1,359.7	**40,721.1**

Issuance in the U.S. Bond Markets

USD Billions

sifma
Invested in America

Year	M	Municipal	Treasury	Mortgage-Related	Corporate Debt	Federal Agency Securities*	Asset-Backed	Total
1996		185.2	612.4	553.4	343.7	277.9	121.9	2,094.5
1997		220.7	540.0	719.3	466.0	323.1	142.2	2,411.3
1998		286.8	438.4	1,259.3	610.7	596.4	186.1	3,377.7
1999		224.4	364.6	1,125.6	629.2	548.0	198.6	3,090.3
2000		198.3	312.4	778.8	574.1	446.6	246.8	2,557.0
2001		286.2	380.7	1,818.1	771.9	941.0	266.8	4,464.7
2002		355.8	571.6	2,519.4	636.0	1,041.5	273.9	5,398.2
2003		380.2	745.2	3,505.0	772.9	1,219.5	287.9	6,910.7
2004		358.1	853.3	2,411.5	774.6	877.8	349.2	5,624.5
2005		407.2	746.2	2,782.1	750.0	635.0	482.5	5,803.0
2006		386.0	788.5	2,730.9	1,057.5	691.8	678.1	6,332.8
2007		429.2	752.3	2,494.3	1,136.2	831.2	634.0	6,277.2
2008		389.3	1,037.3	1,436.8	711.3	984.9	289.3	4,848.9
2009		409.6	2,074.9	2,106.4	940.1	1,086.7	154.0	6,771.7
2010		433.1	2,319.8	2,010.7	1,055.1	1,203.7	108.4	7,130.9
2011		294.7	2,103.3	1,701.4	1,023.4	838.4	126.5	6,087.5
2012		378.9	2,304.5	2,157.7	1,370.2	720.7	233.2	7,165.3
2013		334.9	2,140.0	2,092.0	1,379.9	419.5	262.4	6,628.8
2014		337.5	2,215.4	1,365.3	1,479.3	377.4	309.6	6,084.5
2015		403.1	2,122.5	1,710.0	1,492.5	513.5	255.2	6,496.7

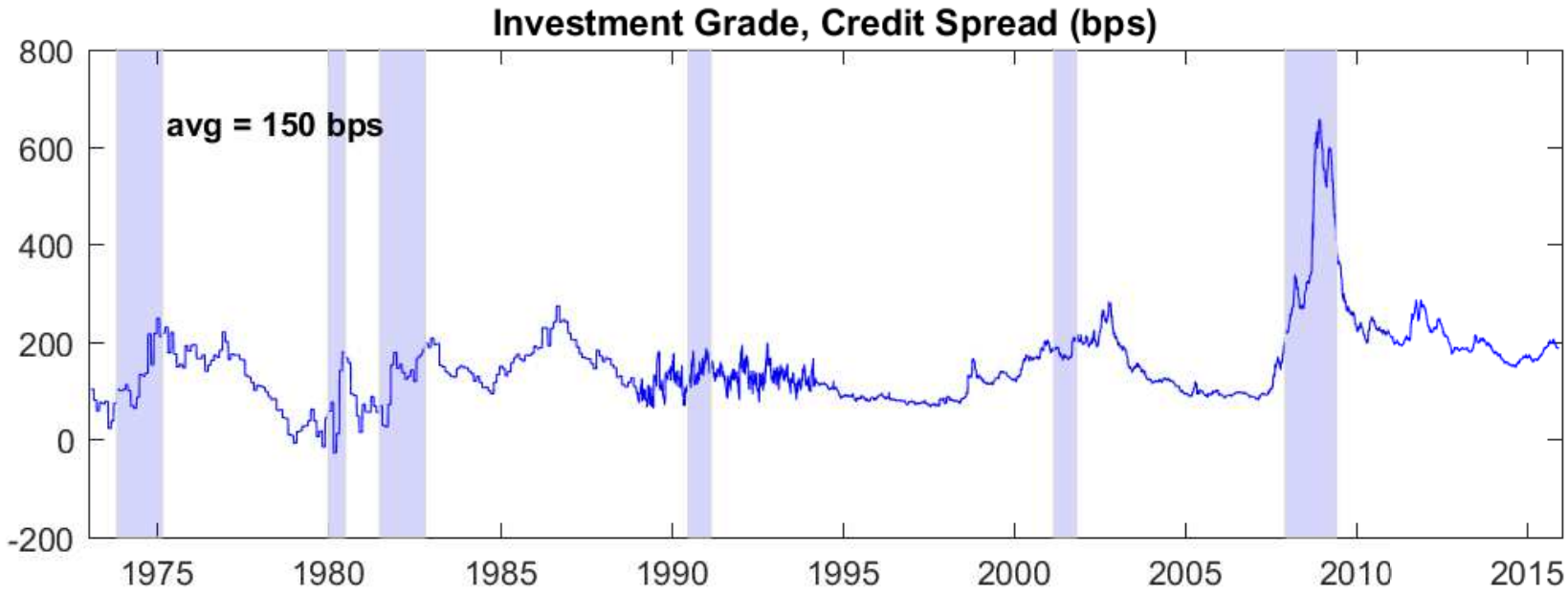
U.S. Bond Markets
Average Daily Trading Volume
USD Billions

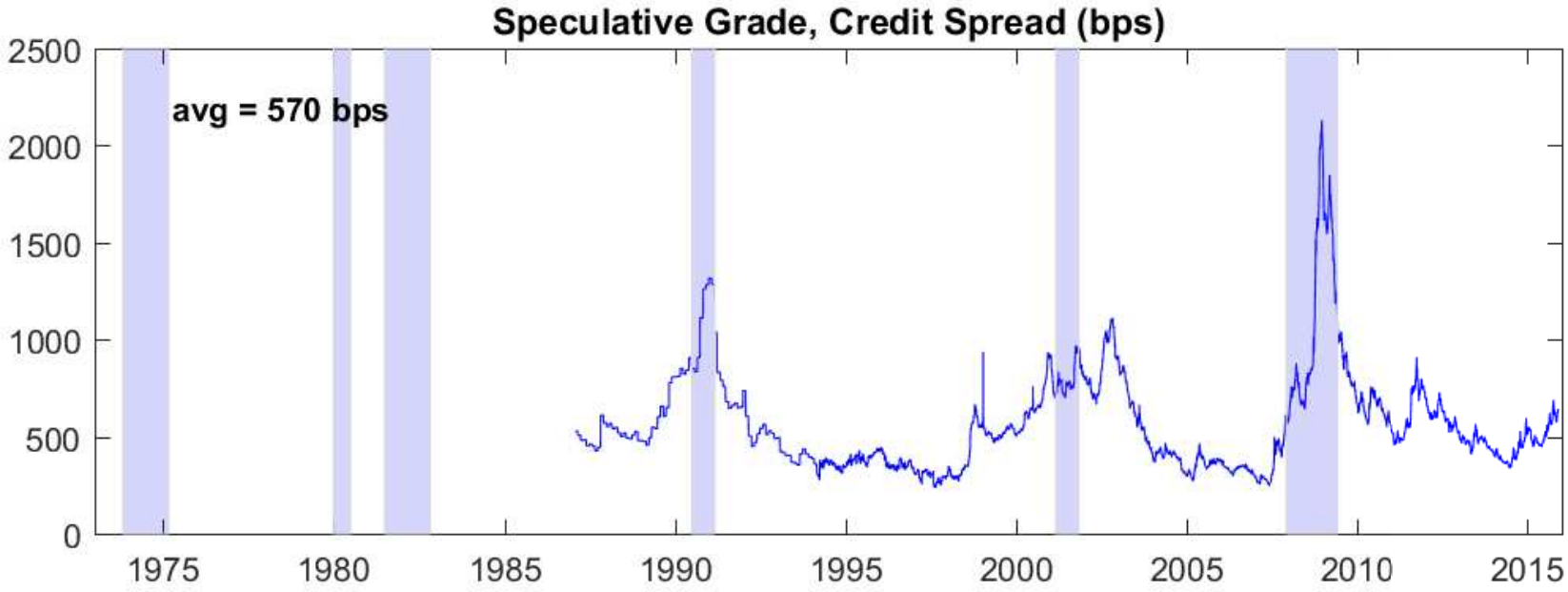
sifma
Invested in America

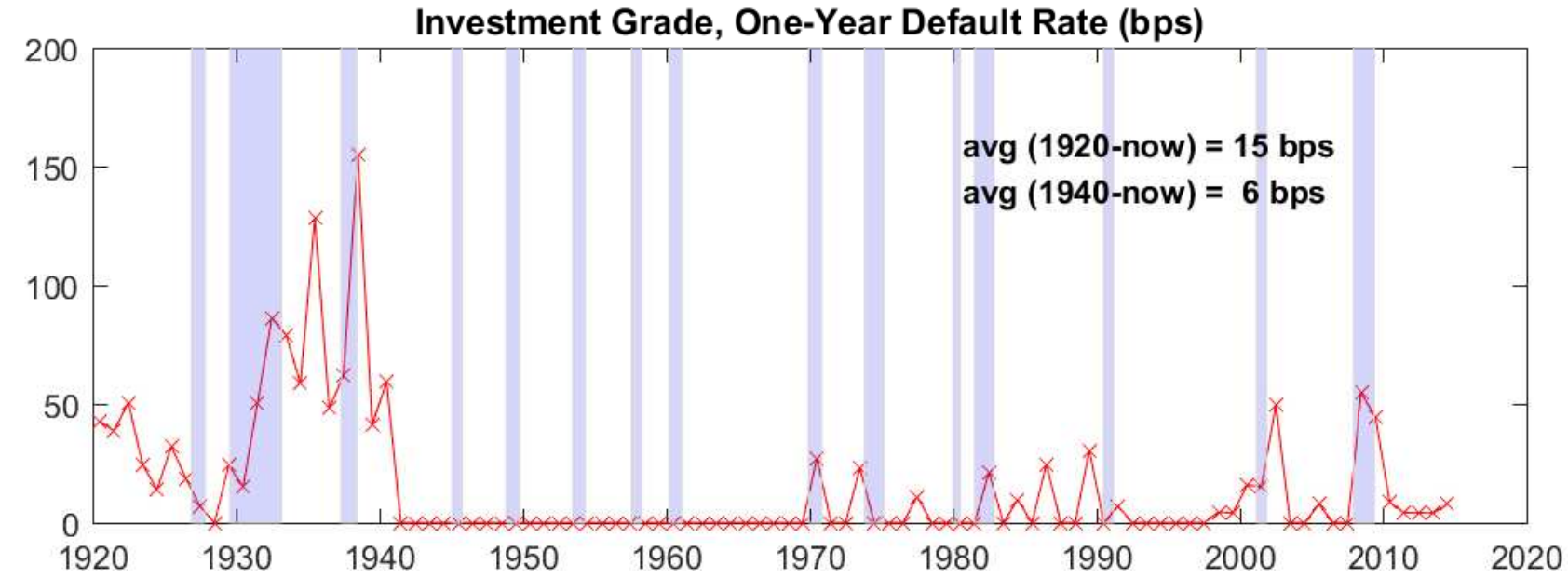
	Municipal	Treasury	Agency MBS	Non-Agency MBS	ABS	Corporate Debt[1]	Federal Agency Securities	Total
1996	1.1	203.7	38.1	-	-	-	31.1	274.0
1997	1.1	212.1	47.1	-	-	-	40.2	300.5
1998	3.3	226.6	70.9	-	-	-	47.6	348.5
1999	8.3	186.5	67.1	-	-	-	54.5	316.5
2000	8.8	206.5	69.5	-	-	-	72.8	357.6
2001	8.8	297.9	112.0	-	-	-	90.2	508.9
2002	10.7	366.4	154.5	-	-	17.8	81.8	631.2
2003	12.6	433.5	206.0	-	-	18.0	81.7	751.8
2004	14.8	499.0	207.4	-	-	17.3	78.8	817.3
2005	16.9	554.5	251.8	-	-	16.6	78.8	918.6
2006	23.1	524.7	254.6	-	-	16.9	74.4	893.7
2007	25.1	570.2	320.1	-	-	16.4	83.0	1,014.9
2008	19.4	553.1	344.9	-	-	14.3	104.5	1,036.1
2009	12.5	407.9	299.9	-	-	19.9	77.7	817.8
2010	13.3	528.2	320.6	-	-	20.5	11.2	893.7
2011	11.3	567.8	243.3	4.4	1.5	20.6	9.6	853.5
2012	11.3	518.9	280.4	4.5	1.5	22.6	9.7	849.0
2013	11.2	545.4	222.8	4.1	1.3	24.7	6.5	816.0
2014	9.9	504.2	178.0	3.7	1.5	26.7	5.3	729.2
2015	8.6	490.1	194.4	3.1	1.4	26.6	4.5	728.7

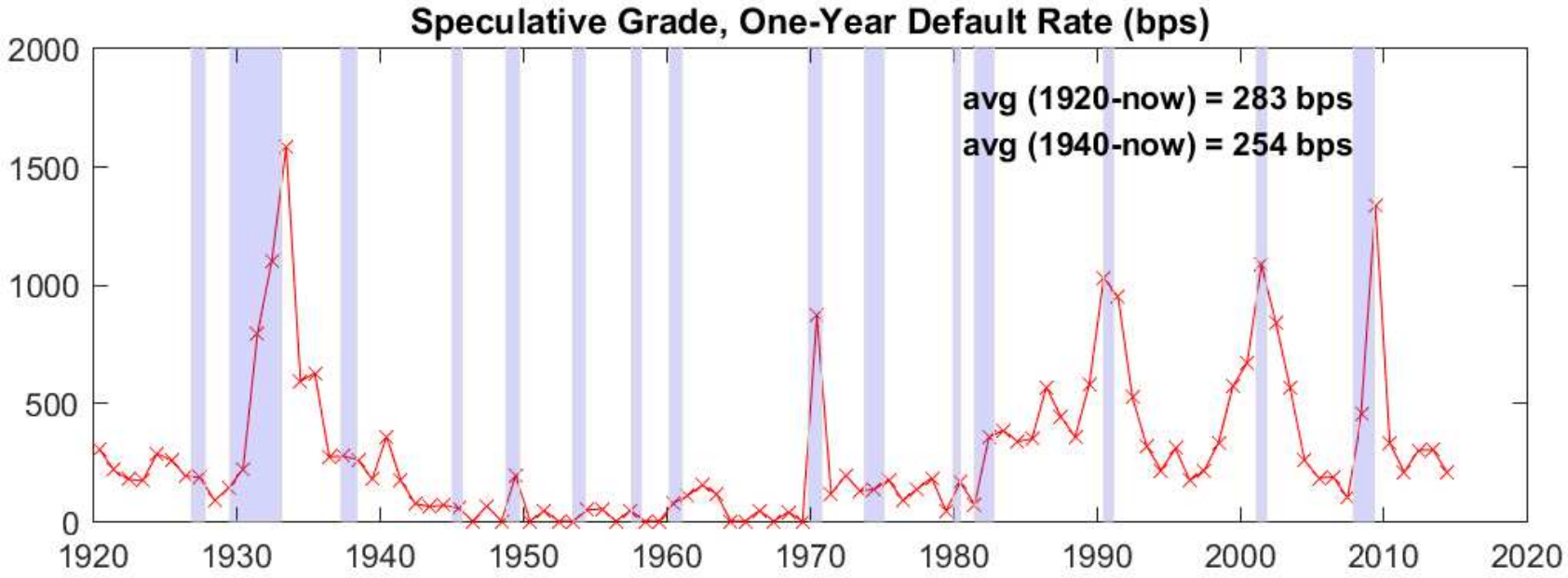
Average daily dollar trading volume in September 2015:

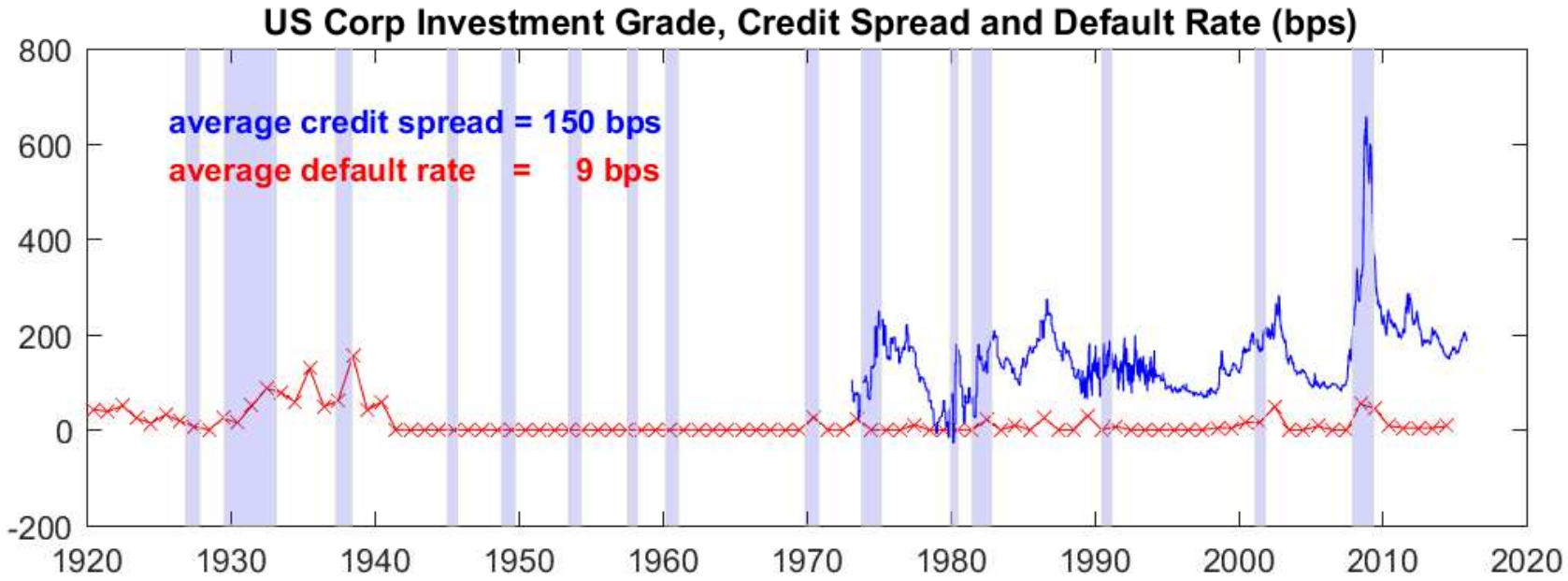
Equity **$321bn**, Treasury **$499bn**, and Corporate Bonds **$25bn**.

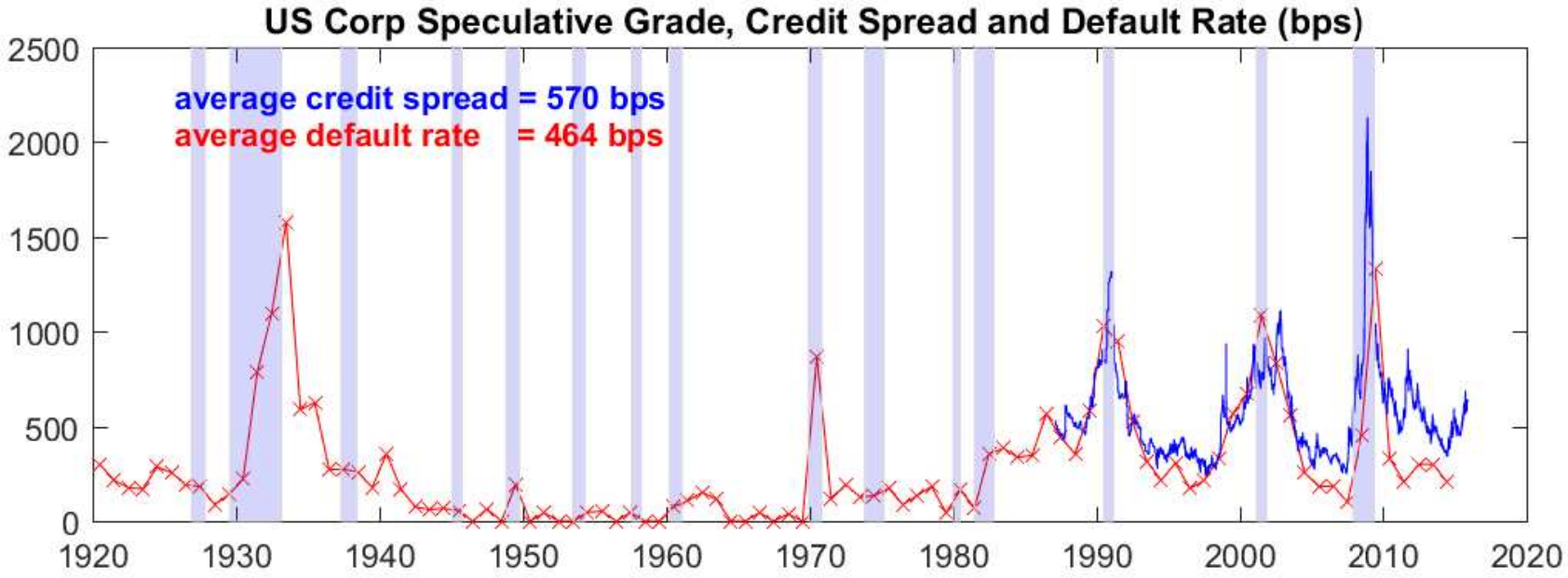

Investment Grade, Credit Spread (bps)
avg = 150 bps
800
600
400
200
0
-200
1975
1980
1985
1990
1995
2000
2005
2010
2015


Speculative Grade, Credit Spread (bps)
avg = 570 bps
2500
2000
1500
1000
500
0
1975
1980
1985
1990
1995
2000
2005
2010
2015


Investment Grade, One-Year Default Rate (bps)
avg (1920-now) = 15 bps
avg (1940-now) = 6 bps
200
150
100
50
0
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020


Speculative Grade, One-Year Default Rate (bps)
avg (1920-now) = 283 bps
avg (1940-now) = 254 bps
2000
1500
1000
500
0
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020


US Corp Investment Grade, Credit Spread and Default Rate (bps)
average credit spread = 150 bps
average default rate = 9 bps
800
600
400
200
0
-200
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020


US Corp Speculative Grade, Credit Spread and Default Rate (bps)
average credit spread = 570 bps
average default rate = 464 bps
2500
2000
1500
1000
500
0
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020

Defaults in 2008, by Industry Distribution

Broad Industry	Percent of Issuer Counts	Percent of Dollar Volume
Banking	8.9%	25.4%
Capital industries	29.7%	5.2%
Consumer Industries	22.8%	5.1%
Energy & Environment	6.9%	2.0%
FIRE	9.9%	53.8%
Media & Publishing	8.9%	6.7%
Retail & Distribution	4.0%	0.7%
Technology	5.0%	0.8%
Transportation	4.0%	0.4%
Utilities	0.0%	0.0%
2008 Total	101	$281.2 bil

- Lehman was the largest default in history: $120.2B.
- 84 of the 101 defaulters were in North American with 74 in the US.
- North American defaulted debt volumes: $226.2B.

Defaults in 2008, by Financial Institutions

Senior Unsecured Bond Recovery Rates for Financial Institution Defaults in 2008[1]

Company	Domain	Default Volume ($Mil)	Sr. Unsecured Bond Recovery
Lehman Brothers Holdings, Inc.	United States	120,164	9.3%
Kaupthing Bank hf	Iceland	20,063	4.0%
Glitnir banki hf	Iceland	18,773	3.0%
GMAC LLC	United States	17,190	69.9%
Washington Mutual Bank	United States	13,600	26.5%
Residential Capital, LLC	United States	12,315	51.7%
Landsbanki Islands hf	Iceland	12,161	3.0%
Washington Mutual, Inc.	United States	5,746	57.0%
GMAC of Canada Ltd	Canada	265	70.7%
Downey Financial Corp.	United States	200	0.5%
Fremont General Corporation	United States	166	46.0%
Luminent Mortgage Capital, Inc.	United States	131	27.3%
Triad Financial Corporation	United States	89	76.5%
Franklin Bank Corp.	United States	80	0.0%
GMAC International Finance B.V.	Netherlands	51	85.5%
Average	35.4%	Median	27.3%

1. Based on 30-day post-default market prices.

Recovery Rates based on debt trading prices 30 days after the default date

EXHIBIT 7

Average corporate debt recovery rates measured by post-default trading prices

	Issuer-weighted			Volume-weighted		
Lien Position	2014	2013	1982-2014	2014	2013	1982-2014
1st Lien Bank Loan	78.4%	75.1%	66.6%	80.6%	67.7%	62.5%
2nd Lien Bank Loan*	10.5%	78.7%	31.8%	10.5%	69.2%	28.5%
Sr. Unsecured Bank Loan	n.a.	n.a.	47.1%	n.a.	n.a.	40.2%
Sr. Secured Bond	59.5%	59.8%	52.8%	76.5%	59.5%	52.4%
Sr. Unsecured Bond	43.3%	43.8%	37.4%	34.3%	29.2%	33.6%
Sr. Subordinated Bond*	46.9%	20.7%	31.1%	28.3%	26.6%	26.0%
Subordinated Bond**	38.8%	26.4%	31.4%	38.0%	33.7%	26.3%
Jr. Subordinated Bond	n.a.	n.a.	24.7%	n.a.	n.a.	17.1%

* The average recovery rates for 2014's and 2013's second lien bank loans and senior subordinated bonds were each based on fewer than five defaults.

** The average recovery rates for 2014's subordinated bonds were based on fewer than five defaults.

Recovery Rates based on ultimate recoveries

EXHIBIT 8

Average corporate debt recovery rates measured by ultimate recoveries, 1987-2014

	Emergence Year			Default Year		
Lien Position	2014	2013	1987-2014	2014	2013	1987-2014
Loans*	81.0%	76.7%	80.2%	68.4%	76.6%	80.2%
Senior Secured Bonds**	57.1%	84.2%	63.0%	59.4%	56.9%	63.0%
Senior Unsecured Bonds***	44.6%	61.3%	48.8%	0.0%	34.4%	48.8%
Subordinated Bonds	0.0%	21.0%	28.2%	0.0%	21.0%	28.2%

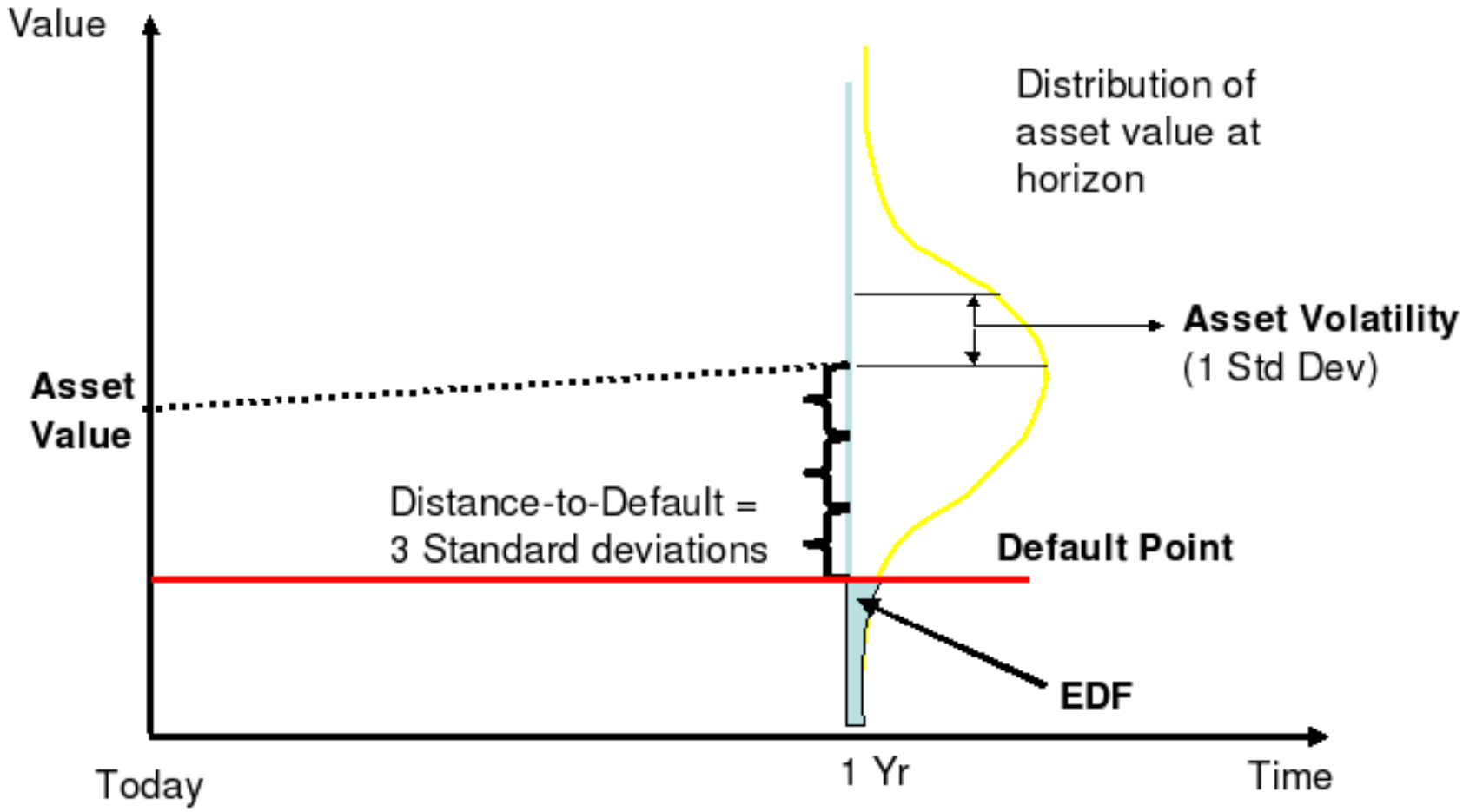
Various Sources to Estimate Default Probability

- We will now focus on modeling and estimating the default probability, while keeping the recovery rate, which is 1 minus the loss rate, at a constant level.
- Information about default probability can be collected from:
  - **Rating:** credit ratings (S&P, Moody's, Fitch) and the historical default rates by rating category.
  - **Equity Market:** firm fundamentals, financial statements and equity-market information (Moody's KMV).
  - **Credit Market:** the market prices of securities with exposure to default risk: corporate bond yield spreads or credit-default swap (CDS) spreads.

Models of Default

- Structural approach: start with the firm's fundamentals, such as firm value or earnings. Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995), and Leland (1994).
- Reduced-form approach: treat default as the outcome of a jump process. Duffie and Singleton (1999).
- For the purpose of pricing a defaultable bond, the main output of a defaultable model is the term structure of probability of default.
- When defaultable securities are pooled together, then the valuation of the pooled security involves crucially on the model-implied probability of correlated default.

Model Default using Structural Approach



Distance to Default:

$$DD = \frac{\ln(V/K) + (\mu - \sigma_A^2/2)\, t}{\sigma_A \sqrt{t}}$$

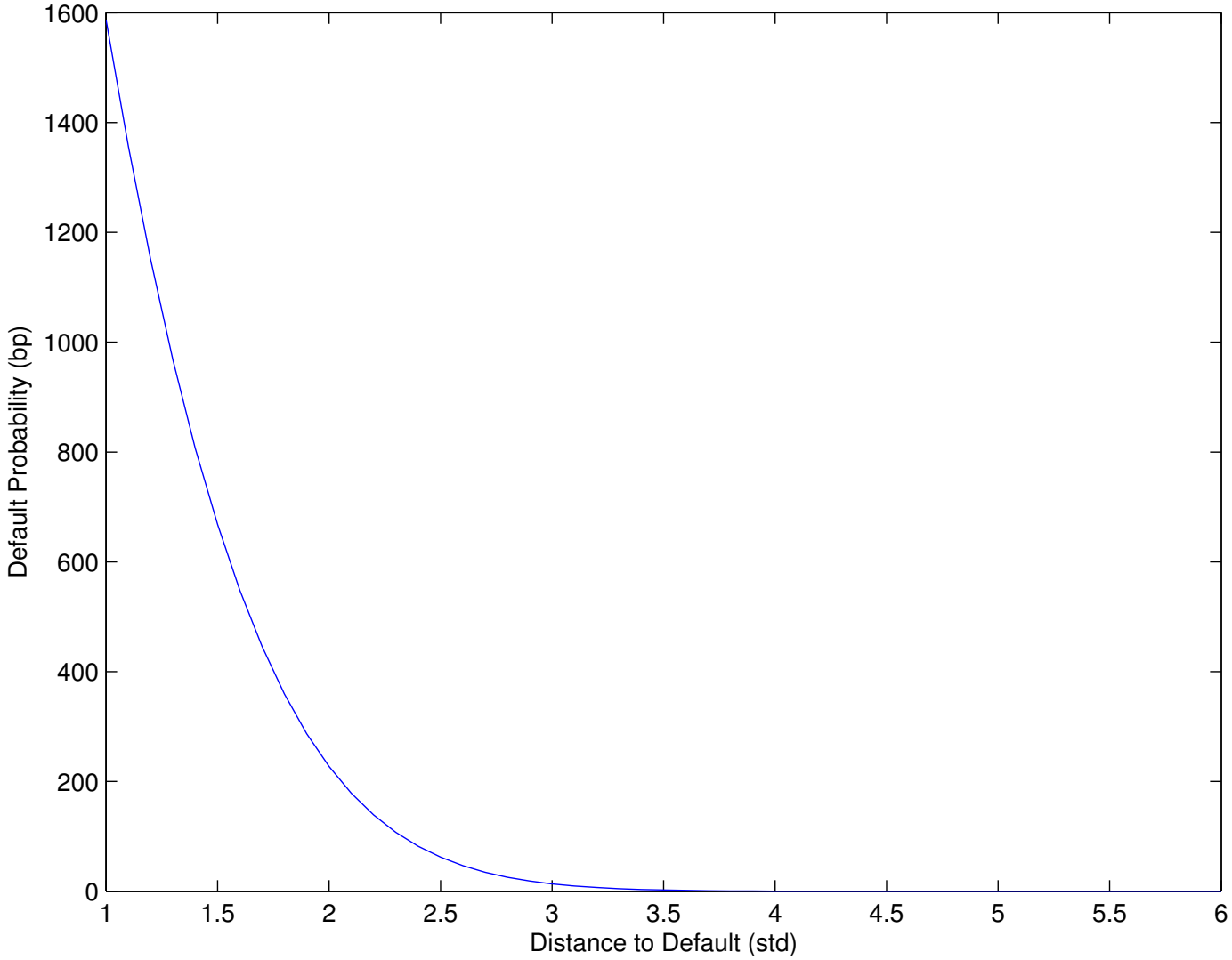
- The market value of assets V
- Asset volatility σ_A
- The book value of liabilities K
- Debt-to-Asset ratio: K/V
- The expected growth rate of asset value μ
- DD = distance to default = number of std the firm is away from default
- t = the time horizon

Calculating Distance to Default

debt ratio K/V	asset growth μ	asset vol σ_A	horizon t	distance to default DD
15%	10%	40%	1yr	4.79
15%	10%	40%	10yr	1.66
50%	10%	40%	1yr	1.78
15%	10%	20%	1yr	9.89
15%	10%	20%	10yr	4.26
15%	20%	40%	1yr	5.04

From Distance to Default to Default Probability

Assuming normal distribution: Default Probability $= N(-DD)$

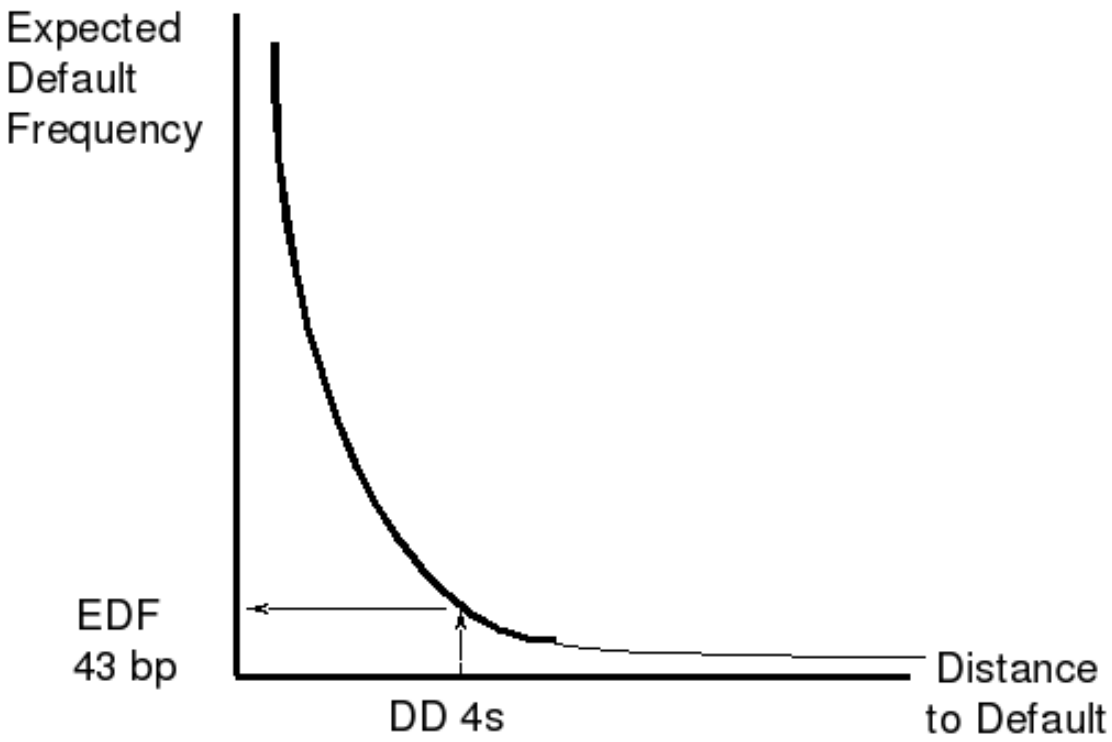


From Distance to Default to Moody KMV's EDF

(Expected Default Frequency)

- The probability from normal distribution is too low and credit risk is not normal.
- Moody's KMV uses actual default rates for companies in similar risk ranges to determine a mapping from DD to EDF.
- This procedure requires a large database of actual defaults.

Moody's EDF


Expected
Default
Frequency
EDF
43 bp
DD 4s
Distance
to Default

Model Default using Reduced-Form Approach

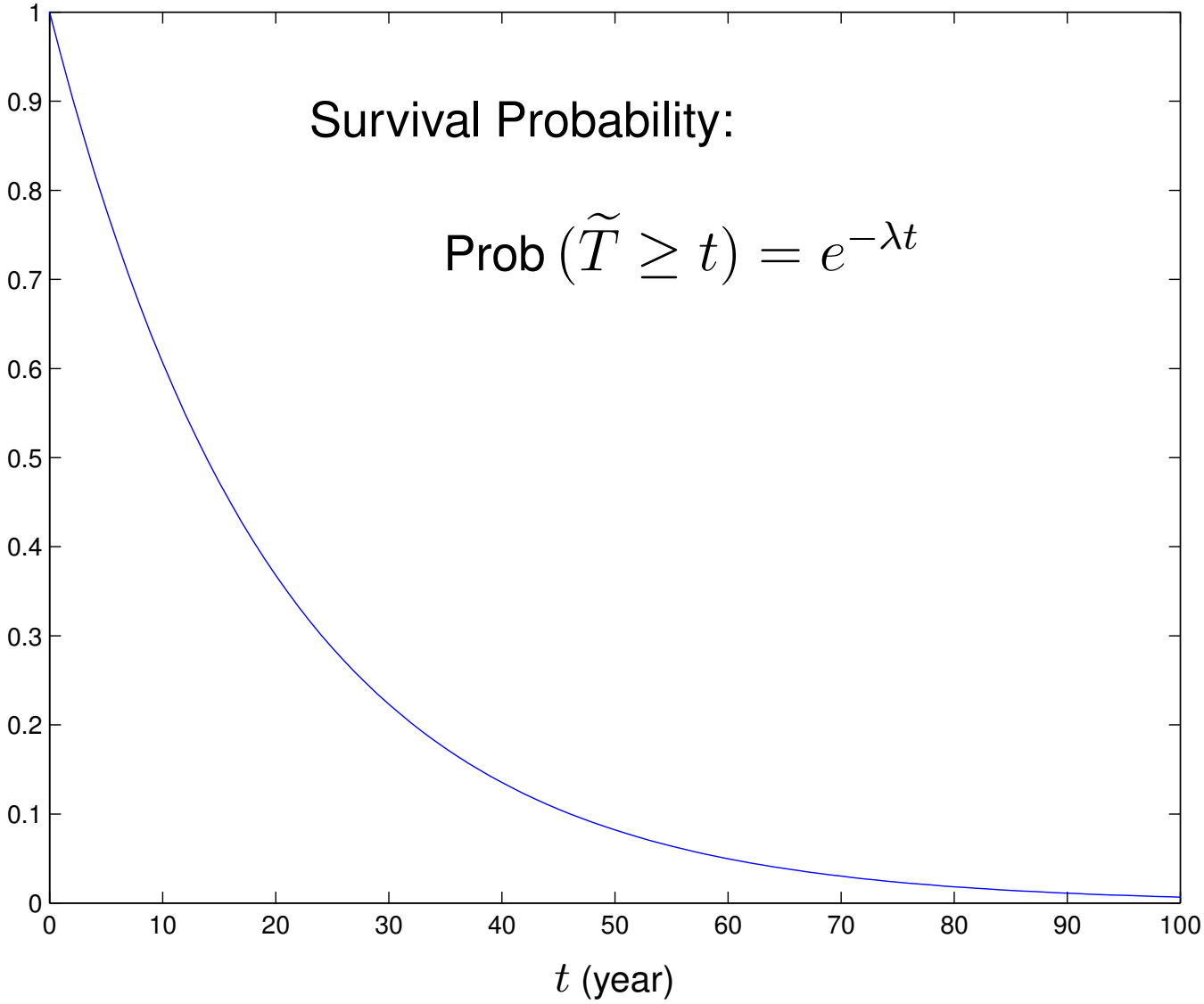
- Let $\widetilde{T}$ be the random default time.
- The probability of survival up to time t:

$$\text{Prob}\left(\widetilde{T} \geq t\right)$$

- The probability of default before time t:

$$\text{Prob}\left(\widetilde{T} < t\right) = 1 - \text{Prob}\left(\widetilde{T} \geq t\right)$$

Constant Default Intensity λ



Pricing a Zero-Coupon Bond

- Assume the constant default intensity λ of a firm is 100 bps.
- **The one-year default probability:** $1 - e^{-\lambda} \approx \lambda$**.**
- Assume zero recovery (100% loss given default):
  - Consider a one-year zero-coupon bond with \$1 face value:

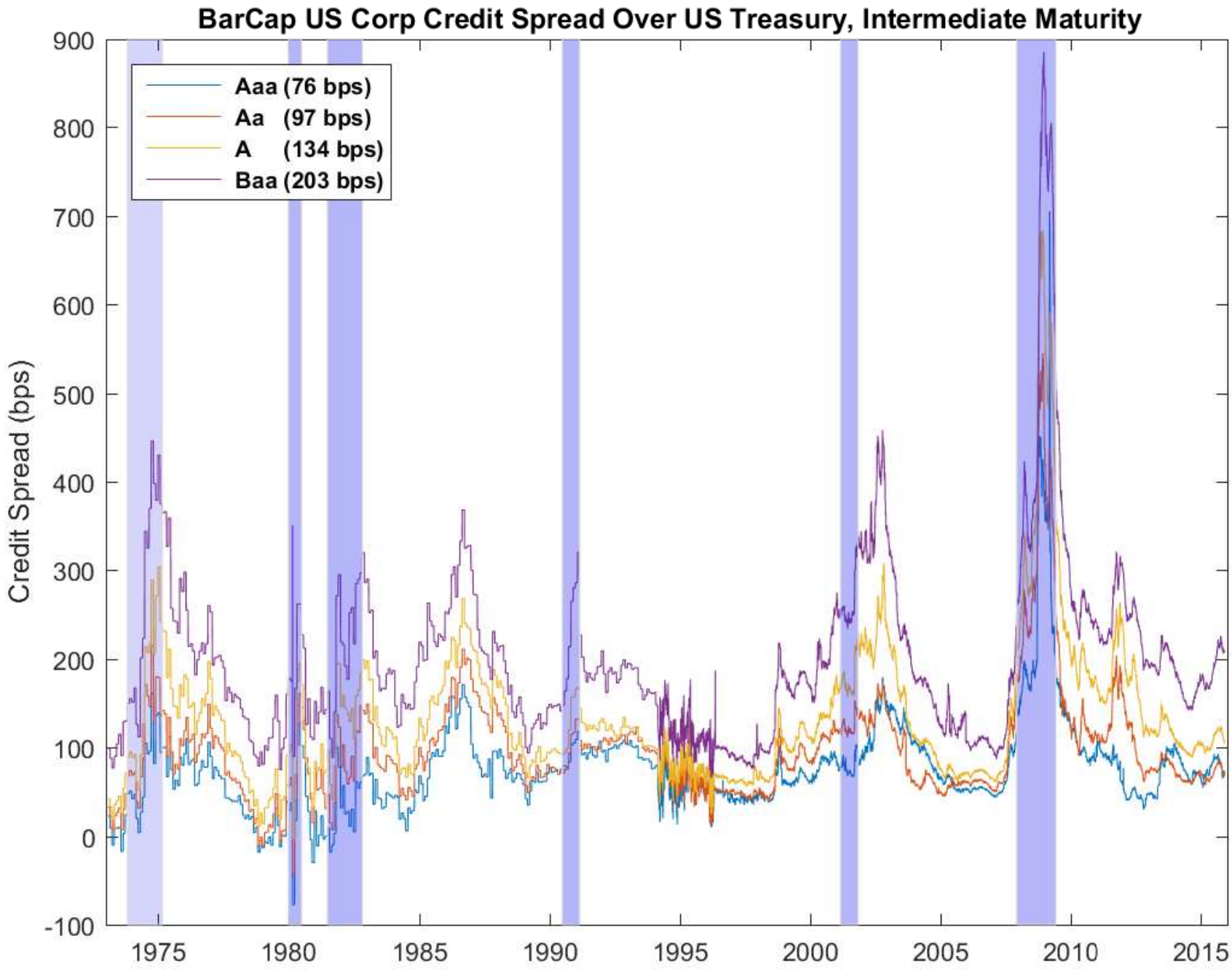
$$P = e^{-r} \text{ Prob } (\widetilde{T} > 1) = e^{-r} \times e^{-\lambda} = e^{-(r+\lambda)}$$

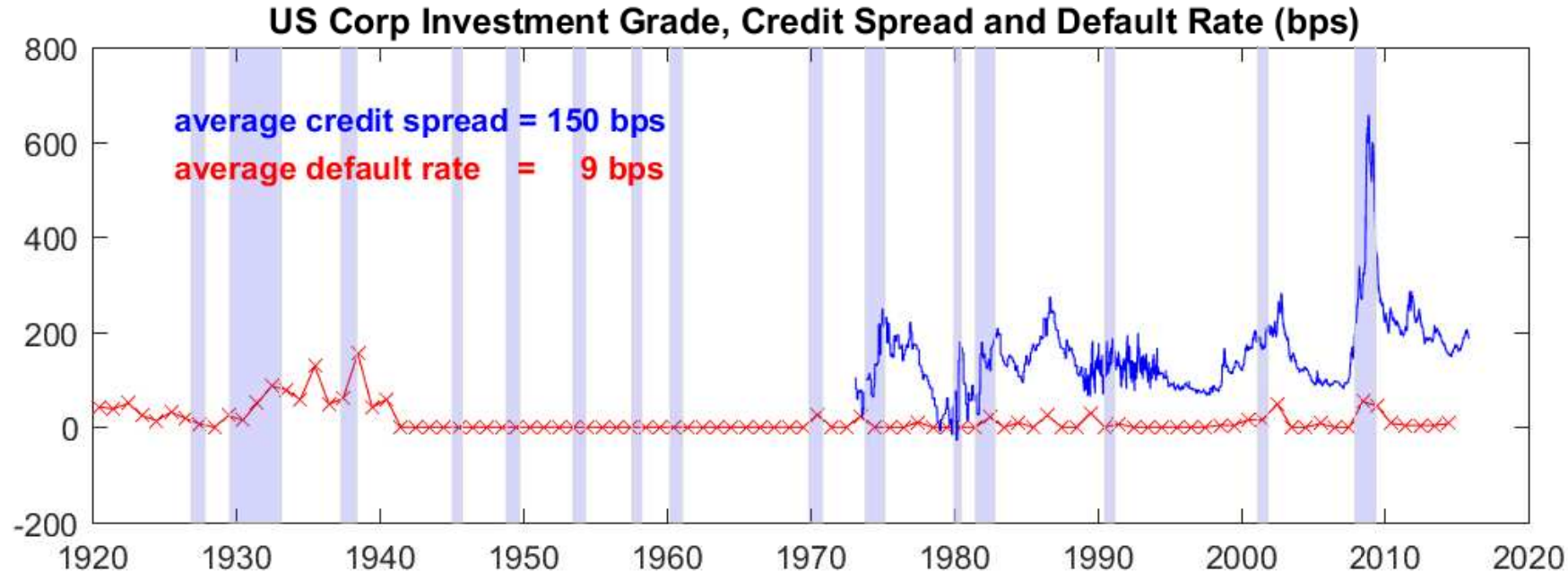
  - The yield to maturity of the defaultable bond: $r + \lambda$
  - **The credit spread of the defaultable bond:** λ
- Assume constant loss given default (L):

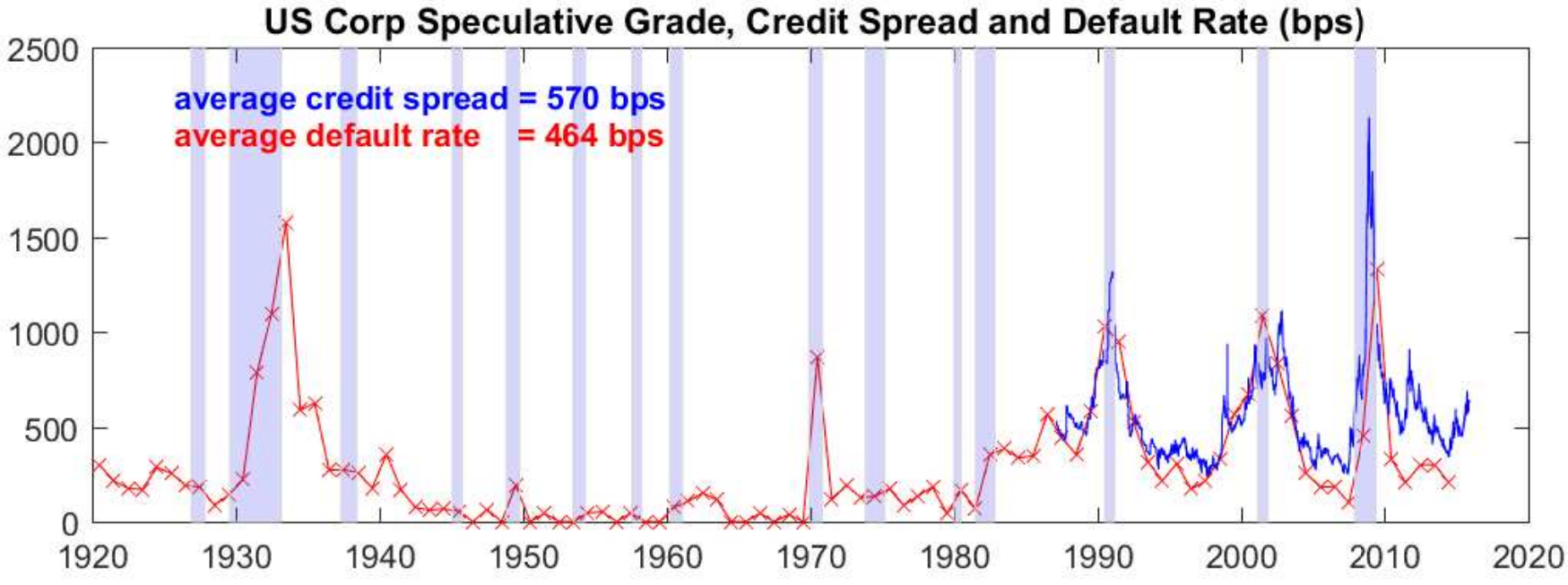
$$P = e^{-r} \text{ Prob } (\widetilde{T} > 1) + e^{-r} \text{ Prob } (\widetilde{T} \leq 1) \times (1 - L)$$

$$= e^{-r} \times e^{-\lambda} + e^{-r} \times (1 - e^{-\lambda}) \times (1 - L)$$

For small λ**, the credit spread is approximately:** $\lambda \times L$.

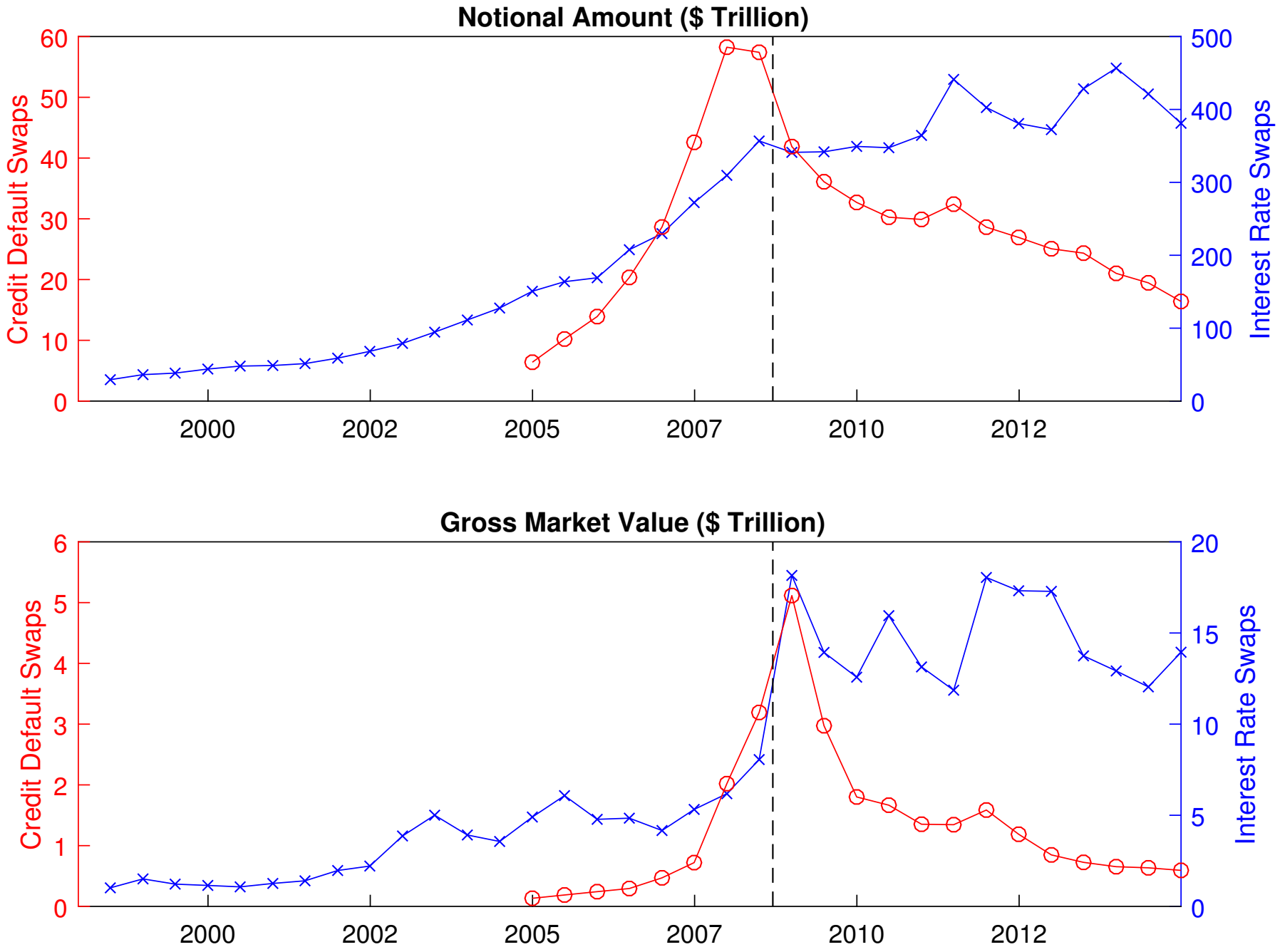

BarCap US Corp Credit Spread Over US Treasury, Intermediate Maturity
Aaa (76 bps)
Aa (97 bps)
A (134 bps)
Baa (203 bps)
Credit Spread (bps)
900
800
700
600
500
400
300
200
100
0
-100
1975
1980
1985
1990
1995
2000
2005
2010
2015


US Corp Investment Grade, Credit Spread and Default Rate (bps)
average credit spread = 150 bps
average default rate = 9 bps
800
600
400
200
0
-200
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020


US Corp Speculative Grade, Credit Spread and Default Rate (bps)
average credit spread = 570 bps
average default rate = 464 bps
2500
2000
1500
1000
500
0
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020

Credit Default Swaps

- The US corporate bond market is among the most illiquid markets. For a market of $8T in 2015, the average daily trading volume is only $25B. By comparison, the average daily volume is $499B for US Treasury and $321B for US Equity.
- In buying a corporate bond, investors take on both duration and credit exposures. To have a pure positive exposure to credit risk, investors have to hedge out the duration risk. To have a pure negative exposure to credit risk, investors have to locate, borrow, and then sell the bonds and buy back the duration exposure.
- The emergence of credit derivatives was in part a response to the limitations of corporate bonds as a vehicle for credit risk.


Notional Amount ($ Trillion)
Credit Default Swaps
Interest Rate Swaps
0
10
20
30
40
50
60
0
100
200
300
400
500
2000
2002
2005
2007
2010
2012
Gross Market Value ($ Trillion)
Credit Default Swaps
Interest Rate Swaps
0
1
2
3
4
5
6
0
5
10
15
20
2000
2002
2005
2007
2010
2012

Valuation: One-Year Credit Default Swap

- Consider a one-year CDS and assume constant interest rate r.
  - The present value of the annuity:

$$\text{CDS} \times P\left(\tilde{T} > 1\right) \times e^{-r}$$

  - The present value of the insurance:

$$\text{Loss} \times P\left(\tilde{T} \leq 1\right) \times e^{-r}$$

- Set CDS so that the two legs have the same present value:

$$\text{CDS} = \frac{P\left(\tilde{T} \leq 1\right) \times \text{Loss}}{1 - P\left(\tilde{T} \leq 1\right)}$$

- **For small** $P(\tilde{T} \leq 1)$, CDS $\approx$ "1yr Default Probability" $\times$ "Loss"

Applying Constant Default Intensity Model

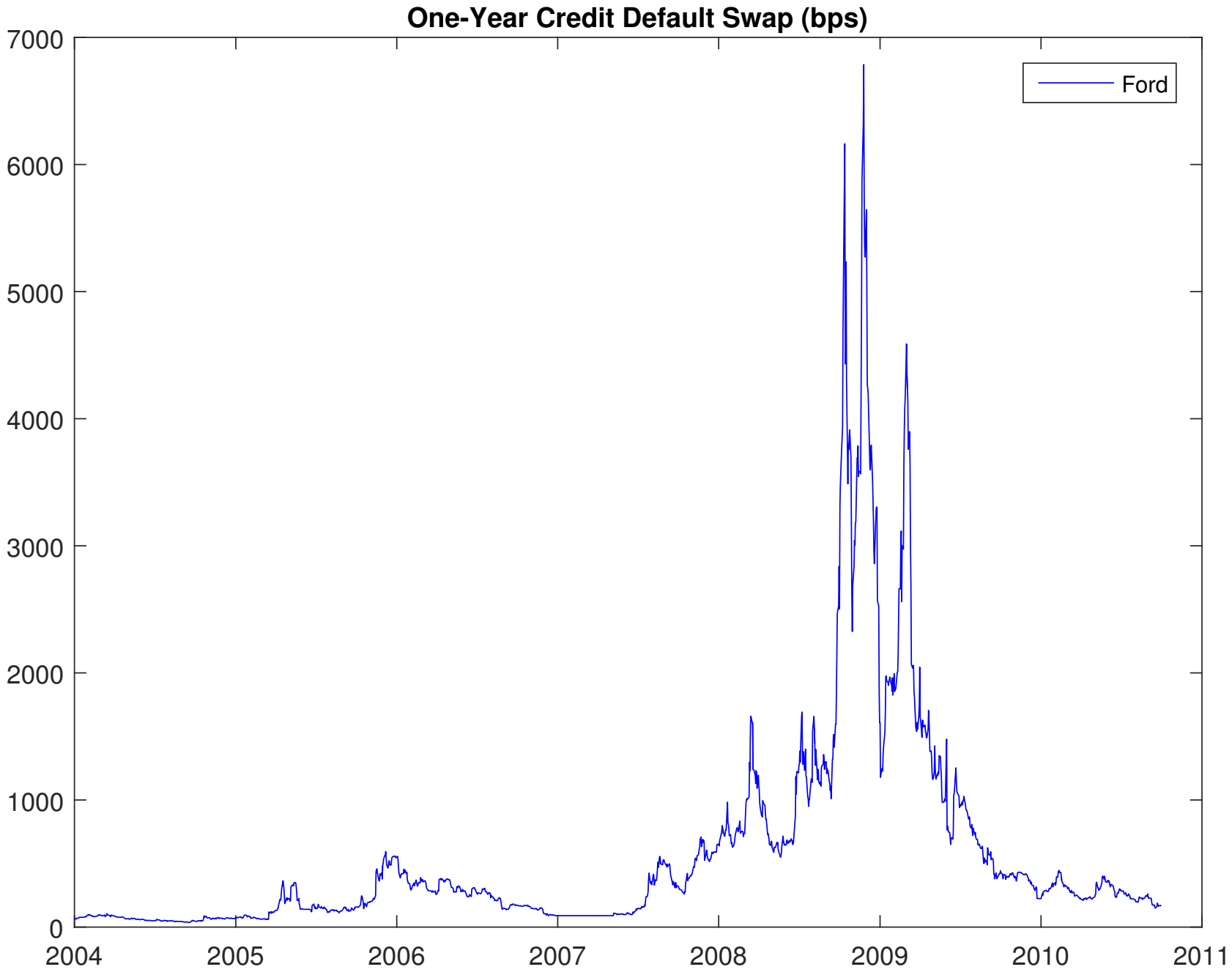
- Let's use the constant default intensity model:

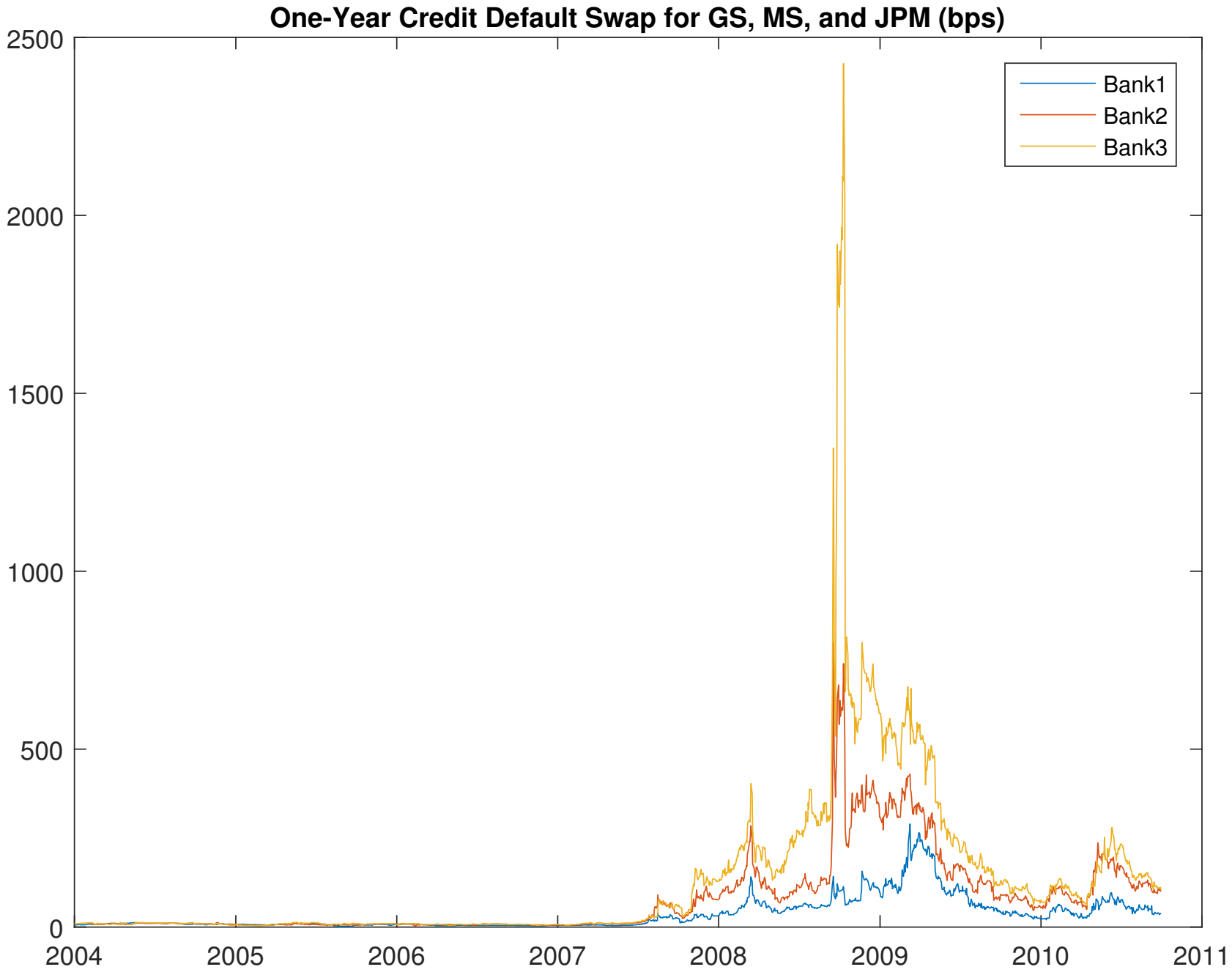
$$\text{one-year default probability} = 1 - e^{-\lambda}$$

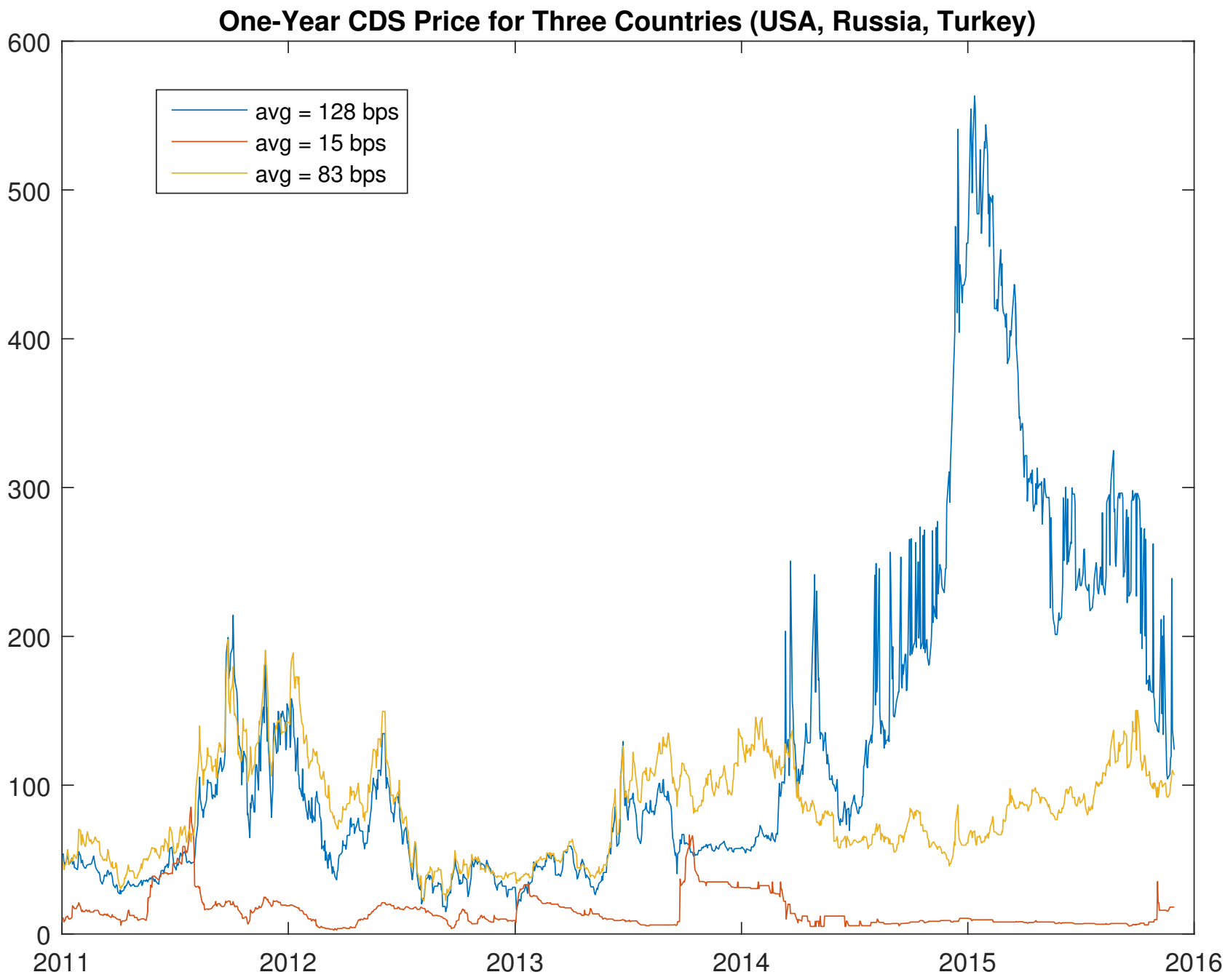
- The one-year CDS price is

$$\text{CDS} = \frac{\left(1 - e^{-\lambda}\right) \times \text{Loss}}{e^{-\lambda}} \approx \lambda \times \text{Loss}\,,$$

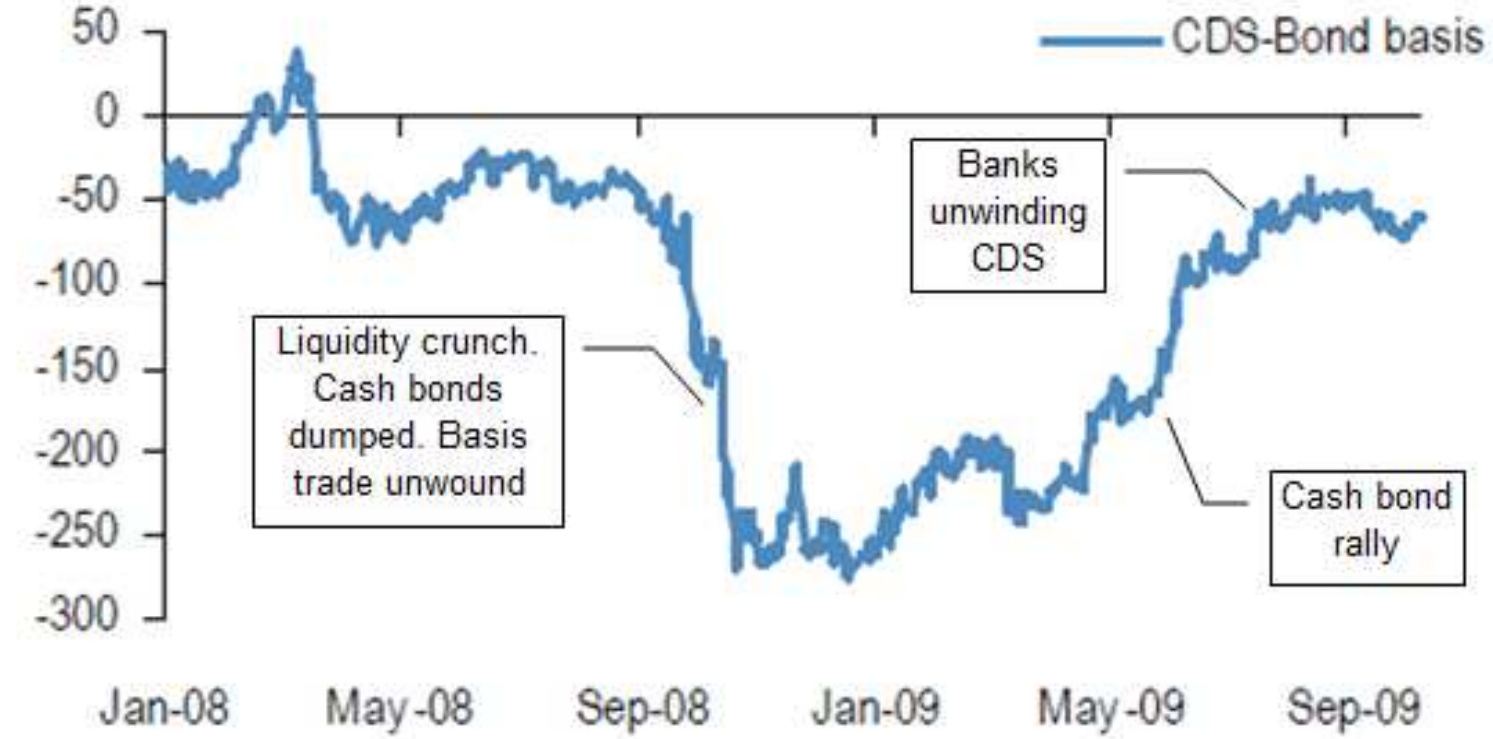
  where the approximation works well for small λ.


One-Year Credit Default Swap (bps)
Ford
7000
6000
5000
4000
3000
2000
1000
0
2004
2005
2006
2007
2008
2009
2010
2011


One-Year Credit Default Swap for GS, MS, and JPM (bps)
2500
2000
1500
1000
500
0
2004
2005
2006
2007
2008
2009
2010
2011
Bank1
Bank2
Bank3


One-Year CDS Price for Three Countries (USA, Russia, Turkey)
avg = 128 bps
avg = 15 bps
avg = 83 bps
600
500
400
300
200
100
0
2011
2012
2013
2014
2015
2016

CDS-Bond Basis



Source: J.P. Morgan.