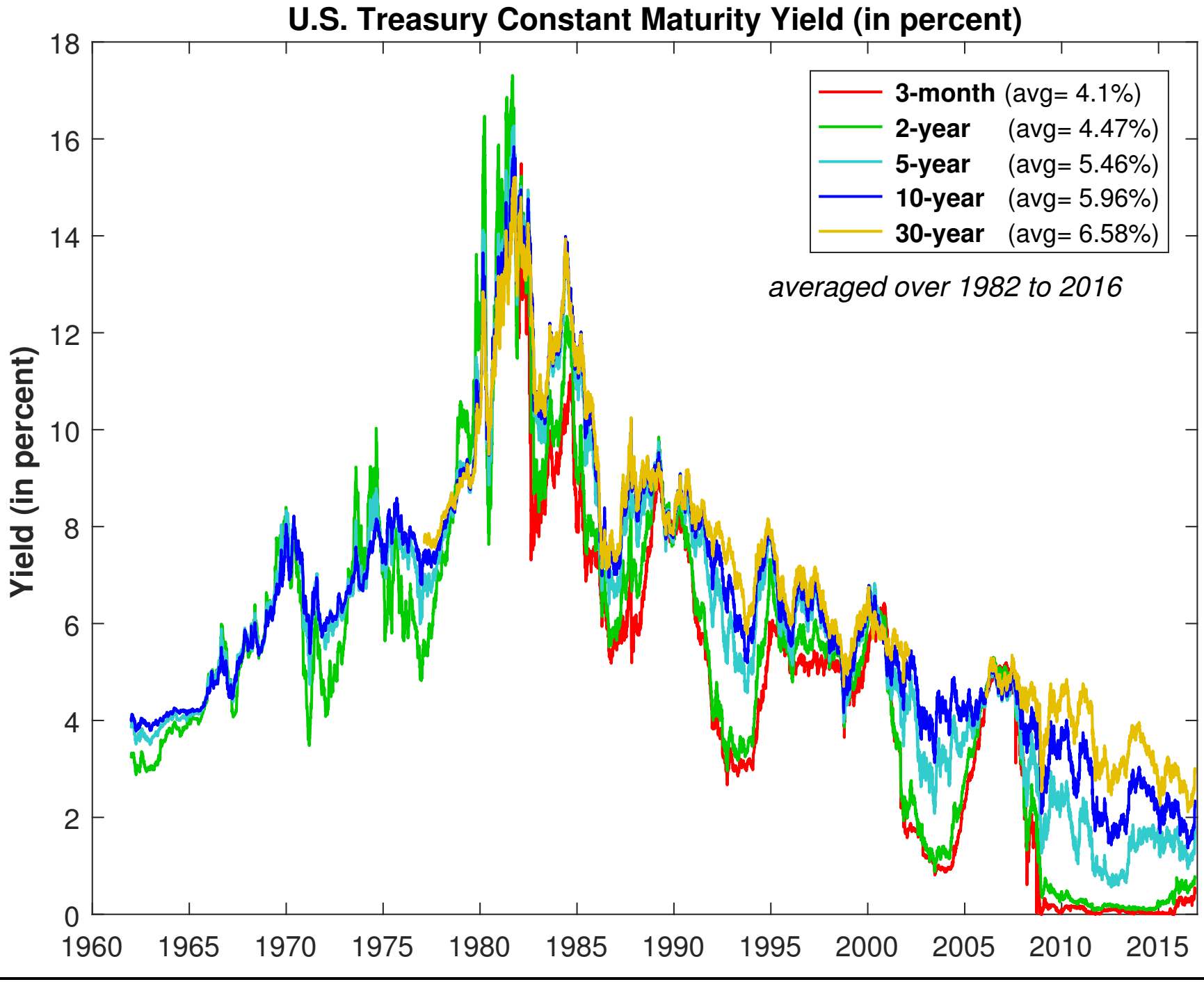
Fixed Income, Yield Curve

15.433 Financial Markets

November 16, 2017

Outline

- Factors influencing the yield curve:
  - Economic factors: monetary policy, expectations on inflation and economic growth, etc.
  - Institutional reasons: next class.
- Statistical analysis of the yield curve: level, slope, and curvature.


U.S. Treasury Constant Maturity Yield (in percent)
Yield (in percent)
18
16
14
12
10
8
6
4
2
0
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
3-month (avg= 4.1%)
2-year (avg= 4.47%)
5-year (avg= 5.46%)
10-year (avg= 5.96%)
30-year (avg= 6.58%)
averaged over 1982 to 2016

Comovement in Yields

- Correlations between yields (daily data from 1982 to 2015):

	3M	2Y	5Y	10Y	30Y
3M	100.0	98.57	96.19	93.61	90.90
2Y	**98.57**	100.0	99.18	97.54	95.47
5Y	96.19	**99.18**	100.0	99.46	98.19
10Y	93.61	97.54	**99.46**	100.0	99.57
30Y	90.90	95.47	98.19	**99.57**	100.0

- Correlations between *daily changes* in yields:

3M	100.0	57.31	46.87	40.18	35.15
2Y	**57.31**	100.0	90.29	82.17	72.90
5Y	46.87	**90.29**	100.0	94.07	85.74
10Y	40.18	82.17	**94.07**	100.0	93.71
30Y	35.15	72.90	85.74	**93.71**	100.0

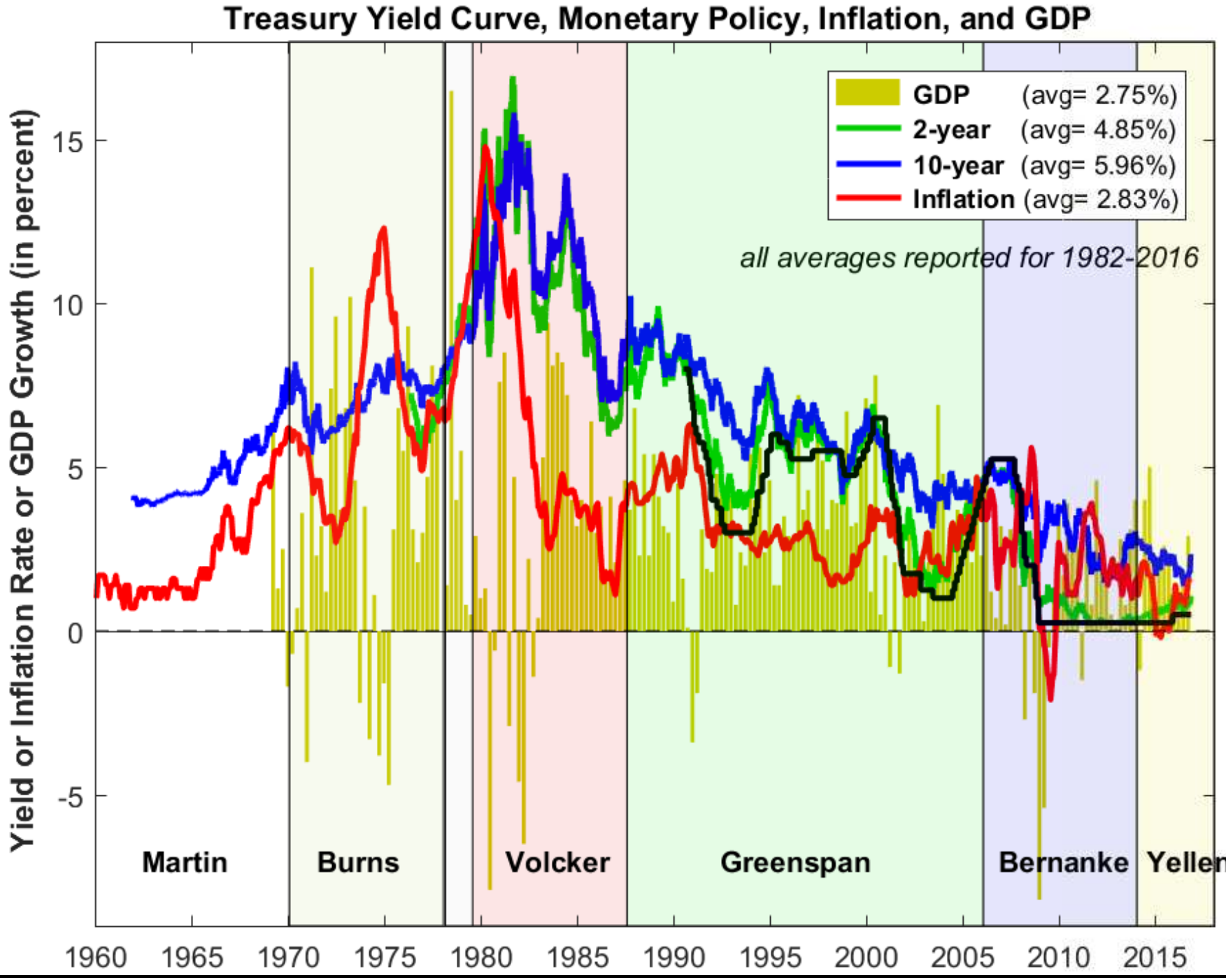
- Yields between the nearest maturities are always more correlated.

  Weaker connections between Treasury bills and the rest of the curve.

The Determinants of the Yield Curve

Some often used explanations (not mutually exclusive):

- Investor's expectations of future interest rates.
- Premiums required by investors to hold long-term bonds: risk premium or liquidity preference.
- Monetary policy: fed funds rate and securities purchasing programs (quantitative easings and operation twist).
- Expectations of future macroeconomic conditions: economic growth and inflation.
- Fiscal policy: budget surplus or deficit.
- Market segmentation; temporary imbalance of supply and demand; holdings by foreign governments.

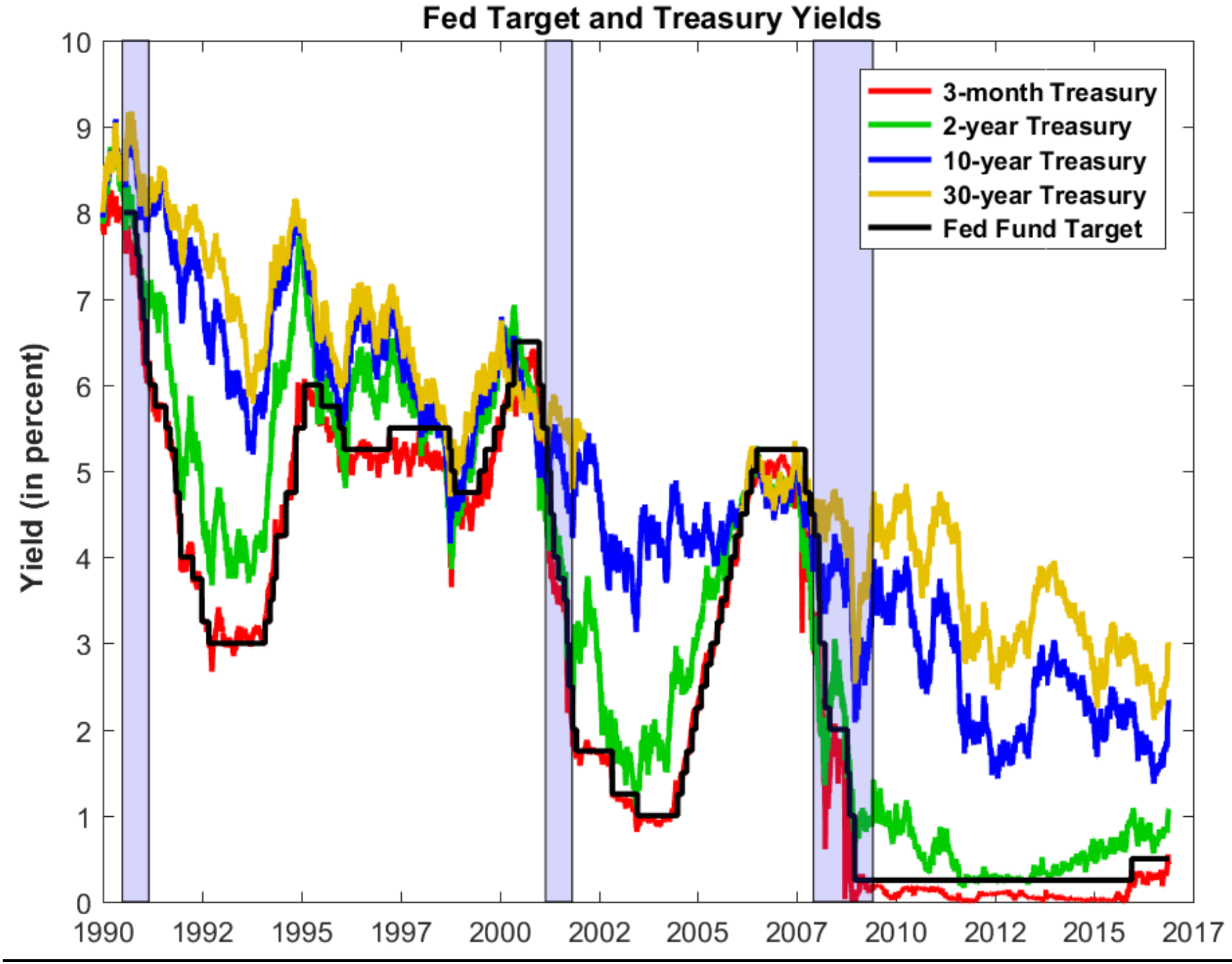

Treasury Yield Curve, Monetary Policy, Inflation, and GDP
GDP (avg= 2.75%)
2-year (avg= 4.85%)
10-year (avg= 5.96%)
Inflation (avg= 2.83%)
all averages reported for 1982-2016
Yield or Inflation Rate or GDP Growth (in percent)
15
10
5
0
-5
Martin
Burns
Volcker
Greenspan
Bernanke
Yellen
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Macroeconomics and the Yield Curve

- The impact of monetary policy on the short-end of the yield curve is direct. Every six weeks, the Federal Open Market Committee (FOMC) meets to decide on the fed funds rate (whether to cut, keep, or increase). This event, watched by all market participants, has a direct impact on the short-end of the yield curve and the overall liquidity of the economy.
- The macroeconomic determinants of long-term interest rates are not as clear. It is typically believed that the long-term interest rates are sensitive to inflation concerns.
- There is also evidence linking the slope of the yield curve to future economic conditions: A sharply upward sloping yield curve has often preceded an economic upturn, a flat yield curve frequently signals an economic slowdown, and an inverted yield curve can be a harbinger of recession.

Fed Funds Rate

- Open market operations–purchases and sales of U.S. Treasury and federal agency securities–are the Federal Reserve's principal tool for implementing monetary policy.
- The federal funds rate is the interest rate at which depository institutions lend balances at the Federal Reserve to other depository institutions overnight.
- Beginning in 1994, the FOMC began announcing changes in its policy stance, and in 1995 it began to explicitly state its target level for the federal funds rate.
- This aspect of monetary policy has an immediate impact on the yield curve (especially the short end), the overall financial markets, and the economy as a whole.

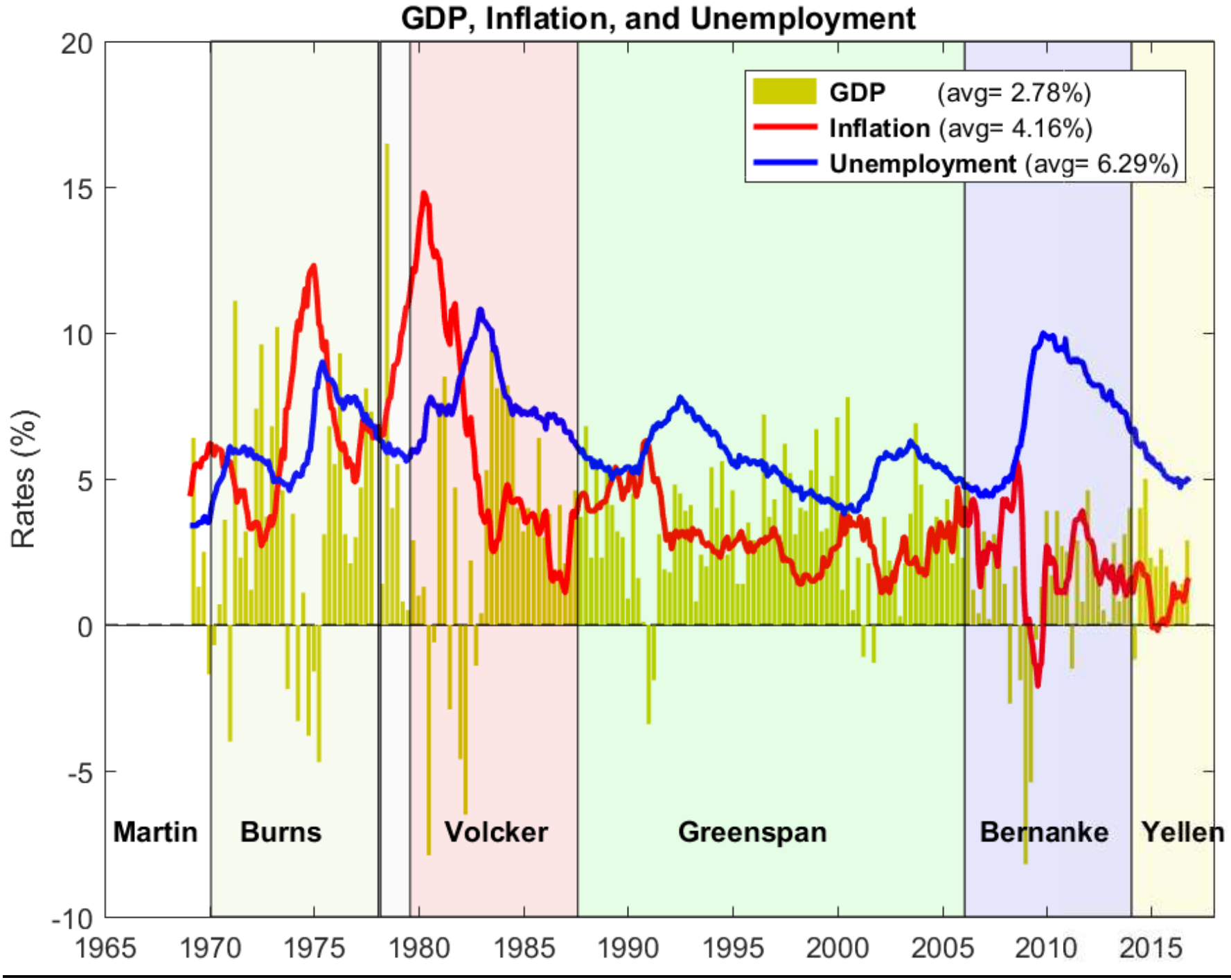

Fed Target and Treasury Yields
Yield (in percent)
0
1
2
3
4
5
6
7
8
9
10
1990
1992
1995
1997
2000
2002
2005
2007
2010
2012
2015
2017
3-month Treasury
2-year Treasury
10-year Treasury
30-year Treasury
Fed Fund Target

Fed Fund Target Rate, Macro Variables, and Yield Curve

- For the Fed, setting the Fed Funds Target Rate is key to an effective monetary policy: price stability and maximum employment.
- The Taylor (1993) Rule:

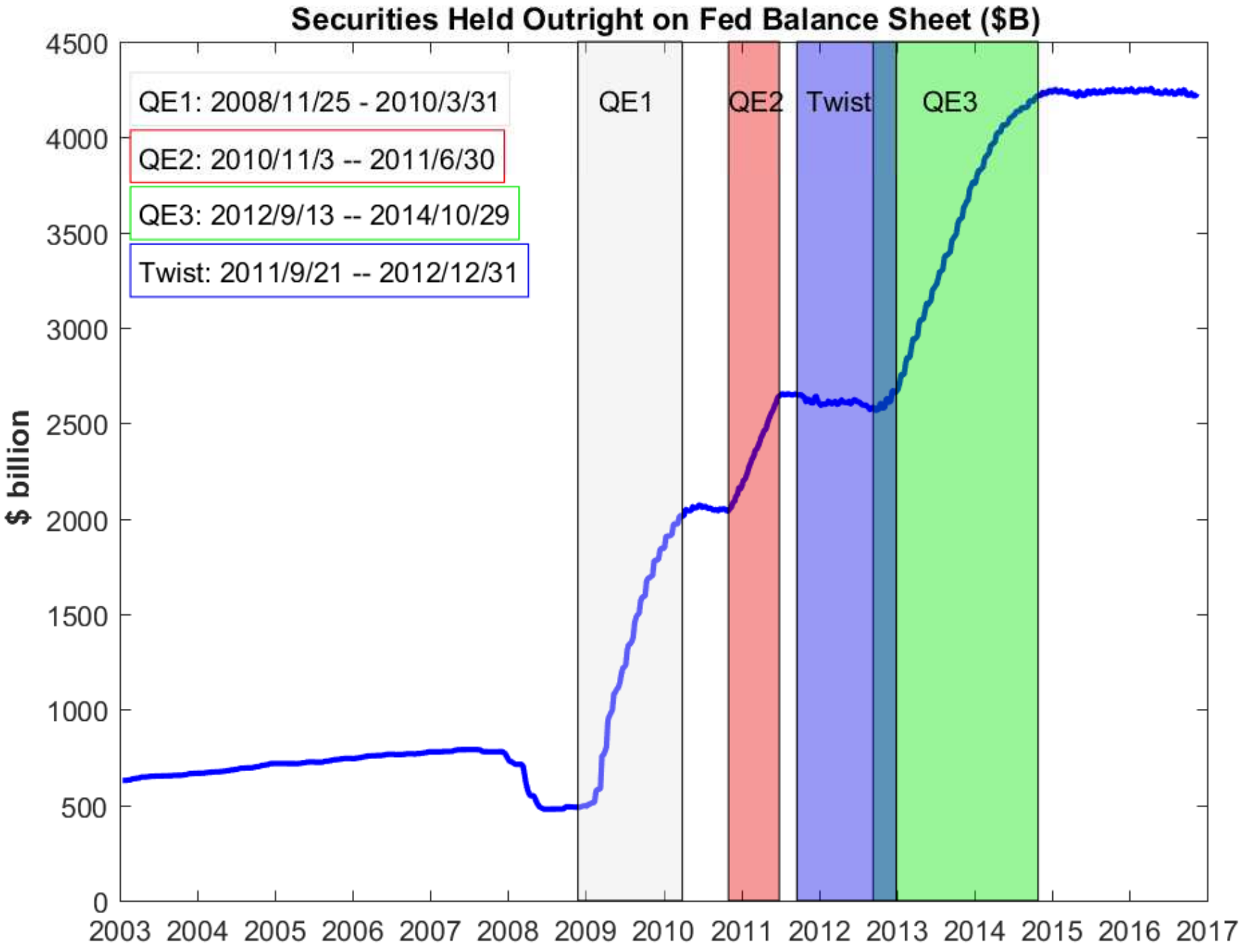
$$r = p + 0.5y + 0.5\,(p - 2) + 2\,,$$

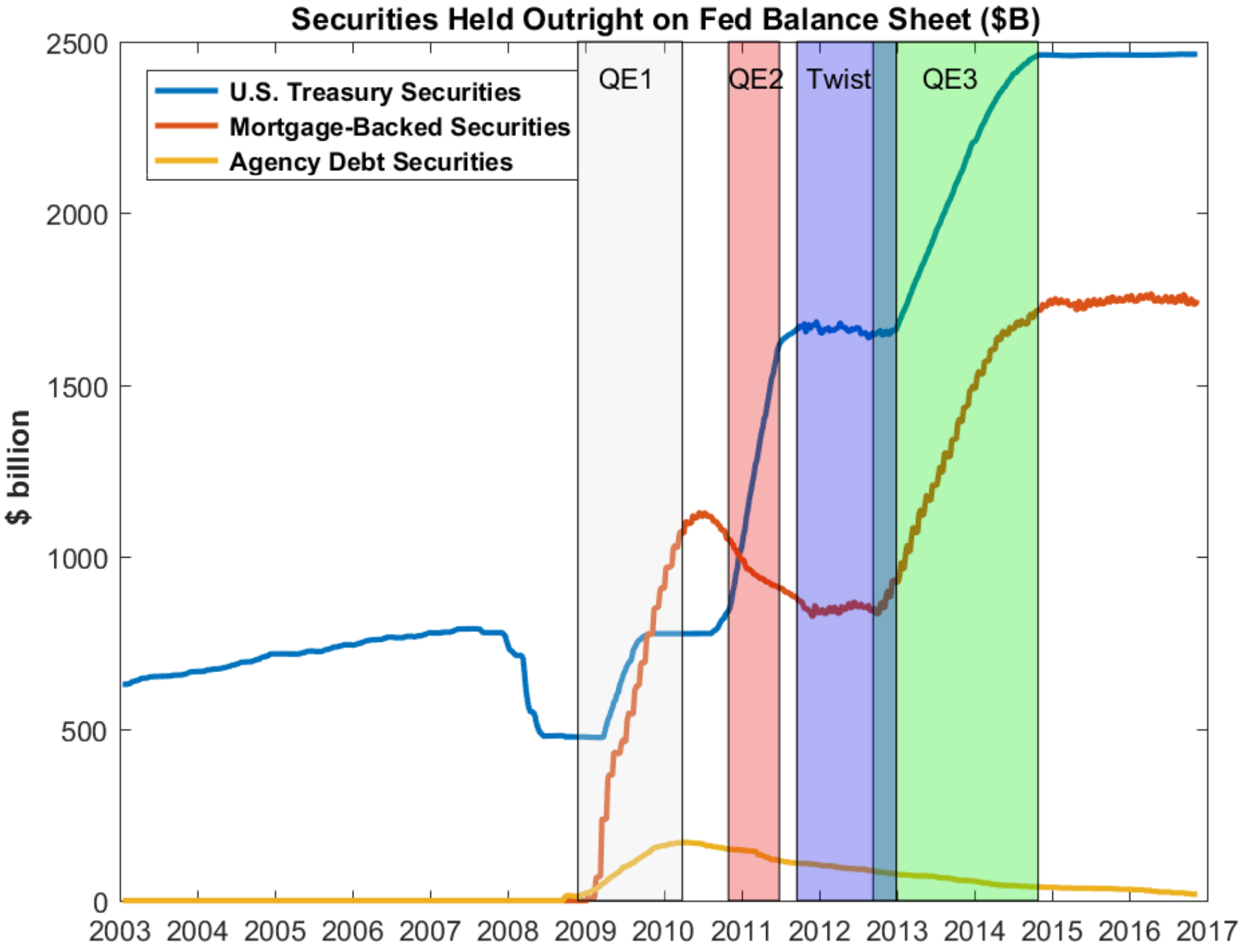
  where r is the fed funds rate, p is the rate of **inflation** over the previous four quarters, and y is the percent deviation of real **GDP** from a target. Other influential macro variables: nonfarm payroll **employment**.
- In anticipation of future Fed actions, the shape of the yield curve (e.g., slope) is closely connected with monetary policy. Federal funds futures are also analyzed to extract expectations of future Fed actions.
- Uncertainties in the target rate affect the markets: Fed transparency and better communications with market participants.

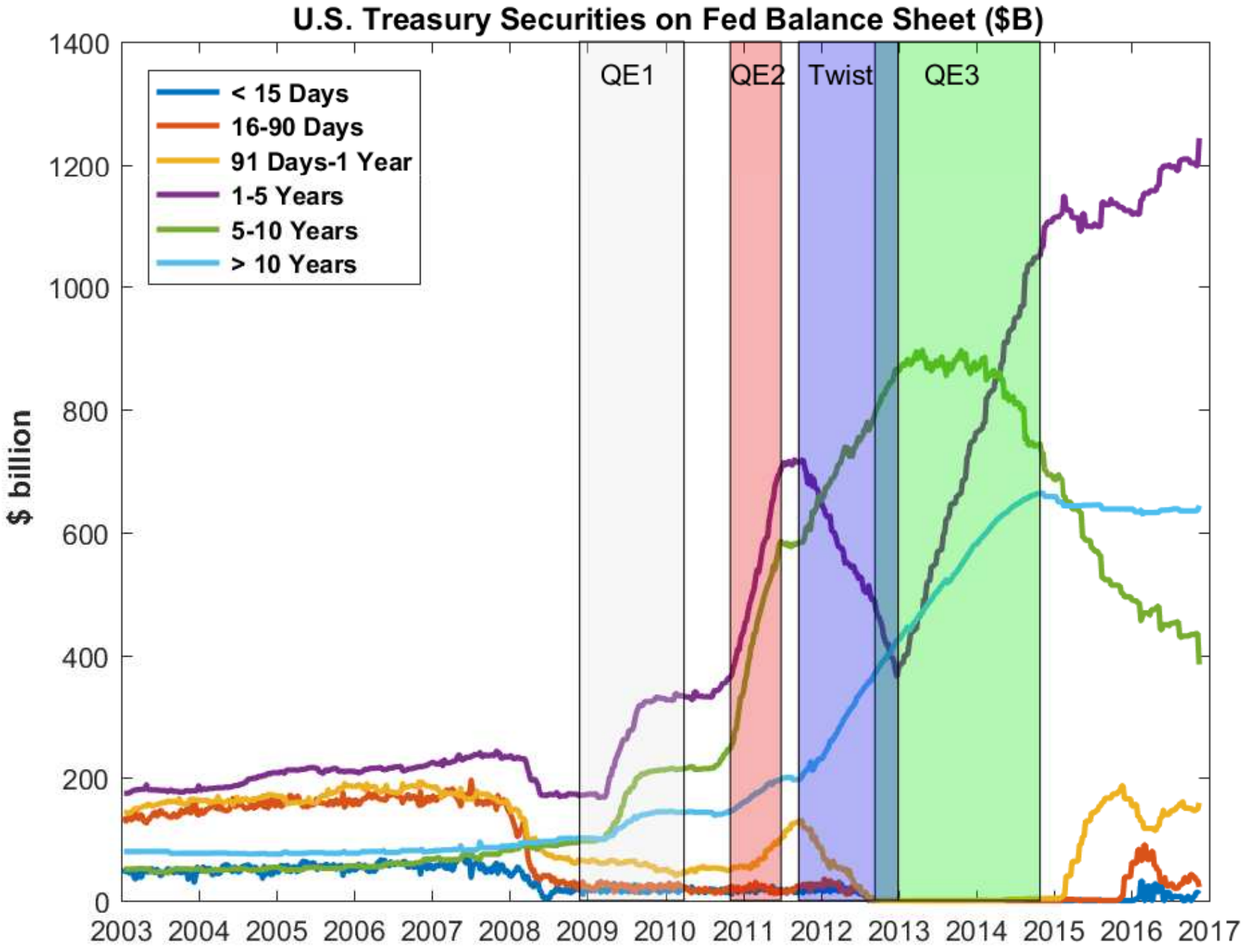

GDP, Inflation, and Unemployment
GDP (avg= 2.78%)
Inflation (avg= 4.16%)
Unemployment (avg= 6.29%)
Rates (%)
20
15
10
5
0
-5
-10
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Martin
Burns
Volcker
Greenspan
Bernanke
Yellen

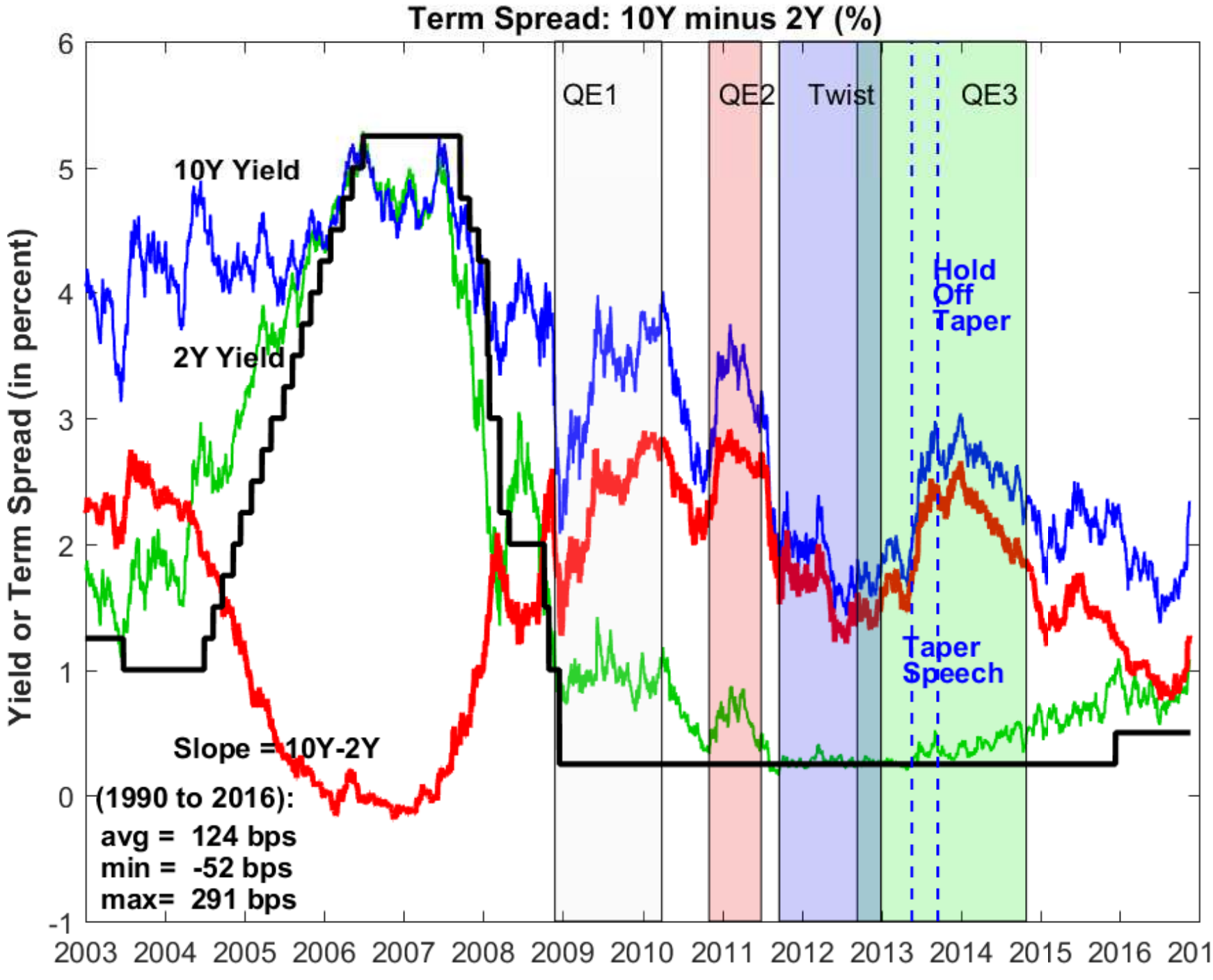
The End of of Orthodoxy: Quantitative Easing and Operation Twist

- When the short-term interest rates reach to zero, what to do to bring down longer-term interest rates?
- *Our purchases of hundreds of billions of dollars of securities were probably the most important and definitely the most controversial tool we would employ.* – Ben Bernanke in *The Courage to Act.*
- QE1: buy GSE debt, mortgage-backed securities, Treasury securities.
- QE2: buy Treasury securities.
- QE3: buy Treasury securities and mortgage-backed securities.
- Operation Twist: buy longer-term, sell shorter-term Treasury securities.
- In buying Treasury securities, the ultimate goal was to precipitate a broad reduction in the cost of credit (e.g., rates on mortgage and corp bonds).


Securities Held Outright on Fed Balance Sheet ($B)
QE1: 2008/11/25 - 2010/3/31
QE2: 2010/11/3 -- 2011/6/30
QE3: 2012/9/13 -- 2014/10/29
Twist: 2011/9/21 -- 2012/12/31
QE1
QE2
Twist
QE3
$ billion
4500
4000
3500
3000
2500
2000
1500
1000
500
0
2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017


Securities Held Outright on Fed Balance Sheet ($B)
U.S. Treasury Securities
Mortgage-Backed Securities
Agency Debt Securities
QE1
QE2
Twist
QE3
$ billion
2500
2000
1500
1000
500
0
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017


U.S. Treasury Securities on Fed Balance Sheet ($B)
$ billion
1400
1200
1000
800
600
400
200
0
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
QE1
QE2
Twist
QE3
< 15 Days
16-90 Days
91 Days-1 Year
1-5 Years
5-10 Years
> 10 Years


Term Spread: 10Y minus 2Y (%)
Yield or Term Spread (in percent)
6
5
4
3
2
1
0
-1
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
QE1
QE2
Twist
QE3
10Y Yield
2Y Yield
Hold
Off
Taper
Taper
Speech
Slope = 10Y-2Y
(1990 to 2016):
avg = 124 bps
min = -52 bps
max= 291 bps

Common Factors in Fixed Income, a Statistical Approach

- As it is true for financial modeling of any markets, the task of first-order importance is to understand the key risk factors affecting the market.
- Market participants have long recognized the importance of identifying the common factors that affect the returns on treasury bonds and related securities.
- To explain the variation in these returns, it is critical to distinguish the systematic risks that have a general impact on the returns of most securities from the specific risk that influence securities individually and hence have a negligible effect on a diversified portfolio.
- In an influential article published in 1991, Litterman and Scheinkman point out that most of the variation in returns on all fixed-income securities can be explained in terms of three "factors," or attributes of the yield curve: level, steepness, and curvature.

An Illustrative Example

- To simplify our analysis, let's start with zero-coupon bonds.
- Once we understand the common factors in zero rates of various maturities, we can readily apply this knowledge to coupon bonds, since they are weighted sums of the zero-coupon bond prices.
- Let's assume that initially the zero rates (annually compounded) are:

	2yr	5yr	10yr	30yr
zero rate	3.5%	4.5%	5%	5.5%
maturity	2	5	10	30
modified duration D	$\frac{2}{1+3.5\%}$	$\frac{5}{1+4.5\%}$	$\frac{10}{1+5\%}$	$\frac{30}{1+5.5\%}$
	1.93	4.78	9.52	28.44

- Using the 2yr zero as the reference bond, the 2-to-5 spread is 100 bps, the 2-to-10 spread is 150 bps, and the 2-to-30 spread is 200 bps.

Directional trade to bet on a parallel shift in level:

The yield on the 2yr zero subsequently increases by 10 bps and all spreads remain the same.

	notional	market value ($ million)			
	($ million)	initial	later	change	approximation
2yr	$(1+3.5\%)^2$	1	$\frac{(1+3.5\%)^2}{(1+3.5\%+10_{\text{bps}})^2}$	-19.30 bps	-1.93 $\times$ 10 bps
5yr	$(1+4.5\%)^5$	1	$\frac{(1+4.5\%)^5}{(1+4.5\%+10_{\text{bps}})^5}$	-47.71 bps	-4.78 $\times$ 10 bps
10yr	$(1+5.0\%)^{10}$	1	$\frac{(1+5.0\%)^{10}}{(1+5.0\%+10_{\text{bps}})^{10}}$	-94.74 bps	-9.52 $\times$ 10 bps
30yr	$(1+5.5\%)^{30}$	1	$\frac{(1+5.5\%)^{30}}{(1+5.5\%+10_{\text{bps}})^{30}}$	-280.22 bps	-28.44 $\times$ 10 bps

Steepener

- Your view: the 2-to-10 spread will increase but not sure of the overall direction of the interest rate.
- Your strategy: steepener.
- Long \$4.9286M of 2yr zero and short \$1M of 10yr zero:

	initial market value (\$ million)	notional amount (\$ million)
2yr	4.9286	$4.9286 \times (1+3.5\%)^2$
10yr	-1	$-(1+5.0\%)^{10}$

- Why 4.9286? Because 4.9286=9.52/1.93 is the ratio of the modified duration D^* of a 10yr zero over that of a 2yr zero.

Two Scenarios:

1. A parallel shift of +10 bps:

	initial ($ Million)	later ($ Million)	change ($ Million)	approximation ($ Million)
2yr	4.9286	$\frac{4.9286\times(1+3.5\%)^2}{(1+3.5\%+10_{\text{bps}})^2}$	-95.10 bps	-1.93 $\times$ 10 bps $\times$ 4.9286
10yr	-1	$-\frac{(1+5.0\%)^{10}}{(1+5.0\%+10_{\text{bps}})^{10}}$	94.74 bps	9.52 $\times$ 10 bps
total			-0.36 bps	

2. The 2yr zero rate decreases by 15 bps and the 10yr zero rate increases by 5 bps.

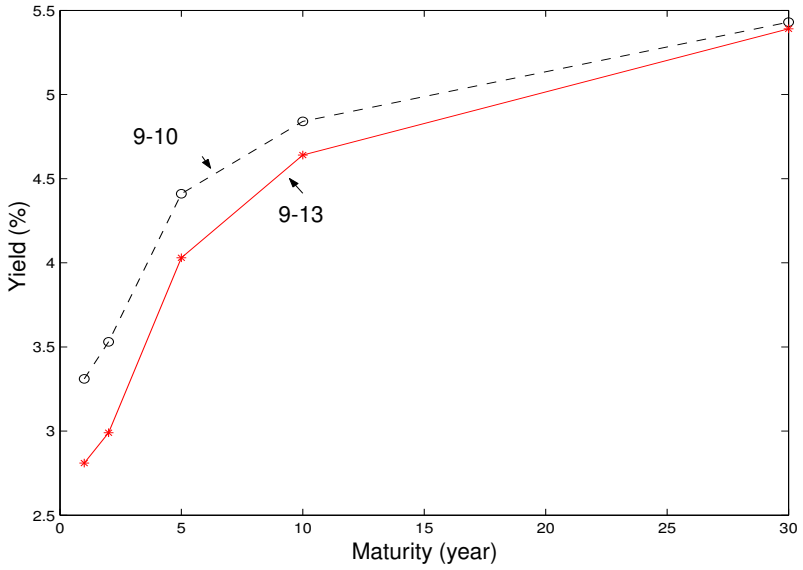
	initial ($ Million)	later ($ Million)	change ($ Million)	approximation ($ Million)
2yr	4.9286	$\frac{4.9286\times(1+3.5\%)^2}{(1+3.5\%-15_{\text{bps}})^2}$	143.17 bps	1.93 $\times$ 15 bps $\times$ 4.9286
10yr	-1	$-\frac{(1+5.0\%)^{10}}{(1+5.0\%+5_{\text{bps}})^{10}}$	47.49 bps	9.52 $\times$ 5 bps
total			190.66 bps	

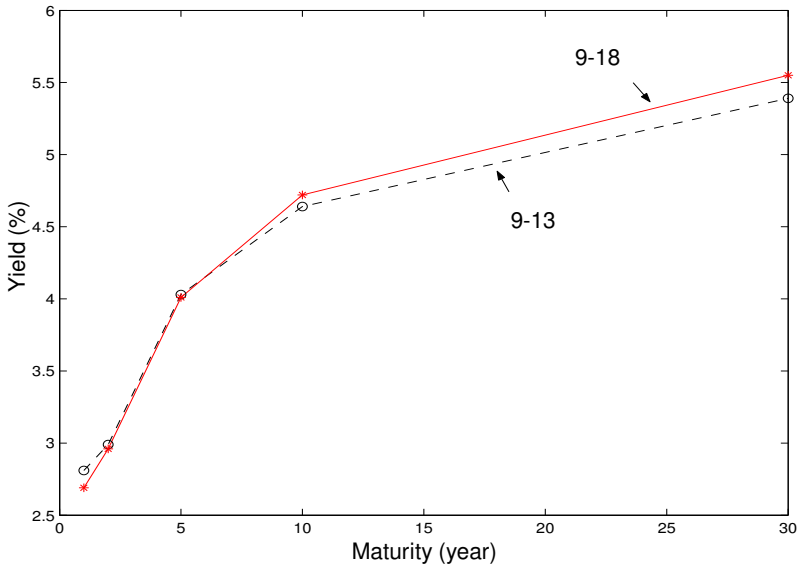
Further Considerations:

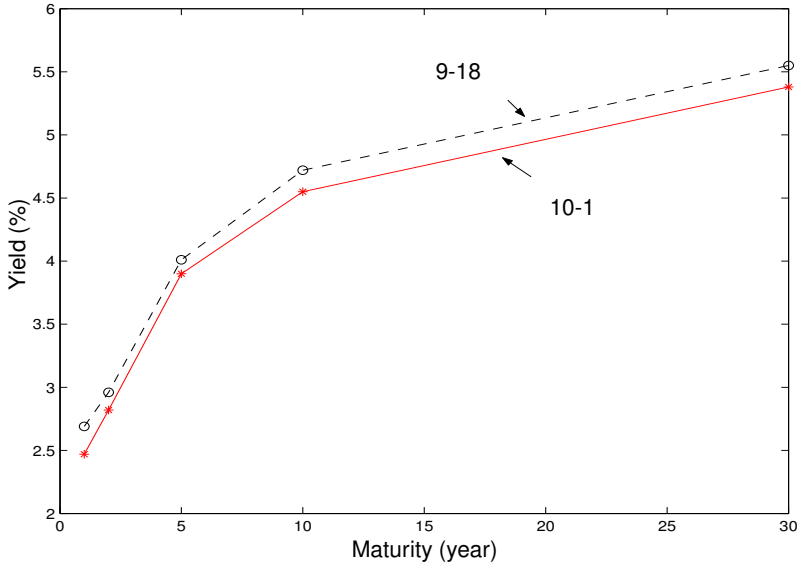
- What if in addition to the steepening described in 2, a parallel movement described in 1 also happened?
- What if in addition to the steepening described in 2, a parallel shift of -10 bps happened?
- What if instead of the steepening described in 2, the 2yr zero increases by 15 bps and the 10yr zero decreases by 5 bps?

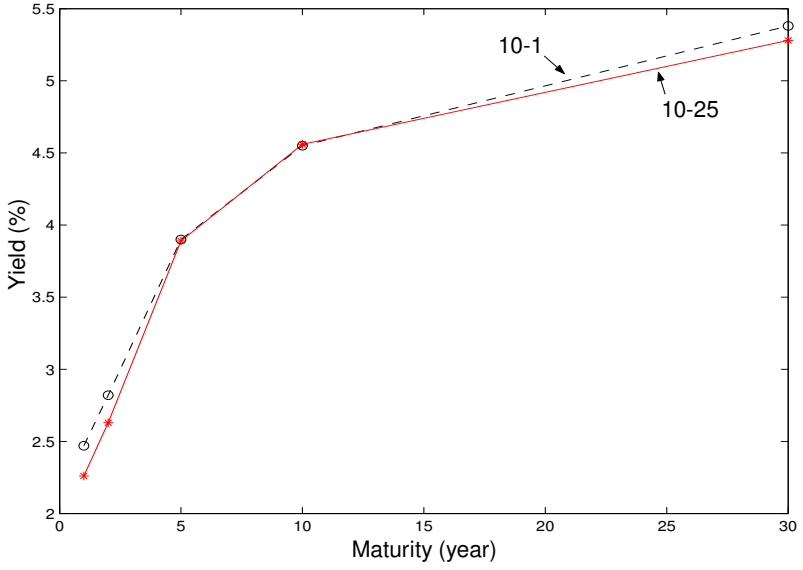
Curvature (or Butterfly):

- Your view: the 2yr and 30yr zeros will move in the same direction while the 10yr zero will move in opposite direction. Not sure about the overall directional of the interest rate, nor about the slope of the yield curve.
- Your strategy: butterfly trade.
- Long 30yr zeros, short 10yr zeros (to hedge against parallel shifts), and long 2yr zeros (to hedge against slope steepening or flattening).


9-10
9-13
Yield (%)
Maturity (year)

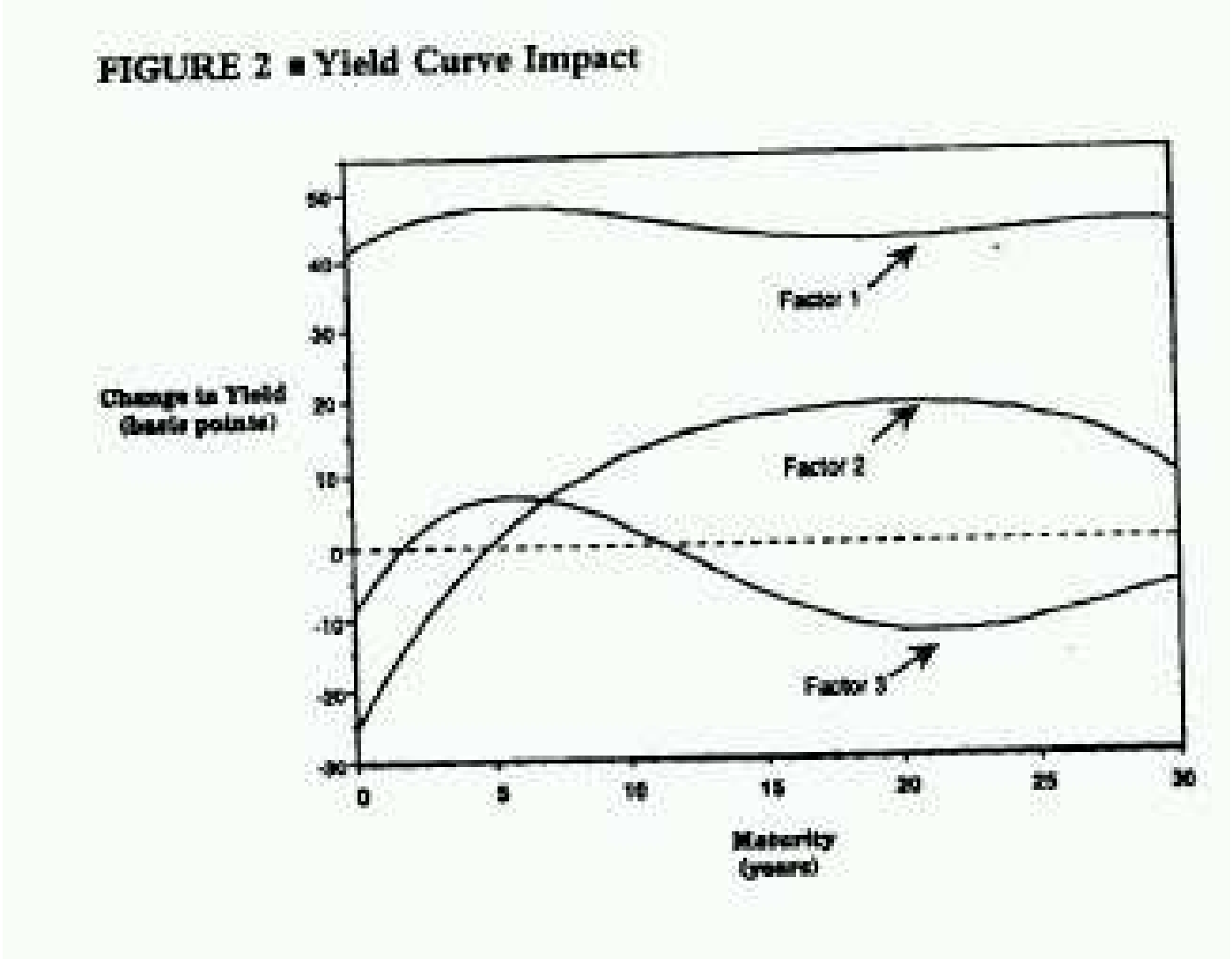

9-18
9-13
Yield (%)
Maturity (year)

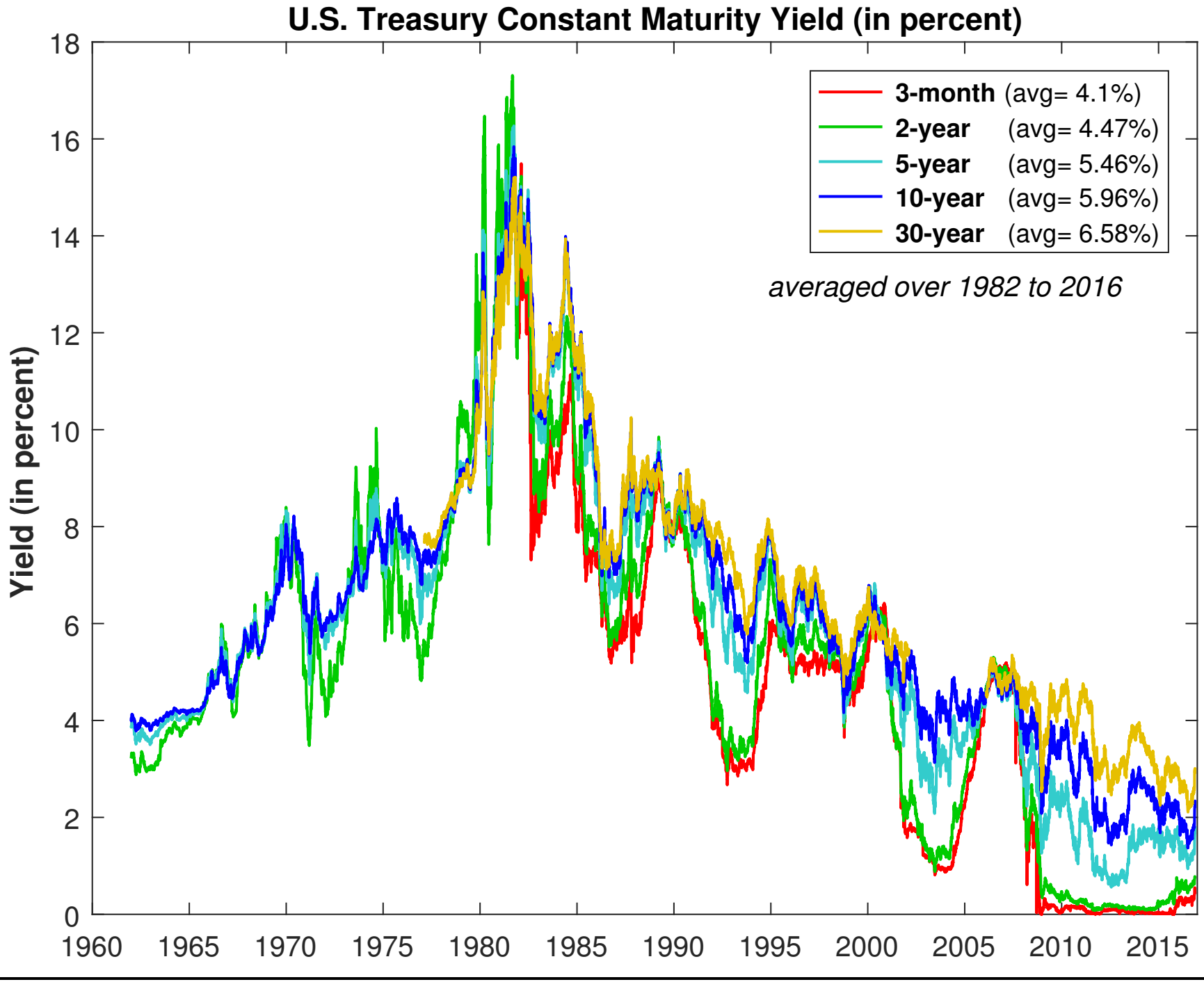

9-18
10-1
Yield (%)
Maturity (year)


10-1
10-25
Yield (%)
Maturity (year)

Three Major Factors in the Fixed-Income Market:

Closely related to the three trading strategies are the three major risk factors reported by Litterman and Scheinkman for the fixed income market:

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Yield (in percent)
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14
12
10
8
6
4
2
0
1960
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1970
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1980
1985
1990
1995
2000
2005
2010
2015
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5-year (avg= 5.46%)
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averaged over 1982 to 2016

cov(Δy) (unit: bps^2, or equivalently, $\times 10^{-8}$)

	3M	1Y	2Y	5Y	10Y	30Y
3M	64.9300	40.4601	32.1459	27.1468	22.3368	17.8525
1Y	40.4601	48.3104	42.5293	39.0606	33.7733	27.6243
2Y	32.1459	42.5293	48.4517	45.1757	39.4565	31.9805
5Y	27.1468	39.0606	45.1757	51.6705	46.6458	38.8411
10Y	22.3368	33.7733	39.4565	46.6458	47.5872	40.7433
30Y	17.8525	27.6243	31.9805	38.8411	40.7433	39.7206

corr(Δy)

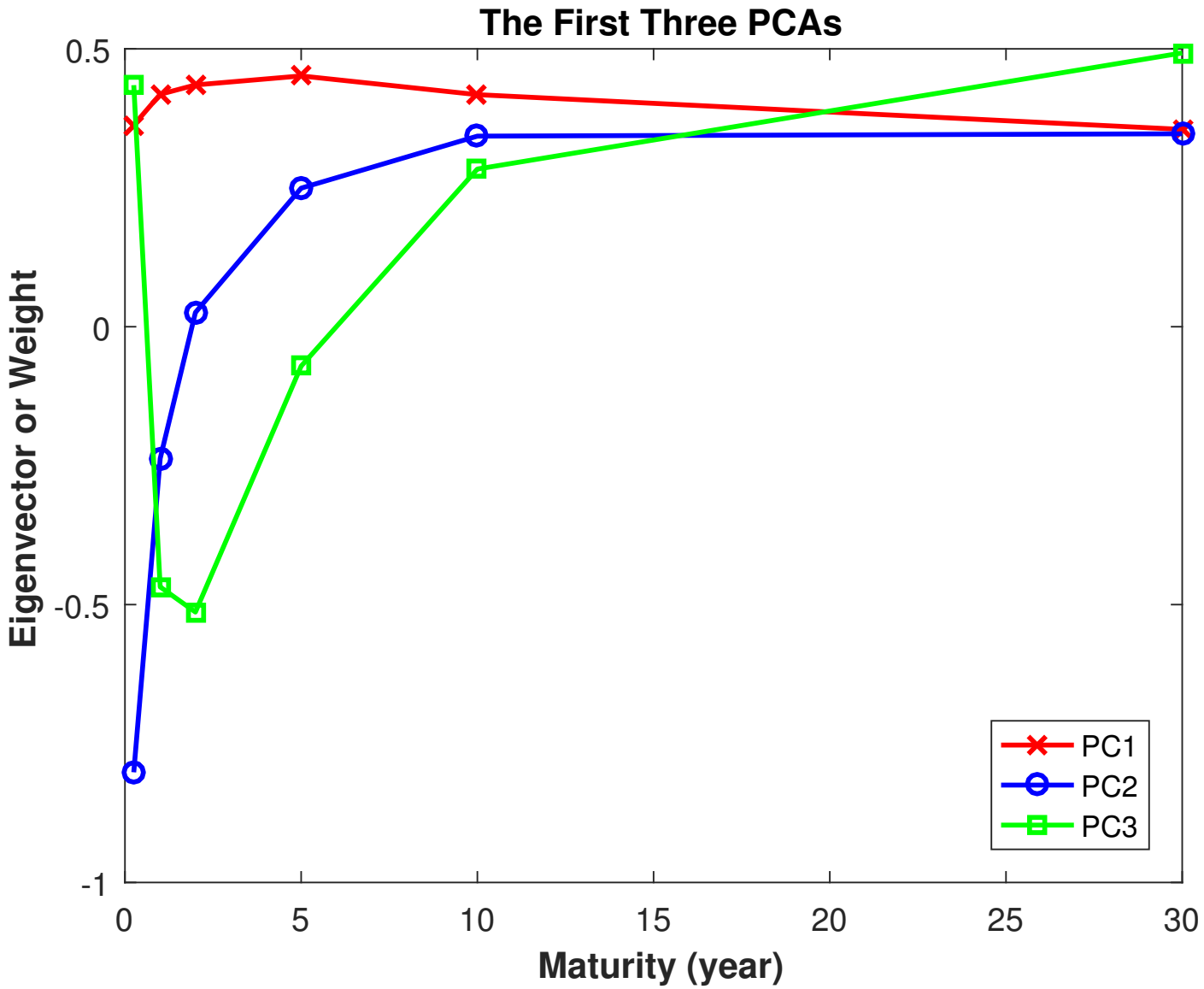
3M	1.0000	0.7224	0.5731	0.4687	0.4018	0.3515
1Y	0.7224	1.0000	0.8790	0.7818	0.7044	0.6306
2Y	0.5731	0.8790	1.0000	0.9029	0.8217	0.7290
5Y	0.4687	0.7818	0.9029	1.0000	0.9407	0.8574
10Y	0.4018	0.7044	0.8217	0.9407	1.0000	0.9371
30Y	0.3515	0.6306	0.7290	0.8574	0.9371	1.0000

Eigenvalues E

E	PC1	PC2	PC3	PC4	PC5	PC6
E (bps^2)	226.99	50.14	13.77	5.45	2.86	1.47
E/sum(E) (%)	75.49	16.68	4.58	1.81	0.95	0.49

Eigenvectors D

D	PC1	PC2	PC3	PC4	PC5	PC6
3M	0.3630	-0.8017	0.4347	0.1876	-0.0365	0.0006
1Y	0.4182	-0.2371	-0.4682	-0.6806	0.2939	0.0016
2Y	0.4351	0.0257	-0.5134	0.3309	-0.6505	0.1176
5Y	0.4513	0.2493	-0.0709	0.4572	0.5076	-0.5124
10Y	0.4176	0.3430	0.2837	0.0418	0.2271	0.7577
30Y	0.3550	0.3472	0.4926	-0.4258	-0.4242	-0.3866


The First Three PCAs
Eigenvector or Weight
Maturity (year)
0.5
0
-0.5
-1
0
5
10
15
20
25
30
PC1
PC2
PC3

Level, Slope, and Curvature

- They name the first factor the *level* factor as it has a similar impact of the yield curve as a parallel shift of the yield curve.
- They call the second factor *steepness*, since shock from this factor lowers the yields of zeros up to five years, and raises the yields for zeros of longer maturities.
- They call the third factor *curvature*, since it increases curvature of the yield curve in the range of maturities below twenty years.

The Relative Importance of the Three Factors:

Table 2 ▪ Implied Zeroes: Relative Importance of Factors ▪ (Percent)

Maturity	Total Variance Explained	Proportion of Total Explained Variance Accounted for by Factor 1	Factor 2	Factor 3
6 months	99.5	79.5	17.2	3.3
1 year	99.4	89.7	10.1	0.2
2 years	98.2	93.4	2.4	4.2
5 years	98.8	98.2	1.1	0.7
8 years	98.7	95.4	4.6	0.0
10 years	98.8	92.9	6.9	0.2
14 years	98.4	86.2	11.5	2.2
18 years	95.3	80.5	14.3	5.2
Average	**98.4**	**89.5**	**8.5**	**2.0**

The first factor is by far the most important, supporting the idea that "first factor" hedging – or its close cousin, duration hedging – takes care of most of the return risk.

$$\Delta y_t = a + \beta^{PC1}\, PC1_t + \beta^{PC2}\, PC2_t + \beta^{PC3}\, PC3_t + \epsilon_t\,.$$

	PC1 β	PC2 β	PC3 β	PC1 R2 (%)	PC2 R2 (%)	PC3 R2 (%)	Total R2 (%)
3M	0.3630	-0.8017	0.4347	46.06	49.63	4.01	99.70
1Y	0.4182	-0.2371	-0.4682	82.18	5.83	6.25	94.26
2Y	0.4351	0.0257	-0.5134	88.67	0.07	7.49	96.23
5Y	0.4513	0.2493	-0.0709	89.46	6.03	0.13	95.62
10Y	0.4176	0.3430	0.2837	83.17	12.39	2.33	97.89
30Y	0.3550	0.3472	0.4926	72.04	15.22	8.41	95.66