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Classes 17 & 18: Risk Management

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1 Why Risk Management?

- **Capital Markets Imperfection:** According to Modigliani and Miller (1958), in perfect capital markets, adding or subtracting financial risk has no impact on the market value of a publicly traded corporation or on the welfare of its shareholders. In the real world, capital markets are imperfect. This imperfection gives rise to the need for risk management.

  At the core of risk management for financial institutions is the concept of "capital adequacy." If new capital could be obtained in perfect financial markets, we would expect a financial firm to raise capital as necessary to avoid the cost of financial distress. In such a setting, purely financial risk would have a relatively small impact, and risk management would not be important. In practice, however, capital is a scarce resource, especially when it is most needed.

  Compared with other types of corporations, financial firms have relatively more liquid balance sheets, made up largely of financial assets. This relative liquidity allows a typical financial firm to operate with a high degree of leverage. For example, major broker-dealers regulated by SEC frequently have a level of accounting capital that is close to the regulatory minimum of 8% of accounting assets, implying a leverage ratio on the order of 12.5 to 1. As we will see later in the class, for Goldman Sachs, the ratio of book assets to book equity was 10.3 to 1 in 2014, with 23% of the liabilities financed by long-term liabilities and 11% financed by Repo (usually overnight and short-term). In 2007, the leverage was even higher: the asset-to-equity ratio at 26.2; and the financing leaned more toward short term: only 18% long-term financing and close to 15% Repo financing.

  Ironically, in light of the relatively high degree of liquidity that fosters high leverage, a significant and sudden financial loss (or reduced access to credit) can cause dramatic illiquidity effects. This, has been the experience for many financial firms during the 2007-08 crisis. Some survived (e.g., Morgan Stanley and Goldman Sachs), some were bought out (e.g., Bear Stearns, Merrill, and Wachovia), and some failed (e.g., Lehman and WaMu). For individual firms, weathering sudden financial losses with adequate capital matters for its own survival. For regulators, it is about the financial stability of the entire system, which has become highly inter-connected through interbank transactions including OTC derivatives trading.

- **Liquidity Mismatch in Assets and Liabilities:** Let's strip the complexity of a financial institution to its bare minimum with this simple example of a bank. It takes in deposits at the short-end of the yield curve and makes loans at the long-end. This maturity transformation is at the core of a bank's profitability. As we will learn in the fixed-income class, the yield curve is typically upward sloping with the spread between the long- and short-term yields averaged to about 100 to 200 basis points. In addition, the longer maturity loans made by banks to firms are usually defaultable, adding another 100 to 200 basis points of credit spread (assuming the loans are investment grade).

  With fractional-reserve banking, the bank is allowed to hold reserves that are only a fraction (e.g., 10%) of their deposit liabilities. For our example, let's assume that the bank is 100% financed by liabilities. It takes in 100 dollars of demand deposits (i.e., liabilities), holds 10 dollars of reserve (i.e., cash or safe assets) and lends out 90 dollars in longer maturity and defaultable loans (i.e., risky assets).

  A run on a bank happens when depositors suspect that the bank has made bad investments in its risky loans and is no longer solvent. They rush to the bank simultaneously to withdraw their deposits. While the demand deposits are highly liquid and can be withdrawn in a moment's notice, the loans sitting on the asset side of the bank's balance sheet are typically of longer maturity and not as liquid. This liquidity mismatch between a bank's assets and liabilities is the root cause of a bank run: an otherwise solvent bank needs to raise capital quickly to meet the simultaneous demands from panicking depositors acting out of fear.

  In a perfect financial market, the bank should be able to raise additional funding using its loans as collateral. But because of information asymmetry regarding the credit worthiness of the loans, potential investors are reluctant to extend funding to the bank (with such a short notice and under a bank run scenario). Moreover, if this bank run happens during a crisis, then capital is even more scarce, making it more difficult for the bank to raise new funding.

So the most likely action of the bank is to sell its long-term assets, often hastily and at fire-sale prices. If multiple banks are facing runs at the same time during a crisis situation, then they would be selling similar long-term assets at severely discounted fire-sale prices. In the U.S., the FDIC deposit insurance has been an effective way to stem out bank runs of this kind. Knowing that their deposits are safely guaranteed by FDIC (up to a certain dollar amount for each depositor), depositors will not rush to the bank to withdraw simultaneously. Consequently, bank runs purely due to liquidity mismatch can be avoided.

- **Equity as a Buffer:** I made the previous example as simple as possible so that we can focus on the heart of the issue: liquidity mismatch. To make the example more realistic, we can further add an equity piece. In doing so, we learn another very important concept: the role of equity as a buffer for risk management.

  Suppose the bank is now financed by 90% liabilities (i.e., demand deposits) and 10% equity. Let's keep the same allocation between risky and riskless assets: 90% risky loans and 10% cash. Now let's see how the 10% equity piece can function as a buffer to cushion the fall of the bank. Suppose the bank has already experienced deposit withdrawal of 10 dollars and has exhausted its 10 dollars of reserves. As the next dollar of withdraw comes in, the bank has to sell a piece of its risky loans. Suppose the fire sale price is 50% of the initial value. To raise 1 dollar of cash, the bank therefore has to sell 2 dollars (book value) of risky loans, incurring a one-dollar loss due to the fire sale. Now the total book value of assets are 88 dollars (90-2), the liabilities are at 79 dollars (90-10-1), and equity absorbs the one-dollar write-down and is at 9 dollars.

  As you can see, equity functions as a buffer to cushion the fall of the bank. Without this equity piece, the bank would have been insolvent. It is obvious that a higher capital ratio (Equity/Asset) adds more buffer and strengthens the financial health of a bank. As you will see later in the class, capital ratios of various kinds are an important part of the regulatory requirements for banks. Although they come in different varieties, depending on how assets and equity are calculated, the essence of these capital ratios is to evaluate the capital adequacy (i.e., the thickness of the buffer) of a bank. In this example, the equity/asset ratio is 10/100=10%. Using the approach of risk-weighted assets (RWA), where cash counts as zero, the RWA of the bank is 90 dollars. Then the capital ratio is 10/90=11.11%. You will find this simple model to be quite handy as we discuss capital ratios for the banking industry.

- **The Balance Sheet of Goldman:** For a financial intermediary such as Goldman Sachs, its balance sheet is certainly more complex than that of a simple bank. But the basic idea is similar.

  So let's start with Goldman's 10K reports. Table 1 summarizes the company's assets, liabilities, and shareholders' equity for a few selected years. You must have learned how to read a financial statement from your accounting classes. So let me focus only on the items that are important for us. I've also changed the names of a few items so that the table would fit in one page.

  Before getting into details, let me mention a few events that are important for Goldman. The company went public in 1999. the first 10K form was published in 1999 with 197 pages. Between 1999 and 2006, the length of the 10K forms fluctuated between 103 pages in 2001 and 298 pages in 2005. The 2007 10K form had 372 pages. On September 21, 2008, Goldman became a bank holding company and the Federal Reserve Board became its primary regulator. Its 2008 form has 731 pages, followed by 411 in 2009, 336 in 2010, 367 in 2011, 480 in 2012, 366 in 2013, and 410 in 2014.

  In conjunction with the increasing thickness of the 10K forms, financial intermediaries like Goldman are facing increasing reporting requirements. Indeed, the financial services industry has been the subject of intense regulatory scrutiny in recent years. The 2010 Dodd-Frank Act significantly altered the financial regulatory regime within which Goldman operates. The implementations of Dodd-Frank and Basel III are still on going, which would have a direct and significant impact on the risk-management practice of this industry.

  - **Assets:** Now let's focus our attention on Table 1. As of December 2014, Goldman holds assets in total of $856 billion. For our purpose, the item that matters the most is "financial instruments owned," which is also the largest item, valued at $312 billion. Going back to our example of a simple bank, this item is similar to the risky loans made by a bank. In the case of Goldman, of course, the collection of risky assets is more diverse. We will focus on this item shortly.

    The two items under "collateralized agreements" are effectively collateralized lending, which are relatively safe in terms of market and credit risk, but are subject to counterparty credit risk. Likewise, items under "receivables" are also sensitive to counterparty credit risk. For the purpose of risk management, measuring and controlling counterparty credit risk is an important component, as you will see later, these items show up in the firm's credit risk weighted assets.

Table 1: Goldman Sachs' Assets, Liabilities, and Shareholders' Equity

Assets in millions	2014	2010	2008	2007
Cash and cash equivalents	57,600	39,788	15,740	10,282
Cash and securities for regulatory and other purposes	51,716	53,731	106,664	119,939
Collateralized agreements:				
Repo Lending and federal funds sold	127,938	188,355	122,021	87,317
Securities borrowed	160,722	166,306	180,795	277,413
Receivables:				
Brokers, dealers and clearing organizations	30,671	10,437	25,899	19,078
Customers and counterparties	63,808	67,703	64,665	129,105
Loans receivable	28,938			
Financial instruments owned	312,248	356,953	328,325	452,595
Other assets	22,599	28,059	30,438	24,067
Total assets	**856,240**	**911,332**	**884,547**	**1,119,796**

Liability and Shareholders' Equity in millions	2014	2010	2008	2007
Deposits	83,008	38,569	27,643	15,370
Collateralized financings				
Repo financing	88,215	162,345	62,883	159,178
Securities loaned	5,570	11,212	17,060	28,624
Other	22,809	38,377	38,683	65,710
Payables:				
Brokers, dealers and clearing organizations	6,636	3,234	8,585	8,335
Customers and counterparties	206,936	187,270	245,258	310,118
Financial instruments sold short	132,083	140,717	175,972	215,023
Unsecured short-term borrowings	44,540	47,842	52,658	71,557
Unsecured long-term borrowings	167,571	174,399	168,220	164,174
Other liabilities and accrued expenses	16,075	30,011	23,216	38,907
Total liabilities	**773,443**	**833,976**	**820,178**	**1,076,996**
Total shareholders' equity	**82,797**	**77,356**	**64,369**	**42,800**

$ in millions	As of December 2014 Financial Instruments Owned	Financial Instruments Sold, But Not Yet Purchased
Commercial paper, certificates of deposit, time deposits and other money market instruments	$ 3,654	$ —
U.S. government and federal agency obligations	48,002	12,762
Non-U.S. government and agency obligations	37,059	20,500
Mortgage and other asset-backed loans and securities:		
Loans and securities backed by commercial real estate	6,582 [1]	1
Loans and securities backed by residential real estate	11,717 [2]	—
Bank loans and bridge loans	15,613	464 [4]
Corporate debt securities	21,603	5,800
State and municipal obligations	1,203	—
Other debt obligations	3,257 [3]	2
Equities and convertible debentures	96,442	28,314
Commodities	3,846	1,224
Subtotal	248,978	69,067
Derivatives	63,270	63,016
Total	**$312,248**	**$132,083**

Figure 1: Goldman's financial instruments, long and short positions.

- **Financial instruments owned:** As shown in Figure 1, the $312 billion of risky assets mostly includes Treasury and agency bonds ($48 billion), foreign government and agency bonds ($37 billion), mortgage and other asset-backed loans and securities ($11 + $6.5 billion), bank loans ($15 billion), corporate debt securities ($21 billion), equity and convertible debentures ($96 billion), and derivatives ($63 billion). So effectively, the risk factors influencing this portion of the balance sheet include interest rate, currency, equity, and commodities.

- **Balance sheet allocation to business segments:** In terms of balance sheet allocation, most of the $312 billion in financial instruments is attributable to two business segments of Goldman. The segment of Institutional Client Services, which "maintain inventory positions to facilitate market-making in fixed income, equity, currency and commodity products," holds majority ($230 billion) of the financial instruments. The segment of Investing & Lending, whose activities include "investing directly in publicly and privately traded securities and in loans, and also through certain investment funds managed by Goldman," holds $47 billion.[1]

[1] Page 69 of Goldman's 2014 10K.

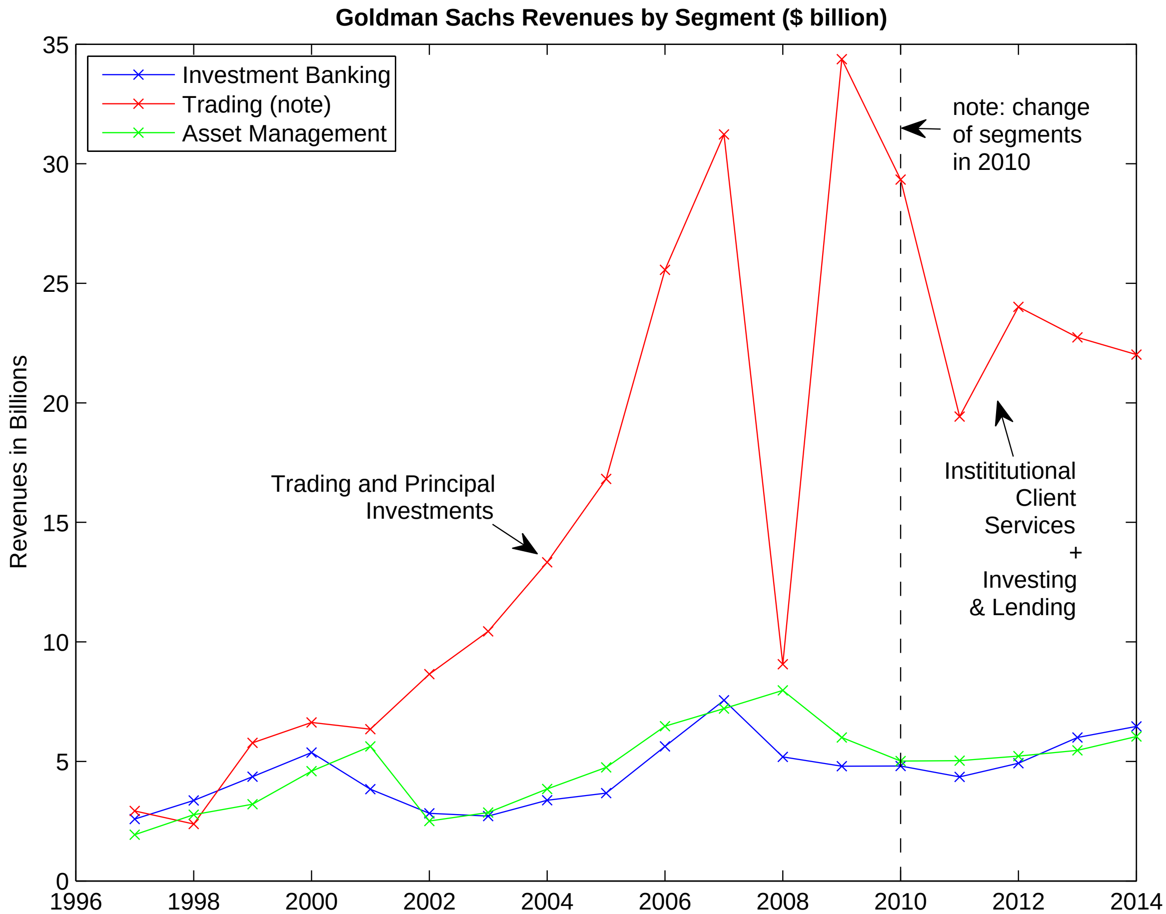

Figure 2: Goldman's Annual Revenues by Business Segment.

From 2009 to 2010, there was a change in how Goldman divide its business segments. My guess is that these two segments belong to the old segment of Trading and Principal Investments. Figure 2 reports the annual revenues by business segment. As you can see, the segment of Trading and Principal Investments (Institutional Client Services + Investing & Lending for post 2010) accounts a large portion of Goldman's revenue and is also the most volatile. Later as we move on to risk management, this segment would be our main focus.

– **Financial instruments sold short:** Figure 1 also reports short positions on financial instruments, valued at $132 billion. On the financial statement, this item shows up in liabilities. For our understanding of the firm's market risk exposure, this item is as important as the $312 billion long positions on financial instruments. It includes short positions on US Treasury and agency bonds ($12 billion), foreign government and agency bonds ($20 billion), corporate debt securities ($5 billion), equity and convertible debentures ($28 billion) and derivatives ($63 billion). The 10K report does not report the correlation between the risk exposure of the long and the short positions. If the long/short positions are paired as hedging positions, then the net risk exposure will be small. To the extreme, we can say that the net exposure is $312 billion minus $132 billion. Otherwise, we need to take a portfolio approach and take into account of the correlations. More on this later.

- **Derivatives Assets and Liabilities:** The value of derivatives assets is $63 billion and derivatives liabilities is $63 billion, which are sizable positions in relation to Goldman's overall positions in financial instruments. Given the inherent leverage of derivatives, the actual risk exposure per dollar position in these derivatives positions is much higher than the other linear instruments on the list. Again, without knowing the underlying correlations between the derivatives assets/liabilities, it is difficult for us to assess the net exposure. If these derivatives positions are the result of market making activities, then most of the $63 billion assets and liabilities in derivatives will net out and the net exposure will be small.

  Figure 3 gives a more detailed description of Goldman's derivatives positions by major product type on a gross basis. For example, the gross value of interest-rate derivatives totals to $786,362 million in assets and $739,607 million in liability with a total notional amount of $47,112,518 million. As of December 2014, the total notional amount of interest-rate OTC derivatives was $505 trillion, making Goldman an important participant in this market. Compared with the $63 billion derivatives assets and $63 billion derivatives liabilities, these gross value numbers are much larger because they exclude the effects of both counterparty netting and collateral, and therefore are not representative of the firm's counterparty exposure.

  Because of these derivatives positions, Goldman are connected to it many counterparties: financial troubles of its counterparties could have a material impact on Goldman (e.g., AIG in 2008) and Goldman's own financial troubles could have a material impact on its counterparties (e.g., Lehman's default on Lehman's derivatives counterparties). For regulators worrying about financial institutions that are too connected to fail, understanding these derivatives positions should be high on their priority list. After all, the super-senior tranches were a huge cause of concerns during the 2007-08 financial crisis.

- **Liabilities:** According to Table 1, the total liabilities of Goldman in 2014 were

$ in millions	As of December 2014 Derivative Assets	Derivative Liabilities	Notional Amount
Derivatives not accounted for as hedges			
Interest rates	$ 786,362	$739,607	$47,112,518
Exchange-traded	228	238	3,151,865
OTC-cleared	351,801	330,298	30,408,636
Bilateral OTC	434,333	409,071	13,552,017
Credit	54,848	50,154	2,500,958
OTC-cleared	5,812	5,663	378,099
Bilateral OTC	49,036	44,491	2,122,859
Currencies	109,916	108,607	5,566,203
Exchange-traded	69	69	17,214
OTC-cleared	100	96	13,304
Bilateral OTC	109,747	108,442	5,535,685
Commodities	28,990	28,546	669,479
Exchange-traded	7,683	7,166	321,378
OTC-cleared	313	315	3,036
Bilateral OTC	20,994	21,065	345,065
Equities	58,931	58,649	1,525,495
Exchange-traded	9,592	9,636	541,711
Bilateral OTC	49,339	49,013	983,784
Subtotal	1,039,047	985,563	57,374,653
Derivatives accounted for as hedges			
Interest rates	14,272	262	126,498
OTC-cleared	2,713	228	31,109
Bilateral OTC	11,559	34	95,389
Currencies	125	16	9,636
OTC-cleared	12	3	1,205
Bilateral OTC	113	13	8,431
Commodities	—	—	—
Exchange-traded	—	—	—
Bilateral OTC	—	—	—
Subtotal	14,397	278	136,134
Gross fair value/notional amount of derivatives	$1,053,444 [1]	$985,841 [1]	$57,510,787
Amounts that have been offset in the consolidated statements of financial condition			
Counterparty netting	(886,670)	(886,670)	
Exchange-traded	(15,039)	(15,039)	
OTC-cleared	(335,792)	(335,792)	
Bilateral OTC	(535,839)	(535,839)	
Cash collateral netting	(103,504)	(36,155)	
OTC-cleared	(24,801)	(738)	
Bilateral OTC	(78,703)	(35,417)	
Fair value included in financial instruments owned/ financial instruments sold, but not yet purchased	$ 63,270	$ 63,016	

Figure 3: Goldman's Derivatives Positions.

at \$773 billion, with \$167 billion financed by long-term borrowings. The other sources of funding, including unsecured short-term borrowings and Repo financing are mostly short term in nature. Going back to the example of a simple bank, these short-term financings correspond to the demand deposits. Unlike the case of demand deposits, which are FDIC insured, there is no insurance on such short-term funding sources. So some of these short-term financings could evaporate in a moment's notice. Some of the short-term fundings are collateralized (e.g., Repo financing), while some are unsecured (e.g., inter-banking lending or commercial paper).

Table 2: Assets-to-Equity and Financing

	2014	2010	2008	2007
assets (\$m)	856,240	911,332	884,547	1,119,796
equity (\$m)	82,797	77,356	64,369	42,800
assets-to-equity ratio	10.3x	11.8x	13.7x	26.2x
total liabilities (\$m)	773,443	833,976	820,178	1,076,996
long-term borrowings (\$m)	167,571	174,399	168,220	164,174
other long-term financings (\$m)	7,249	13,848	17,460	33,300
% of long-term liabilities	22.60%	22.57%	22.64%	18.34%
total liabilities (\$m)	773,443	833,976	820,178	1,076,996
Repo financing (\$m)	88,215	162,345	62,883	159,178
% of Repo financing	11.41%	19.47%	7.66%	14.78%

Table 2 shows that in 2014, long-term liabilities account for 22.60% of Goldman's total liabilities, while in 2007, the number was only 18.34%. In recent years, financial firms such as Goldman have experienced disruptions in the credit markets, including reduced access to credit and higher costs of obtaining credit. As such, it is important for them to maintain stable funding in the form of long-term debt. On the other hand, because of the positive term spread (long term yields minus short term yields), long-term financing is more costly.

As we see in Table 2, Repo financing accounted for 11.41% of Goldman's liability in 2014 and 14.78% in 2007. This form of short-term (usually overnight) and collateralized (e.g., Treasury and agency bonds, corporate bonds, and equity) financing is an important source of funding for most investment banks.

– **Leverage:** With total assets at \$856 billion, total liabilities at \$773 billion, and shareholders' equity at \$82 billion, the leverage of a financial firm such as Goldman is markedly different from that of a non-financial firm. As shown in Table 2, the assets-to-equity ratio was around 10 to 1 in 2014 and 26 to 1 in 2007.

- **Runs on Financial Institutions:** We talked about how a bank run could happen because of the liquidity mismatch between assets and liabilities. After going through the balance sheet of Goldman, it is obvious that the same kind of liquidity mismatch exists in a financial intermediary like Goldman. In particular, long-term liabilities as a percentage of Goldman's total liabilities is 21.67% in 2014 and 15.24% in 2007. In other words, Goldman relies on short-term financing which could evaporate quickly if the markets are no longer confident of Goldman's solvency. Such was the case for Lehman in 2008. After Lehman's default, the solvency of Morgan Stanley and Goldman was seriously questioned by market participants. They had to go out and raise new capital: Morgan Stanley from Japan's Mitsubishi bank on a weekend in the form of a check of $9 billion and Goldman Sachs from Warren Buffett.

  Whenever there is liquidity mismatch in assets and liabilities, there is potential of a run. The 2008 run on money market funds is one such example. Money market funds are an important component of the shadow banking system and are an important source of short-term financing for financial institutions such as Goldman. Money market funds hold commercial paper issued by financial firms such as Goldman and Lehman, and also lend to these dealers in the Triparty Repo market.

  Usually, the assets held by market funds are short term, highly liquid, and of minimum credit risk. This includes short-term Treasury securities and highly rated commercial paper. They mimic bank accounts by allowing check-writing and by fixing the price of a share at $1 – meaning investors could reasonably expect to suffer no losses. Many individual investors keep some cash in money funds, usually in connection with a broader brokerage account. Institutions, including corporations, municipal governments, and pension funds, also find money funds to be a convenient place to park their cash.

  In 2008, one of the money market funds, the Reserve Primary Fund took more risks than many, in an attempt to achieve higher returns and attract more investors. It had invested about $785 million in Lehman's commercial paper, which became worthless after the Lehman default on Monday, September 15, 2008. A run on the fund quickly began, with about $40 billion withdraw (2/3 of the fund's value) by the end of the day on Tuesday.

  The run was not only on this fund alone, it was quickly developing into a run on the entire industry of prime money market funds. In the three weeks between September 10 and October 1, $439 billion would run from the prime funds, while $362 billion would flow in to the government-only funds (funds invested at least 99.5% in cash, short-term Treasury securities, and Repos collateralized by Treasury securities). This run on money market funds also dried up the commercial paper's market, cutting an important source of short-term funding for financial and non-financial companies.

In 2007 and 2008, we also witnessed the runs on financial institutions such as Bear Stearns, Lehman, Merrill, Morgan Stanley, and even Goldman Sachs. Again, one common characteristics of these firms is the liquidity mismatch between their assets and liabilities. Such firms usually rely heavily on short-term liabilities such as inter-bank lending (Fed Funds and Euro-Dollar), Repo financing (Triparty Repo via money market funds), and commercial paper. Unlike commercial banks such as J.P. Morgan, these firms do not have a broad deposit base. The short-term funding sources they rely upon are subject to runs, especially during financial crises, and the runs on money market funds certainly did not help. Moreover, if a bank was suspected to be the next Lehman, it would have even more trouble funding itself through the short-term funding sources in Fed Funds, Repo, or commercial paper. At the same time, its long-term assets are deteriorating and its counterparties are requesting for more collateral for existing liabilities connected with derivatives positions.

As you've read in the popular press, it has been a death spiral in real time. By the way, the Mitsubishi story was in Andrew Ross Sorkin's book on "Too big to fail," which reads like a thriller (if you are looking for entertainment on a weekend).

2 Market Risk Measurement

- **Value-at Risk:** For financial institutions, the larger economic consequences of market risk are felt over relatively short time horizons, often over a few weeks, if not days. Discussions between regulators and their constituent financial institutions have results in a widely applied measure of market risk called value-at-risk.

  For a portfolio of securities (long and short positions), VaR is the potential loss in value due to adverse market movements over a defined time horizon with a specified confidence level.

  - **The scope of the VaR calculation:** Going back to the Goldman's balance sheet, the items listed in Figure 1 will be the scope over which the VaR calculation is done. Moreover, only those financial instruments in Goldman's trading book will be included in the VaR calculation while financial instruments held in

Goldman's banking book are excluded from the VaR calculation. The firm has the discretion in choosing where to allocate a security: to its trading book or banking book. Securities in the banking book are held to maturity, while those in trading books are more frequently traded. So the VaR calculation will cover only the portion of the financial assets listed in Figure 1 that are allocated to the bank's trading book.

- **Confidence level and time horizon:** The typical confidence level p is 99% or 95%, focusing on the 1% or 5% worst-case scenario. To go further out in the tail, sometimes banks also calculation VaR with a confidence level of 99.6%, which is linked to the 0.4% worst-case scenario.

  For a typical broker-dealer or proprietary trading operation, the larger economic consequences of market risk are felt over relatively short time horizons. So the typical time horizon is over two weeks (10 days) or one day.

- **Goldman's VaR:** For both risk management purposes and regulatory capital calculations, Goldman uses a single VaR model which captures risks including those related to interest rates, equity prices, currency rates and commodity prices. The VaR used for regulatory capital requirements (regulatory VaR) differs from risk management VaR due to different time horizons and confidence levels: 10-day and 99% for regulatory VaR and one-day and 95% for risk management VaR. These two VaR calculations also differ in the scope of positions on which VaR is calculated. For our analysis, we will focus on the VaR reported by Goldman for risk management purpose: one-day and 95%.

• **Calculating VaR:** The original intention of the VaR measure is to capture the tail events: the amount of portfolio loss when a 5% left-tail event happens over a day. The actual implementation of the VaR measure, however, relies heavily on the assumption of a normal distribution.

Let's start with a simple example of a portfolio consisting entirely of the S&P 500 index. Suppose that the current market value of the portfolio is \$100 million. Using the historical return data available up to day t, the EWMA model gives us a volatility forecast σ_{t+1} for the next day's stock return R_{t+1}. Standing at day t, the value of the portfolio at the end of day $t+1$ would be $\$100\,\text{M} \times (1 + R_{t+1})$. As discussed in the volatility class, the mean of R_{t+1} is negligible for this one-day horizon. So let's focus on the impact of volatility on the profit/loss of this portfolio.

Focusing on the potential loss, we are interested in how much we would lose if a 5%

tail event happens. Assuming normal distribution, a 5% tail corresponds to a critical value of -1.645σ; a 1% tail corresponds to a critical value of -2.326σ. Using these number, the loss in portfolio value associated with a 5% worst-case scenario would be

$$\text{VaR} = \$100\,\text{M} \times 1.645 \times \sigma_{t+1}$$

For daily returns on the S&P 500 index, the volatility is about 1%. So VaR= $1.645 M.

As you can see, although VaR was designed to capture the tail events, the actual implementation of VaR uses a normal distribution. As a result, calculating VaR bolts down to calculating volatility:

$$\text{VaR} = \text{portfolio value} \times 1.645 \times \text{daily portfolio sigma}\,.$$

Moreover, given how VaR is phrased, one might mistaken VaR as a predictor of the future. In practice, VaR is a measure of the past because the portfolio volatility is estimated using historical returns. In fact, if you calculate the VaR for a risky portfolio right before the any of the crisis, you will not be able to pick up anything above and beyond what the volatility estimate can give you. In this sense, VaR is a more reactive measure: reacting to market volatility.

Figure 4 plots the time-series of VaR for a hypothetical portfolio consisting entirely of the S&P 500 index. Suppose that this portfolio has a market value of $100 million on January 2, 2008. For comparison, I also plot the time-series of the daily EWMA volatility of the S&P 500 index multiplied by 1.645. On January 2, 2008, two time-series started at the same level because VaR = $\$100M \times 1.645 \times \text{Sigma}$.

As shown in Figure 4, the time variation of VaR has two driving forces: the market value of the portfolio and the portfolio volatility. As the year progressed, this passively managed portfolio kept losing its market value. As a result, the blue line (VaR) is lower than the red line (volatility). By late October and early November, it is obvious that the portfolio has lost quite a bit of its value because the difference between the blue line and the red line became quite large. Overall, however, it is obvious that the time variation in VaR tracks the volatility movement quite closely.

- **Calculating VaR for a Portfolio:** As shown in Figure 1, the trading portfolio of a large financial intermediary such as Goldman could be large and complex. In one of its 10K forms, Goldman mentioned 6 million individual positions, 70,000 market factors and 1 million computing hours in its risk management calculations:

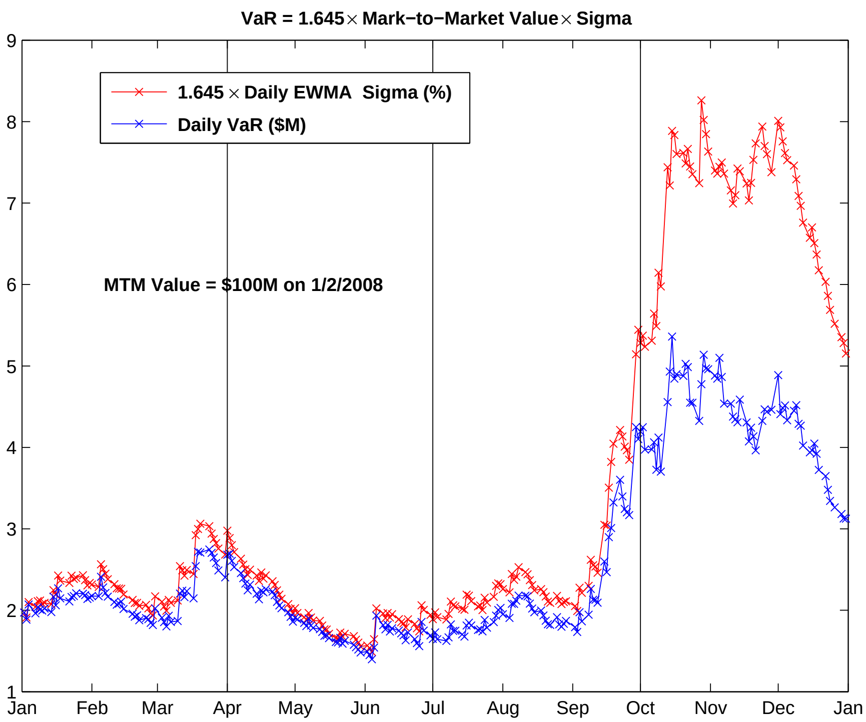

Figure 4: Time series of daily VaR for a portfolio of the S&P 500 Index with an initial market value of $100 million on Jan. 2, 2008.

"We also rely on technology to manage risk effectively. While judgment remains paramount, the speed, comprehensiveness and accuracy of information can materially enhance or hinder effective risk decision making. We mark to market approximately 6 million positions every day. And, we rely on our systems to run stress scenarios across multiple products and regions. In a single day, our systems use roughly 1 million computing hours for risk management calculations.

When calculating VaR, we use historical simulations with full valuation of approximately 70,000 market factors. VaR is calculated at a position level based on simultaneously shocking the relevant market risk factors for that position. We sample from 5 years of historical data to generate the scenarios for our VaR calculation. The historical data is weighted so that the relative importance of the data reduces over time. This gives greater importance to more recent observations and reflects current asset volatilities, which improves the accuracy of our estimates of potential loss. As a result, even if our inventory positions were unchanged, our VaR would increase with increasing market volatility and vice versa."

- **Risk Factors:** The first task of a risk manager is to identify risk factors that are important for risk management purposes. Suppose there are N risk factors. For this N risk factors, the risk manager calculates the covariance-covariance matrix using the EWMA approach. On day t, Σ_{t+1} is the covariance-variance matrix calculated using return data up to day t:

$$\Sigma_{t+1} = \begin{pmatrix} (\sigma_1)^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \dots & \rho_{1N}\sigma_1\sigma_N \\ \rho_{21}\sigma_2\sigma_1 & (\sigma_2)^2 & \rho_{23}\sigma_2\sigma_3 & \dots & \rho_{2N}\sigma_2\sigma_N \\ \rho_{31}\sigma_3\sigma_1 & \rho_{32}\sigma_3\sigma_2 & (\sigma_3)^2 & \dots & \rho_{3N}\sigma_3\sigma_N \\ \dots & \dots & \dots & \dots & \dots \\ \rho_{N1}\sigma_N\sigma_1 & \rho_{N2}\sigma_N\sigma_2 & \rho_{N3}\sigma_N\sigma_3 & \dots & (\sigma_N)^2 \end{pmatrix},$$

  where ρ_{ij} is the correlation between risk factor i and j and σ_i is the volatility for risk factor i. To simplify the notation, I dropped the time-subscripts for ρ and σ, which are EWMA estimates using data up to time t and time-stamped by $t+1$.

- **Risk Mapping:** Given the N risk factors, the next step is to map the individual positions in the firm's portfolio into positions on the risk factor. For example, a $100 million position in AAPL maps to $100 million position in the risk factor for the US equity market. After this risk mapping is done, the risk manager will have a vector of portfolio weights on day t:

$$W_t = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \cdots \\ w_N \end{pmatrix},$$

where w_i is the portfolio weight associated with risk factor i. Again, I dropped the time subscripts for w_i to simplify the notation.

– **Portfolio Volatility and VaR:** Armed with the variance-covariance matrix Σ and the portfolio weights W, the portfolio volatility can be calculated using the matrix operation:

$$\sigma_{t+1}^2 = W_t' \times \Sigma_{t+1} \times W_t \,,$$

where W_t' is the transpose of W_t. This might be a good time for you to get yourself familiar with matrix operations such as mmult and transpose in Excel. Once the portfolio volatility is obtained, the portfolio VaR is

$$\text{VaR} = \text{portfolio value} \times 1.645 \times \text{daily portfolio sigma}\,.$$

If we are interested in calculating VaR for positions related only to interest rates, we can construct an interest rate portfolio weight W^{IR} by turning off the portfolio weights on other risk factors (i.e., making the weights zero). We can then calculate the volatility associated with only the interest rate exposure:

$$\left(\sigma_{t+1}^{\text{IR}}\right)^2 = \left(W_t^{\text{IR}}\right)' \times \Sigma_{t+1} \times W_t^{\text{IR}}$$

- **Goldman's VaR, Magnitude:** Table 3 reports Goldman's daily average VaR for a few selected years. Goldman also reports VaR separately for the risk exposures in interest rates, equity, currency, and commodities. As you can see, the individual VaR's do not add up to equal to the total VaR because of the diversification effect. Only when these four risk factors are perfectly correlated, would we expect to see the four individual VaR's to sum up to equal to the total VaR.

  The VaR numbers for Goldman are in the range of $100 million. Recall that in calculating the these VaRs, the key ingredients are the portfolio value and the portfolio

Table 3: Goldman's Average Daily VaR

Financial Instruments in millions	2014	2010	2008	2007
Long	312,248	356,953	328,325	452,595
Short	132,083	140,717	175,972	215,023
Long - Short ($m)	180,165	216,236	152,353	237,572
Average Daily VaR in millions	2014	2010	2008	2007
Total	72	134	180	138
Interest Rates	51	93	142	85
Equity Prices	26	68	72	100
Currency Rates	19	32	30	23
Commodity Prices	21	33	44	26

volatility:

$$\text{VaR} = \text{portfolio value} \times 1.645 \times \text{daily portfolio sigma}\,.$$

If we know one of them, then knowing VaR can help us back out the other. The problem is that neither the portfolio value or the portfolio sigma is reported by Goldman. Still, let's do some guess work.

Let's first suppose that the long/short positions are paired positions and the net exposure is long minus short. So for 2014, the number is $180,165 million. Suppose that 10% of these positions have been allocated by Goldman to its trading book and fall under the scope of VaR calculation. So portfolio value = $18 billion. Then

$$\text{daily portfolio sigma} = \frac{\text{VaR}}{\text{portfolio value} \times 1.645} = \frac{\$72}{\$18,016.5 \times 1.645} = 24\,\text{basis points}\,.$$

Repeat the same exercise for 2007 (again assuming the trading portfolio consists only 10% of the long-short positions):

$$\text{sigma} = \frac{\text{VaR}}{\text{portfolio value} \times 1.645} = \frac{\$138}{\$23,757.2 \times 1.645} = 35\,\text{basis points}\,.$$

For 2008, the inferred volatility is higher, around 72 basis points. For 2010, it is 31 basis points.

To assess these levels of daily volatility, let's compare them with numbers that we are familiar with. As you know, the equity market has a daily volatility around 100 basis points. For the fixed income market, the standard deviation of the daily changes in the 10-year yields is around 7 basis points. Assuming a duration of 8 years for 10-year bonds, the daily volatility of 10-year treasure bond is about 56 basis points. The typical annual volatility for a currency portfolio is about 9%, making the daily volatility of a currency portfolio at about 57 basis points.

Now back to our inferred volatility of around 24 basis points in 2014, which seems low compared to the numbers we are familiar with. There could be several reasons for this. The diversification benefit across the asset classes will further reduce the overall portfolio volatility. The hedging activities within the trading book will reduce the portfolio volatility. Finally, it is also possible that the trading book is smaller than the 10% assumption we made earlier. Or it could be that the trading book of Goldman is of very low volatility. In any case, this is not meant to be a serious exercise looking into the trading book of Goldman.

- **More on the Portfolio:** In estimating the portfolio value of Goldman, we assumed that the long/short positions are paired and think of the net exposure as long minus short. Let's do a little better than that.

  Using the 2014 number, it is long \$312B and short \$132B. So the portfolio weight on the long portfolio R_t^L is $w^L = 312/(312-132) = 173\%$, the weight on the short portfolio R_t^S is $w^S = -73\%$, and the total portfolio is

$$R_t = w^L\, R_t^L + w^S\, R_t^S = 173\%\, R_t^L - 73\%\, R_t^S$$

  The volatility of the portfolio is

$$\mathrm{var}(R_t) = (w^L)^2\, \mathrm{var}(R_t^L) + (w^S)^2\, \mathrm{var}(R_t^S) + 2\rho\, w^L\, w^S\, \mathrm{std}(R_t^L)\, \mathrm{std}(R_t^S)\,,$$

  where ρ is the correlation between these the long and short portfolios. It is difficult for us to assess the magnitude of ρ without seeing the book. So let's think of different scenarios.

  Suppose $\rho = 1$ and $\mathrm{std}(R_t^L) = \mathrm{std}(R_t^S) = \sigma$, the volatility of the portfolio becomes

$$\mathrm{var}(R_t) = (w^L)^2\, \sigma^2 + (w^S)^2\, \sigma^2 + 2w^L\, w^S\, \sigma^2 = (w^L + w^s)^2 \sigma^2\,.$$

  We are back to the earlier assumption that the long/short positions are paired and the net exposure is \$312 billion minus \$132 billion.

Suppose ρ is not 1 but close to one. It is very likely that there are hedging activities between the long/short portfolios, but the hedging will not take out all of the risk. As a result, the portfolio volatility would be higher because of the leverage involved in the long/short portfolio.

Take the extreme case of $\rho = 0$, and again assuming $\text{std}(R_t^L) = \text{std}(R_t^S) = \sigma$, the portfolio volatility is $\sqrt{(w^L)^2 + (w^S)^2}\sigma$, which is 1.88σ for the 2014 case. This is not surprising because leverage increases portfolio volatility. Again, these are not meant to be a serious investigation into the trading book of Goldman. Rather, I am using them as useful exercises on calculating the volatility of a portfolio.

- **Goldman's VaR, Time-Variation:** Let's also take a look at the time-variation of Goldman's VaR to see if there is anything we can learn. Figure 5 plots Goldman's daily VaR in 2008 (from Goldman's 10K), along with the VaR of a hypothetical portfolio consisting entirely of the S&P 500 index. I set the hypothetical portfolio to have an initial market value of \$8 billion so that the portfolio VaR at the beginning of the year matches the VaR number for Goldman's portfolio. Of course, unlike the passive portfolio in the S&P 500 index, the Goldman's portfolio is actively managed and most likely, the positions were adjusted to the market conditions at the time.

  As shown in Figure 5, for 2008, the Goldman's VaR bottomed to a level close to \$130 million in mid-February (with a visible spike in mid-January). In the last quarter of 2008, Goldman's VaR peaked to a level around \$240 million. As discussed earlier, the VaR of a portfolio increases for two reasons: increasing portfolio volatility or increasing market value of the portfolio. Overall, it is difficult for us to learn too much from this time-series plot of Goldman's VaR. The visible spike in mid-January was interesting (no significant increase in the stock market volatility on the same day), and was probably due to a sudden increase in Goldman's portfolio volatility.

  For a risk manager, sudden spikes in VaR could be alarming as well as informative. It was reported in the media that in December 2006, Goldman's various indicators, including VaR and other risk models, began suggesting that something was wrong. Not hugely wrong, but wrong enough to warrant a closer look. As a result of that effort, Goldman started to reduce their exposure to mortgage-back securities in late 2006.

  In a large financial firm such as Goldman, trading and market-making take place in a decentralized fashion on various trading desks. In calculating the VaR number, individual positions scattered in different parts of the firm are aggregated and compiled

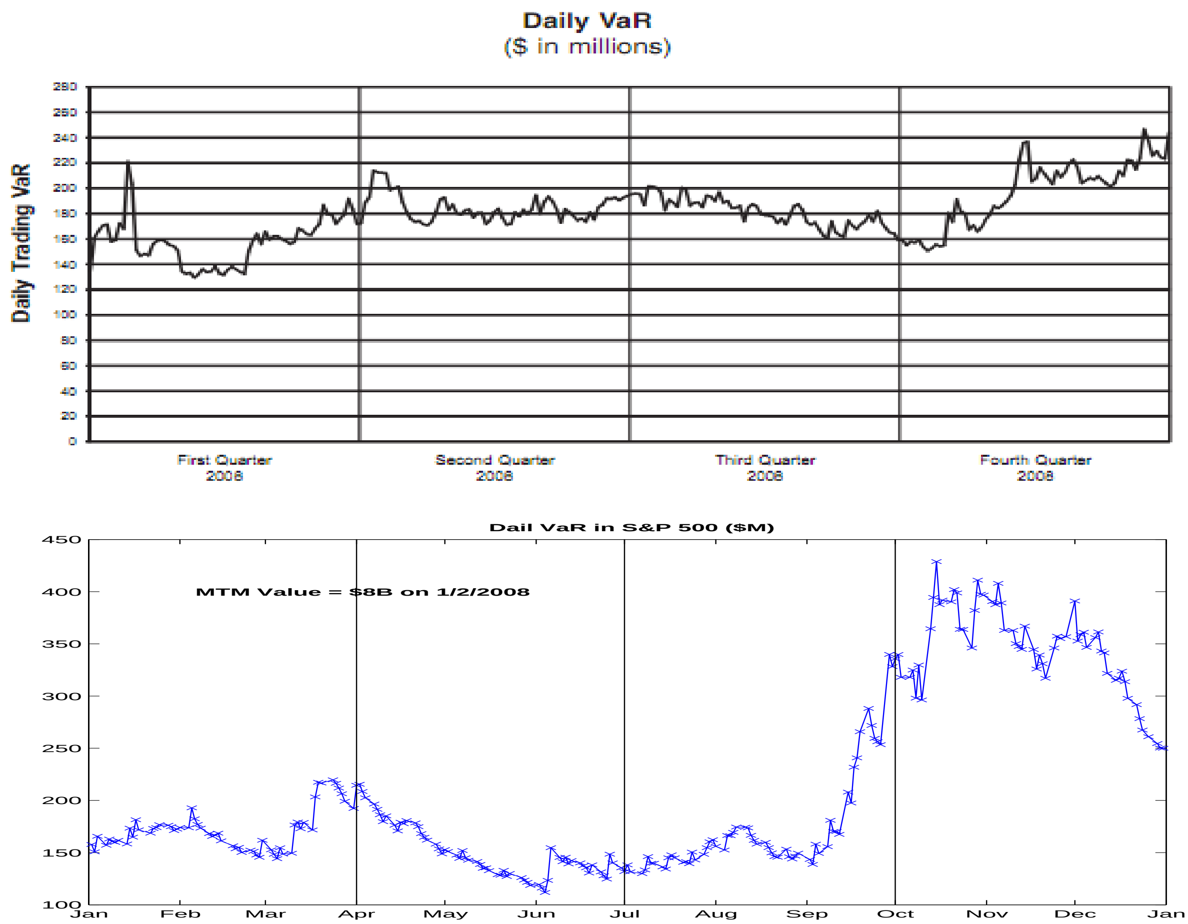

Figure 5: Time-Series of Daily VaR of Goldman Sachs in 2008 vs. Daily VaR of $8 billion in the S&P 500 Index in 2008

into one large portfolio. At the market close, executives of the firm have information of the firm's overall portfolio value as well as its loss and profit from the days before; the portfolio volatility as well as its increase or reduction from the days before. This effort itself is meaningful for the firm, and how to make the VaR measure useful relies crucially on the judgment of a good risk manager.

It would be naive for a risk manager to believe that a VaR of $100 million means that the potential portfolio loss (of a 5% worst-case scenario) is somehow in the neighborhood of $100 million. If this is how VaR is being used in practice, then, quoting the hedge fund manager David Einhorn, VaR is *"relatively useless as a risk-management tool and potentially catastrophic when its use creates a false sense of security among senior managers and watchdogs. This is like an air bag that works all the time, except when you have a car accident."*

- **Days Exceeding VaR:** On each business day, Goldman compares its daily trading net revenues with the VaR calculated at the end of the prior business day and report, in each year's 10K form, the number of days the firm incurs trading losses in excess of the 95% one-day VaR. Figure 6 plots this VaR exception from 1999 through 2014. As a comparison, the VaR exception numbers for a hypothetical portfolio of the S&P 500 index are also plotted in Figure 6.

  Let's start with bottom panel of Figure 6. Given the definition of 95% VaR, the expectation is that the VaR limits would be exceed 5% of the days in a year: $5\% \times 252 = 12.6$. In some years, because of the tail fatness, the days of VaR exception were above 12.6 days (e.g., 2007 and 2008). In general, the numbers fluctuate around 12.6 days per year. The top panel reports the days of VaR exception for Goldman. The results are quite peculiar: most of the years, the numbers were either 0 or 1. Only during the 2007-08 crisis, did these numbers became meaningfully large.

3 Regulatory Requirements

The regulatory requirements for banks makes a very long list and requires exhaustive and patient learning. The landscape of regulatory requirements is still in transition with new rules and requirements phasing in over the next few years. Their effectiveness remains to be evaluated. In the meanwhile, the increasing regulatory requirements have certainly created more risk compliance jobs.

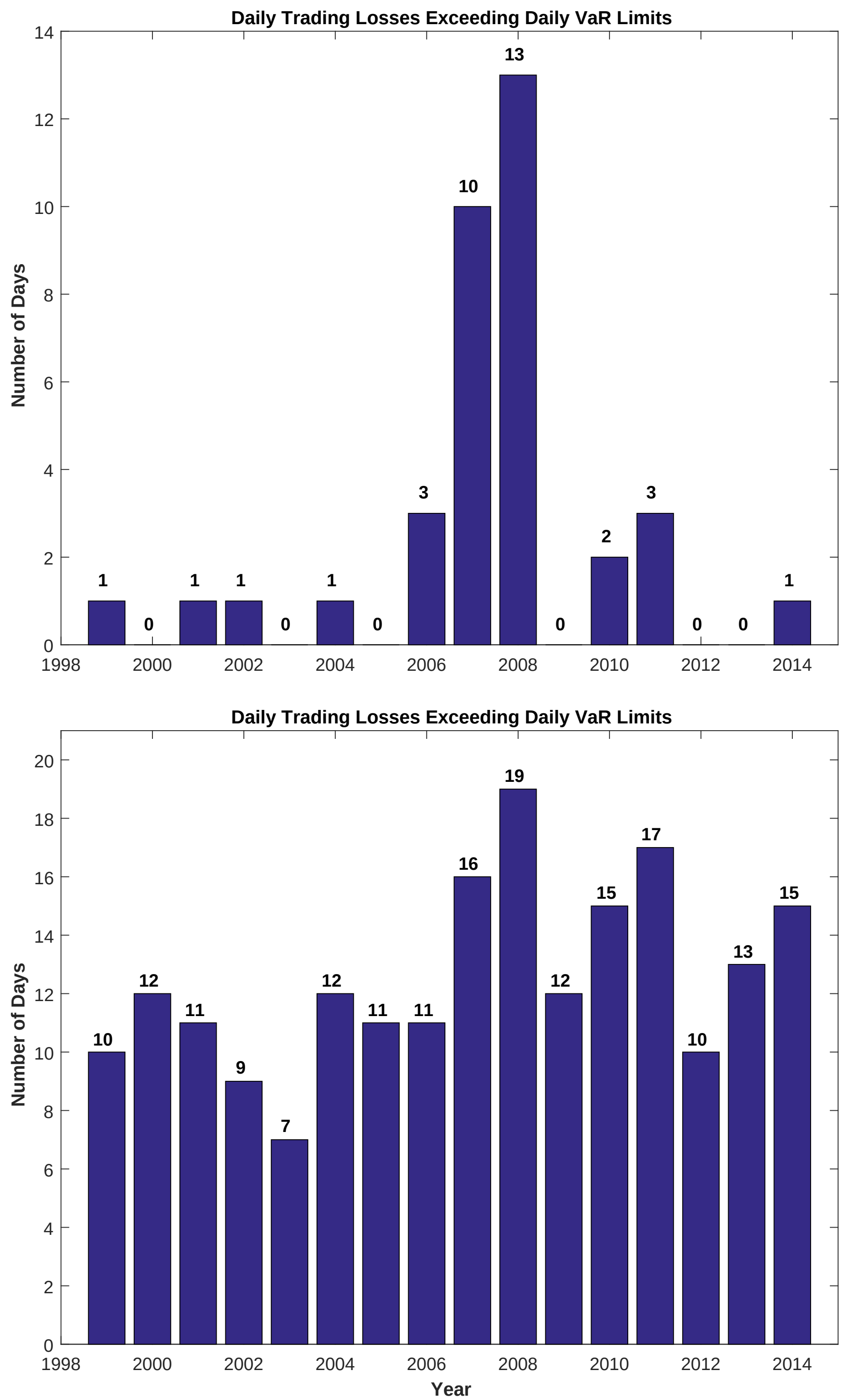

Figure 6: The Number of VaR Exception Days per Year.

- **Capital Adequacy:** As we learned in the example of a simple bank, equity acts as a buffer to cushion the downfall a bank during stressful situations. An important component of the regulatory requirements is expressed as capital ratios that compare measures of regulatory capital to risk weighted assets (RWAs). Capital ratios are ratios of Capital to Assets. Let's take a look at the regulatory measures of these two items separately.

- **Risk Weighted Assets:** Going back to our simple example, the bank holds 10 dollars in cash and 90 dollars in risky loans. For regulatory purpose, the 10 dollars in cash is safe and carries a zero weight in RWA. For the risky loans, there are two kinds of risks: credit and market risk. The bank incurs credit risk because the firms the bank lends to might default. The associated risk weights depend on the type of counterparty (e.g., sovereign, bank, broker-dealer or other entity), the credit worthiness of the counterparty (Aaa, A, Baa, etc), and whether or not the loan is collateralized. In the case of the loan, the bank also incurs market risk because the fluctuations of interest rates. If the bank also holds equity or loans in foreign currencies, then stock market risk and currency risk will also affect the bank's asset. Overall, the bank's RWAs is the sum of its credit RWAs and market RWAs, and most of the regulatory capital ratios are calculated as a ratio to this RWA number.

  Figure 7 reports Goldman's RWA in 2014, which including three components, credit, market, and operational RWAs. The actual calculations of these number requires some training, which I am not at all an expert. But the Table is a good starting point for us to understand the various components of RWA and their relative importance.

  From Figure 7, we can see that the regulatory landscape is still in transition. For example, Goldman reported its RWAs in 2014 under two sets of capital frameworks: Basel III Advanced Rules and Standardized Capital Rules.

- **Regulatory Capital and Capital Ratios:** There are also various ways of measuring regulatory capital, including the newly proposed Common Equity Tier 1 (CET1) capital. Figure 8 is a good starting point to understand the differences in these capital measures. Essentially, what matters in capital requirement is the quantity as well as quality of the capital.

  Capital requirements are expressed as capital ratios of the various regulatory capitals to RWAs. Figure 9 the minimum ratios under the Revised Capital Framework as of December 2014 and January 2015, as well as the minimum ratios that expected by Goldman to apply at the end of the transitional provisions beginning January 2019.

$ in millions	As of December 2014 Basel III Advanced	 Standardized
Credit RWAs		
Derivatives	**$122,501**	**$180,771**
Commitments, guarantees and loans	**95,209**	**89,783**
Securities financing transactions [1]	**15,618**	**92,116**
Equity investments	**40,146**	**38,526**
Other [2]	**54,470**	**71,499**
Total Credit RWAs	**327,944**	**472,695**
Market RWAs		
Regulatory VaR	**10,238**	**10,238**
Stressed VaR	**29,625**	**29,625**
Incremental risk	**16,950**	**16,950**
Comprehensive risk	**8,150**	**9,855**
Specific risk	**79,918**	**79,853**
Total Market RWAs	**144,881**	**146,521**
Total Operational RWAs	**97,488**	**—**
Total RWAs	**$570,313**	**$619,216**

1. Represents resale and repurchase agreements and securities borrowed and loaned transactions.

2. Includes receivables, other assets, and cash and cash equivalents.

Figure 7: Credit, Market, and Operational Risk Weighted Assets Reported by Goldman Sachs.

$ in millions	As of December 2014
Common shareholders' equity	$ 73,597
Deductions for goodwill and identifiable intangible assets, net of deferred tax liabilities	(2,787)
Deductions for investments in nonconsolidated financial institutions	(953)
Other adjustments	(27)
Common Equity Tier 1	**69,830**
Perpetual non-cumulative preferred stock	9,200
Junior subordinated debt issued to trusts	660
Other adjustments	(1,257)
Tier 1 capital	**78,433**
Qualifying subordinated debt	11,894
Junior subordinated debt issued to trusts	660
Other adjustments	(9)
Tier 2 capital [1]	**12,545**
Total capital	**$ 90,978**

Figure 8: Regulatory Capital.

	December 2014 Minimum Ratio [1]	January 2015 Minimum Ratio [1]	January 2019 Minimum Ratio
CET1 ratio	4.0%	4.5%	8.5% [4]
Tier 1 capital ratio	5.5%	6.0%	10.0% [4]
Total capital ratio	8.0% [3]	8.0% [3]	12.0% [4]
Tier 1 leverage ratio [2]	4.0%	4.0%	4.0%

Figure 9: Minimum Capital Ratios and Capital Buffers.

The framework of RWA has been subject to much criticism, especially given that banks are allowed to use their own risk models to calculate Market RWAs. Tier 1 leverage ratio moves away from the RWA framework, and measures the ratio of Tier 1 capital to the average adjusted total assets.

- **Liquidity Adequacy:** We also learned in our simple example that the liquidity mismatch between the assets and liabilities is the root cause of runs on financial institutions. The more recent regulatory effort in Basel III pays special attention to this liquidity issue and proposed two liquidity measures.

  – **LCR:** The measure of Leverage Coverage Ratio (LCR) is to promote the short-term resilience of the liquidity risk profile of banks. It does so by ensuring that banks have an adequate stock of unencumbered high-quality liquidity assets that can be converted easily and immediately in private markets into cash to meet their liquidity needs.

  – **NSFR:** Another proposed measure in Basel III is Net Stable Funding Ratio (NSFR), which requires that long-term financing resources (e.g., equity and any liability maturing after one year, retail deposits, deposits from non-financial corporates and public entities) must exceed long-term commitments.

- **The Last Taxi Cab in the Train Station:** I heard this story from Prof. Doug Diamond who was the Fischer Black Visiting Professor of Finance at MIT Sloan in 2015.

  On a cold and rainy night, the last train arrived at a small town in ... Europe. There was just one passenger getting off from the train and he is tired and hungry and eager to go home. There was one taxi cab waiting at the train station. The passenger got in and asked to be taken to his home, which is only a few miles away from the train station. But the taxi driver told him that he cannot take him there. According to the local law, there must always be one taxi cab waiting at the train station.

  It is one of those story that sounds crazy and yet not totally crazy. Going back to the regulatory requirements on capital and liquidity adequacy, it is possible that banks are required to hold liquidity that goes unused, just like the last taxi cab at the train station. But this does not necessarily mean that the unused liquidity was not useful. In a way, the presence of the unused liquidity deters the run on the financial institution. For a bank, the calculation would be how costly it is to hold the unused liquidity vs the cost of a run. For regulators, the concern is not on just one bank, but the liquidity and stability of the entire financial system. As such, they would want to focus on the liquidity adequacy of those highly connected financial institutions.