

# Class 3: The Yield Curve

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**Jun Pan**

**Shanghai Advanced Institute of Finance (SAIF)  
Shanghai Jiao Tong University**

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- Risk factors affecting the bond market:
  - ▶ Level: Parallel shifts of the yield curve; duration hedging.
  - ▶ Slope: Differential movements between the long- and short-term interest rates.
- Principal component analysis (PCA):
  - ▶ Identify common risk components in the data.
  - ▶ Quantify the risk factors in terms of their importance.
- PCA Applications: financial markets with common risk factors.
  - ▶ Bond markets: across maturities.
  - ▶ Domestic equity markets: SMB, HML, momentum, etc.
  - ▶ International equity markets: across countries.
  - ▶ Currency markets: across currencies.
  - ▶ Commodity futures: across maturities.

## An Illustrative Example

- To simplify our analysis, let's start with zero-coupon bonds.
- Coupon bonds: weighted sums of the zero-coupon bond prices.
- Initially, the zero rates are:

|                   | 2yr                 | 5yr                 | 10yr               | 30yr                 |
|-------------------|---------------------|---------------------|--------------------|----------------------|
| zero rate         | 3.5%                | 4.5%                | 5%                 | 5.5%                 |
| maturity          | 2                   | 5                   | 10                 | 30                   |
| modified duration | $\frac{2}{1+3.5\%}$ | $\frac{5}{1+4.5\%}$ | $\frac{10}{1+5\%}$ | $\frac{30}{1+5.5\%}$ |
|                   | 1.93                | 4.78                | 9.52               | 28.44                |

- Using the 2yr zero as the reference bond,
  - ▶ 5-to-2 spread: 100 bps.
  - ▶ 10-to-2 spread: 150 bps.
  - ▶ 30-to-2 spread: 200 bps.

## Directional Trade to Bet on a Parallel Shift in Level

- The yield on the 2yr zero subsequently increases by 10 bps.
- All spreads remain the same.

|      | notional<br>(\$ million) | market value (\$ million) |  |             |                                |
|------|--------------------------|---------------------------|--|-------------|--------------------------------|
|      |                          | initial                   | later  | change      | approximation                  |
| 2yr  | $(1 + 3.5\%)^2$          | 1                         | $\frac{(1+3.5\%)^2}{(1+3.5\%+10\text{bps})^2}$       | -19.30 bps  | $-1.93 \times 10 \text{ bps}$  |
| 5yr  | $(1 + 4.5\%)^5$          | 1                         | $\frac{(1+4.5\%)^5}{(1+4.5\%+10\text{bps})^5}$       | -47.71 bps  | $-4.78 \times 10 \text{ bps}$  |
| 10yr | $(1 + 5.0\%)^{10}$       | 1                         | $\frac{(1+5.0\%)^{10}}{(1+5.0\%+10\text{bps})^{10}}$ | -94.74 bps  | $-9.52 \times 10 \text{ bps}$  |
| 30yr | $(1 + 5.5\%)^{30}$       | 1                         | $\frac{(1+5.5\%)^{30}}{(1+5.5\%+10\text{bps})^{30}}$ | -280.22 bps | $-28.44 \times 10 \text{ bps}$ |

# Steepener

- Your view:
  - ▶ The 10-to-2 spread will increase.
  - ▶ No view on the overall direction of the interest rate.
- Your strategy: steepener.
  - ▶ Long \$4.9286M of 2yr zero.
  - ▶ Short \$1M of 10yr zero.

|      | initial market value<br>(\$ million) | notional amount<br>(\$ million) |
|------|--------------------------------------|---------------------------------|
| 2yr  | 4.9286                               | $4.9286 \times (1 + 3.5\%)^2$   |
| 10yr | -1                                   | $-(1 + 5.0\%)^{10}$             |

- Why 4.9286? Because  $4.9286 = 9.52 / 1.93$ .

## Two Scenarios:

- ① A parallel shift of +10 bps:

|       | initial<br>(\$ Million) | later<br>(\$ Million)  | change<br>(\$ Million ) | approximation<br>(\$ Million)               |
|-------|-------------------------|--|-------------------------|---|
| 2yr   | 4.9286                  | $\frac{4.9286 \times (1+3.5\%)^2}{(1+3.5\%+10\text{bps})^2}$ | -95.10 bps              | $-1.93 \times 10 \text{ bps} \times 4.9286$ |
| 10yr  | -1                      | $-\frac{(1+5.0\%)^{10}}{(1+5.0\%+10\text{bps})^{10}}$        | 94.74 bps               | $9.52 \times 10 \text{ bps}$                |
| Total |                         |  | <b>-0.36 bps</b>        |   |

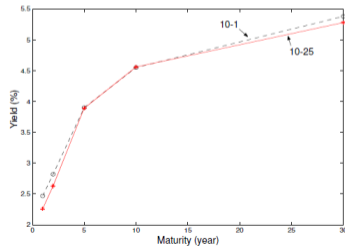
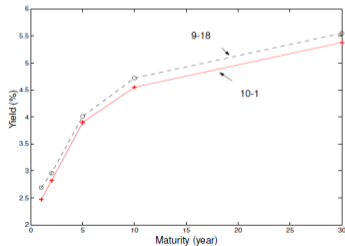
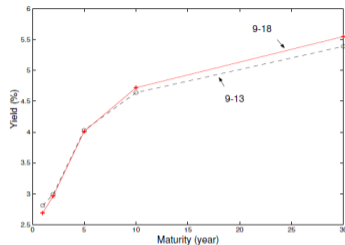
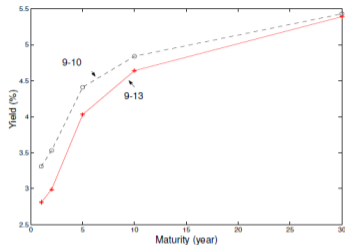
- ② The 2yr zero rate decreases by 15 bps and the 10yr zero rate increases by 5 bps.

|       | initial<br>(\$ Million) | later<br>(\$ Million)  | change<br>(\$ Million ) | approximation<br>(\$ Million)              |
|-------|-------------------------|--|-------------------------|--|
| 2yr   | 4.9286                  | $\frac{4.9286 \times (1+3.5\%)^2}{(1+3.5\%-15\text{bps})^2}$ | 143.17 bps              | $1.93 \times 15 \text{ bps} \times 4.9286$ |
| 10yr  | -1                      | $-\frac{(1+5.0\%)^{10}}{(1+5.0\%+5\text{bps})^{10}}$         | 47.49 bps               | $9.52 \times 5 \text{ bps}$                |
| Total |                         |  | <b>190.66 bps</b>       |  |

## Curvature (or Butterfly):

- Your view: the 2yr and 30yr zeros will move in the same direction while the 10yr zero will move in opposite direction. Not sure about the overall directional of the interest rate, nor about the slope of the yield curve.
- Your strategy: butterfly trade.
- Long 30yr zeros, short 10yr zeros (to hedge against parallel shifts), and long 2yr zeros (to hedge against slope steepening or flattening).

# Yield Curve Movement after 911

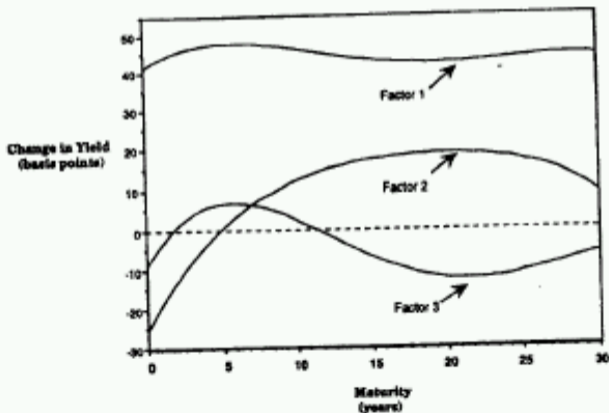




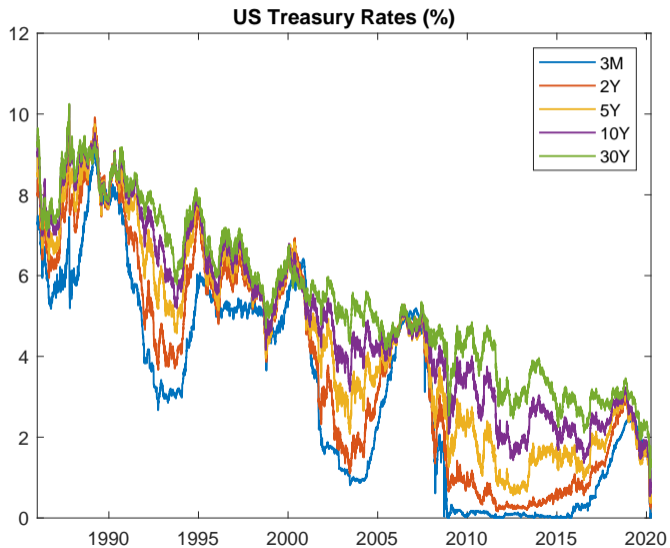
## Three Major Factors in the Fixed-Income Market:

Closely related to the three trading strategies are the three major risk factors reported by Litterman and Scheinkman for the fixed income market:

**FIGURE 2 ■ Yield Curve Impact**



# Treasury Yields



## Variance-Covariance Matrix

$\text{cov}(\Delta y)$  (unit:  $\text{bps}^2$ , or equivalently,  $\times 10^{-8}$ )

|     | 3M      | 1Y      | 2Y      | 5Y      | 10Y     | 30Y     |
|-----|---------|---------|---------|---------|---------|---------|
| 3M  | 64.9300 | 40.4601 | 32.1459 | 27.1468 | 22.3368 | 17.8525 |
| 1Y  | 40.4601 | 48.3104 | 42.5293 | 39.0606 | 33.7733 | 27.6243 |
| 2Y  | 32.1459 | 42.5293 | 48.4517 | 45.1757 | 39.4565 | 31.9805 |
| 5Y  | 27.1468 | 39.0606 | 45.1757 | 51.6705 | 46.6458 | 38.8411 |
| 10Y | 22.3368 | 33.7733 | 39.4565 | 46.6458 | 47.5872 | 40.7433 |
| 30Y | 17.8525 | 27.6243 | 31.9805 | 38.8411 | 40.7433 | 39.7206 |

$\text{corr}(\Delta y)$

|     | 3M     | 1Y     | 2Y     | 5Y     | 10Y    | 30Y    |
|-----|--------|--------|--------|--------|--------|--------|
| 3M  | 1.0000 | 0.7224 | 0.5731 | 0.4687 | 0.4018 | 0.3515 |
| 1Y  | 0.7224 | 1.0000 | 0.8790 | 0.7818 | 0.7044 | 0.6306 |
| 2Y  | 0.5731 | 0.8790 | 1.0000 | 0.9029 | 0.8217 | 0.7290 |
| 5Y  | 0.4687 | 0.7818 | 0.9029 | 1.0000 | 0.9407 | 0.8574 |
| 10Y | 0.4018 | 0.7044 | 0.8217 | 0.9407 | 1.0000 | 0.9371 |
| 30Y | 0.3515 | 0.6306 | 0.7290 | 0.8574 | 0.9371 | 1.0000 |

# Principal Component Analysis

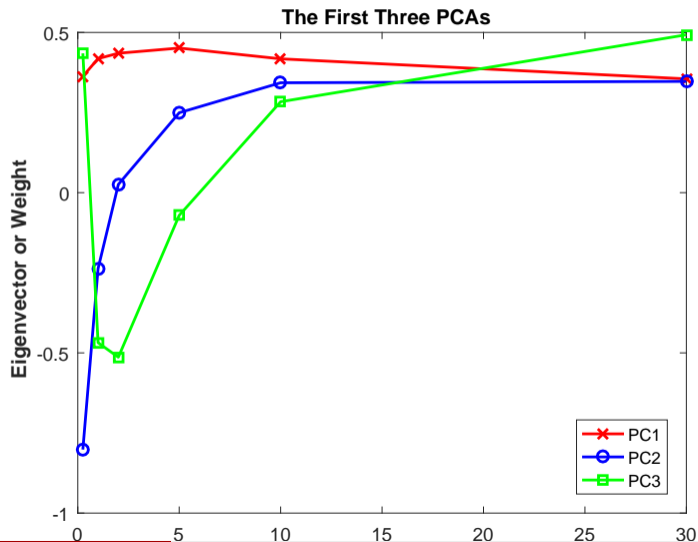
## Eigenvalues E

| E                     | PC1    | PC2   | PC3   | PC4  | PC5  | PC6  |
|-----------------------|--------|-------|-------|------|------|------|
| E (bps <sup>2</sup> ) | 226.99 | 50.14 | 13.77 | 5.45 | 2.86 | 1.47 |
| E/sum(E) (%)          | 75.49  | 16.68 | 4.58  | 1.81 | 0.95 | 0.49 |

## Eigenvectors D

| D   | PC1    | PC2     | PC3     | PC4     | PC5     | PC6     |
|-----|--------|---------|---------|---------|---------|---------|
| 3M  | 0.3630 | -0.8017 | 0.4347  | 0.1876  | -0.0365 | 0.0006  |
| 1Y  | 0.4182 | -0.2371 | -0.4682 | -0.6806 | 0.2939  | 0.0016  |
| 2Y  | 0.4351 | 0.0257  | -0.5134 | 0.3309  | -0.6505 | 0.1176  |
| 5Y  | 0.4513 | 0.2493  | -0.0709 | 0.4572  | 0.5076  | -0.5124 |
| 10Y | 0.4176 | 0.3430  | 0.2837  | 0.0418  | 0.2271  | 0.7577  |
| 30Y | 0.3550 | 0.3472  | 0.4926  | -0.4258 | -0.4242 | -0.3866 |

# The First Three PCAs



## Level, Slope, and Curvature

- They name the first factor the *level* factor as it has a similar impact of the yield curve as a parallel shift of the yield curve.
- They call the second factor *steepness*, since shock from this factor lowers the yields of zeros up to five years, and raises the yields for zeros of longer maturities.
- They call the third factor *curvature*, since it increases curvature of the yield curve in the range of maturities below twenty years.

## The Relative Importance of the Three Factors:

**Table 2 • Implied Zeroes: Relative Importance of Factors • (Percent)**

| <u>Maturity</u> | <u>Total Variance Explained</u> | <u>Proportion of Total Explained Variance Accounted for by</u> |                 |                 |
|-----------------|---------------------------------|--|-----------------|-----------------|
|                 |                                 | <u>Factor 1</u>  | <u>Factor 2</u> | <u>Factor 3</u> |
| 6 months        | 99.5                            | 79.5   | 17.2            | 3.3             |
| 1 year          | 99.4                            | 89.7   | 10.1            | 0.2             |
| 2 years         | 98.2                            | 93.4   | 2.4             | 4.2             |
| 5 years         | 98.8                            | 98.2   | 1.1             | 0.7             |
| 8 years         | 98.7                            | 95.4   | 4.6             | 0.0             |
| 10 years        | 98.8                            | 92.9   | 6.9             | 0.2             |
| 14 years        | 98.4                            | 86.2   | 11.5            | 2.2             |
| 18 years        | 95.3                            | 80.5   | 14.3            | 5.2             |
| <b>Average</b>  | <b>98.4</b>                     | <b>89.5</b>  | <b>8.5</b>      | <b>2.0</b>      |

The first factor is by far the most important, supporting the idea that “first factor” hedging – or its close cousin, duration hedging – takes care of most of the return risk.

$$\Delta y_t = a + \beta^{PC1} PC1_t + \beta^{PC2} PC2_t + \beta^{PC3} PC3_t + \epsilon_t.$$

|     | PC1<br>$\beta$ | PC2<br>$\beta$ | PC3<br>$\beta$ | PC1<br>R2 (%) | PC2<br>R2 (%) | PC3<br>R2 (%) | Total<br>R2 (%) |
|-----|----------------|----------------|----------------|---------------|---------------|---------------|-----------------|
| 3M  | 0.3630         | -0.8017        | 0.4347         | 46.06         | 49.63         | 4.01          | 99.70           |
| 1Y  | 0.4182         | -0.2371        | -0.4682        | 82.18         | 5.83          | 6.25          | 94.26           |
| 2Y  | 0.4351         | 0.0257         | -0.5134        | 88.67         | 0.07          | 7.49          | 96.23           |
| 5Y  | 0.4513         | 0.2493         | -0.0709        | 89.46         | 6.03          | 0.13          | 95.62           |
| 10Y | 0.4176         | 0.3430         | 0.2837         | 83.17         | 12.39         | 2.33          | 97.89           |
| 30Y | 0.3550         | 0.3472         | 0.4926         | 72.04         | 15.22         | 8.41          | 95.66           |



## Daily Currency Returns from 2000 through 2016

| std (%)  | GBP  | EUR  | AUD  | CAD  | CNY  | INR  | JPY  | CHF  | THB  | RUB  |
|----------|------|------|------|------|------|------|------|------|------|------|
|          | 0.60 | 0.63 | 0.82 | 0.59 | 0.10 | 0.38 | 0.64 | 0.70 | 0.36 | 0.77 |
| corr (%) | GBP  | EUR  | AUD  | CAD  | CNY  | INR  | JPY  | CHF  | THB  | RUB  |
| GBP      |      | 64.4 | 53.1 | 45.3 | 12.9 | 25.2 | 12.3 | 49.7 | 18.6 | 23.9 |
| EUR      | 64.4 |      | 55.3 | 46.0 | 10.4 | 23.3 | 27.5 | 78.2 | 21.1 | 25.7 |
| AUD      | 53.1 | 55.3 |      | 62.3 | 12.8 | 33.9 | 2.9  | 39.2 | 23.4 | 32.8 |
| CAD      | 45.3 | 46.0 | 62.3 |      | 9.4  | 27.0 | 1.0  | 32.7 | 19.9 | 33.3 |
| CNY      | 12.9 | 10.4 | 12.8 | 9.4  |      | 16.8 | 5.6  | 8.8  | 16.1 | 10.7 |
| INR      | 25.2 | 23.3 | 33.9 | 27.0 | 16.8 |      | -4.0 | 15.1 | 25.5 | 26.6 |
| JPY      | 12.3 | 27.5 | 2.9  | 1.0  | 5.6  | -4.0 |      | 37.5 | 17.6 | -1.1 |
| CHF      | 49.7 | 78.2 | 39.2 | 32.7 | 8.8  | 15.1 | 37.5 |      | 17.7 | 19.3 |
| THB      | 18.6 | 21.1 | 23.4 | 19.9 | 16.1 | 25.5 | 17.6 | 17.7 |      | 15.9 |
| RUB      | 23.9 | 25.7 | 32.8 | 33.3 | 10.7 | 26.6 | -1.1 | 19.3 | 15.9 |      |

## The First Three PCs

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The first three PC's of currency returns

|          | PC 1   | PC 2    | PC3     |
|----------|--------|---------|---------|
| E/sum(E) | 44.32% | 17.07%  | 12.27%  |
| GBP      | 0.3502 | 0.0715  | 0.1382  |
| EUR      | 0.4191 | 0.2531  | 0.0037  |
| AUD      | 0.5410 | -0.2512 | 0.4398  |
| CAD      | 0.3259 | -0.1870 | 0.1890  |
| CNY      | 0.0122 | -0.0032 | -0.0046 |
| INR      | 0.1154 | -0.1104 | 0.0113  |
| JPY      | 0.1226 | 0.5569  | -0.3908 |
| CHF      | 0.4100 | 0.4488  | -0.1497 |
| THB      | 0.0882 | 0.0084  | -0.0390 |
| RUB      | 0.3138 | -0.5558 | -0.7582 |

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## The First Three PCs

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The first three PC's of currency returns

|          | PC 1   | PC 2    | PC3     |
|----------|--------|---------|---------|
| E/sum(E) | 54.03% | 20.25%  | 10.10%  |
| GBP      | 0.3748 | -0.0312 | 0.2595  |
| EUR      | 0.4538 | 0.1878  | 0.2925  |
| AUD      | 0.5621 | -0.4869 | -0.4163 |
| CAD      | 0.3324 | -0.2946 | -0.1879 |
| JPY      | 0.1499 | 0.6699  | -0.7030 |
| CHF      | 0.4524 | 0.4371  | 0.3799  |

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| Fama-French 25 Portfolios |        |         |         |
|---------------------------|--------|---------|---------|
|                           | PC 1   | PC 2    | PC3     |
| E/sum(E)                  | 83.84% | 4.39%   | 3.19%   |
| A1                        | 0.2874 | -0.5850 | -0.6434 |
| A2                        | 0.2456 | -0.2909 | 0.4001  |
| A3                        | 0.2378 | -0.1526 | 0.0693  |
| A4                        | 0.2225 | -0.1552 | 0.1332  |
| A5                        | 0.2468 | -0.1586 | 0.0062  |
| B1                        | 0.2048 | -0.1685 | 0.2221  |
| B2                        | 0.2013 | -0.0520 | 0.2160  |
| B3                        | 0.1981 | -0.0093 | 0.1173  |
| B4                        | 0.2033 | 0.0110  | 0.0517  |
| B5                        | 0.2353 | -0.0044 | -0.0768 |
| C1                        | 0.1938 | -0.0751 | 0.1897  |
| C2                        | 0.1732 | 0.0673  | 0.1030  |
| C3                        | 0.1765 | 0.1135  | 0.0537  |
| C4                        | 0.1873 | 0.1154  | 0.0207  |
| C5                        | 0.2283 | 0.1122  | -0.1330 |
| D1                        | 0.1555 | 0.0565  | 0.1347  |
| D2                        | 0.1606 | 0.1471  | 0.0868  |
| D3                        | 0.1707 | 0.1824  | 0.0360  |
| D4                        | 0.1810 | 0.1743  | -0.0947 |
| D5                        | 0.2289 | 0.2343  | -0.1251 |
| E1                        | 0.1264 | 0.1461  | 0.0572  |
| E2                        | 0.1298 | 0.2066  | 0.0048  |
| E3                        | 0.1380 | 0.2427  | -0.0676 |
| E4                        | 0.1662 | 0.2704  | -0.1142 |
| E5                        | 0.2050 | 0.2846  | -0.3807 |

| Momentum 25 Portfolios |        |         |         |
|------------------------|--------|---------|---------|
|                        | PC 1   | PC 2    | PC 3    |
| E/sum(E)               | 82.40% | 5.53%   | 4.06%   |
| A1                     | 0.2791 | 0.0183  | 0.4184  |
| A2                     | 0.2498 | -0.0775 | 0.3119  |
| A3                     | 0.2311 | -0.1137 | 0.2692  |
| A4                     | 0.2267 | -0.2113 | 0.1814  |
| A5                     | 0.2150 | -0.3085 | 0.0656  |
| B1                     | 0.2629 | 0.1010  | 0.1670  |
| B2                     | 0.2212 | -0.0021 | 0.1058  |
| B3                     | 0.1945 | -0.0576 | 0.0161  |
| B4                     | 0.1906 | -0.1614 | -0.0098 |
| B5                     | 0.1963 | -0.2603 | -0.1152 |
| C1                     | 0.2480 | 0.2102  | 0.0695  |
| C2                     | 0.2057 | 0.0630  | -0.0203 |
| C3                     | 0.1844 | -0.0147 | -0.0639 |
| C4                     | 0.1633 | -0.0974 | -0.1426 |
| C5                     | 0.1654 | -0.2323 | -0.2459 |
| D1                     | 0.2399 | 0.2932  | -0.0217 |
| D2                     | 0.1898 | 0.1263  | -0.1283 |
| D3                     | 0.1665 | 0.0267  | -0.1472 |
| D4                     | 0.1569 | -0.0562 | -0.2211 |
| D5                     | 0.1499 | -0.2015 | -0.3146 |
| E1                     | 0.2049 | 0.6397  | -0.1310 |
| E2                     | 0.1573 | 0.1926  | -0.1621 |
| E3                     | 0.1430 | 0.0898  | -0.2025 |
| E4                     | 0.1228 | -0.0192 | -0.2724 |
| E5                     | 0.1243 | -0.1436 | -0.3605 |