Class 3: Modeling the Yield Curve

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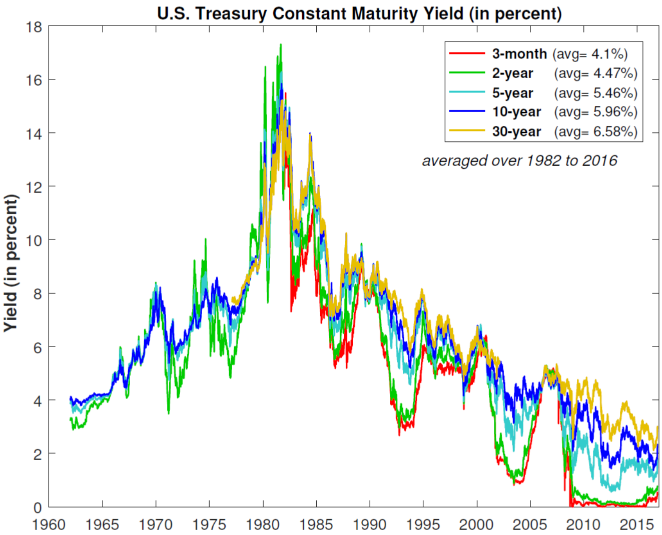
Outline

- Term-structure modeling is employed by investment banks and hedge funds to
  - Arbitrage trading across the yield curve.
  - Pricing and hedging securities with interest-rate exposures.
  - Security designs: interest-rate derivatives; collateralized mortgage obligations.
- The main insight:
  - Various parts of the yield curve are inter-connected.
  - Focus on the common risk factors: level, slope, volatility.
- Some well-known models and their challenges:
  - Merton (Ho-Lee) model, Black-Karasinski, and Black-Derman-Toy.
  - Vasicek, Cox-Ingersoll-Ross (CIR), and affine models.
  - Tractable enough to allow for fast pricing and hedging calculations.
- Our focus today:
  - How to calibrate the model to the data?
  - Use term structure models to identify trading opportunities.

US Treasury Yield Curve



The Vasicek Model

- The Vasicek model is a continuous-time term-structure model:

$$dr_t = \kappa\,(\bar{r} - r_t)\;dt + \sigma\,dB_t$$

- For this class, let's use the discrete-time version of the model. Let r_t be the three-month T-bill rate at time t, and $r_{t+\Delta}$ be the three-month T-bill rate at the next Δ instant:

$$r_{t+\Delta} - r_t = (\bar{r} - r_t)\,\kappa\,\Delta + \sigma\,\sqrt{\Delta}\,\epsilon_{t+\Delta}$$

- At any time t, the short rate is subject to a new shock $\epsilon_{t+\Delta}$, which is standard normally distributed. Shocks are independent across time.

The Parameters for the Model

- $\bar{r}$ controls the normal level of the interest rates, or the long-run mean of the interest rates:

$$E(r_t) = \bar{r}$$

- σ controls the conditional variance:

$$\text{var}\left(r_{t+\Delta} \middle| r_t\right) = \sigma^2 \frac{1 - \exp(-2\kappa\Delta)}{2\kappa} \approx \sigma^2 \Delta$$

- κ controls the rate at which the interest rate reverts to its long-run mean $\bar{r}$.
  - When κ is big, any deviation from the long-run mean will be pulled back to its normal level $\bar{r}$ pretty quickly.
  - When κ is small, it takes a long time for the interest rate to come back to its normal level. Interest rates are very persistent in this situation.

Bond Pricing

Suppose that the time-t three-month T-bill rate is r_t and suppose that we know κ, σ, and $\bar{r}$. According to the Vasicek model, the price of T-year zero-coupon bond with face value of \$1 is determined by

$$P_t = e^{A + B\, r_t}\,,$$

where

$$B = \frac{e^{-\kappa T} - 1}{\kappa}$$

$$A = \bar{r}\left(\frac{1 - e^{-\kappa T}}{\kappa} - T\right) + \frac{\sigma^2}{2\kappa^2}\left(\frac{1 - e^{-2\kappa T}}{2\kappa} - 2\frac{1 - e^{-\kappa T}}{\kappa} + T\right)$$

Calibrating the Model, using Time-Series Data

- To be useful, the model parameters need to be calibrated to the data.
- Suppose you are given a time-series of three-month T-bill rates, observed with monthly frequency.
- The Vasicek model is equivalent to

$$r_{t+1} = a + b\, r_t + c\, \epsilon_{t+1} \,,$$

where $a = \kappa \bar{r} \Delta$, $b = 1 - \kappa \Delta$, and $c = \sigma \sqrt{\Delta}$.
- If the interest rates are observed in monthly frequency, then $\Delta = 1/12$.
- If we know that $\kappa = 0.1$, $\sigma = 0.01$, and $\bar{r} = 5\%$, then $a = \kappa \bar{r}/12 = 0.005$, $b = 1 - \kappa/12 = 0.9917$, and $c = \sigma\sqrt{1/12} = 0.007071$.
- Conversely, if you know how to estimate a, b, c, you can back out κ, σ, and $\bar{r}$.

Calibrating the model, using the Yield Curve

- Instead of using the historical time-series data to estimate the model, the industry practice is to calibrate the model to the yield curve.
- Given r, κ, $\bar{r}$, and σ, the model can price bonds of any maturities.
- On any given day, we observe prices and yields of various maturities. We can take advantage of these market-traded prices by forcing the model to price such bonds as precisely as possible.
- In other words, κ, $\bar{r}$, and σ are calibrated to today's yield curve. Tomorrow, we repeat the same exercise and end up with a different set of model parameters.

Relative Value Investing

Excerpts from Chifu Huang's Guest Lecture at MIT Sloan in March 2011

- Relative value investing takes the view that deviations from any reasonable/good model is created by transitory supply and demand imbalances originated from
  - Clientele effects and institutional rigidity;
  - Derivatives hedging;
  - Accounting/tax rules;
- These imbalances dissipate over time as
  - Economics of substitution takes hold.
  - Imbalances reverse themselves as market conditions change.

Relative Value Investing

Excerpts from Chifu Huang's Guest Lecture at MIT Sloan in March 2011

- Do not make judgment on level of interest rates or slope of the curve.
- Assume that a few points on the yield curve are always fair. For example:
  - 10-year rate: capturing the level of long-term interest rates.
  - 2-year rate: together with 10-yr rate, capturing the slope of the curve.
  - 1-month rate: capturing short term interest rate/expectation on monetary policy in the near term.
- Predicting level of interest rates of other maturities or their cheapness/richness "relative to" the presumed fair maturities.
- Buying/selling cheap/rich maturities hedged with fair maturities to make the portfolio insensitive to changes of the level and the slope of the yield curve and to changes of monetary policy.

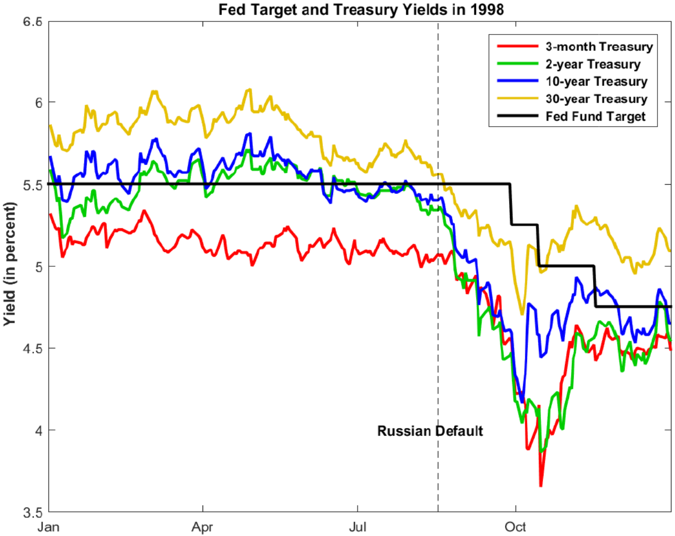
Market Price vs. Model Price

Crisis Behavior of 30-yr Cheapness/Richness

Aug-Sep 1998 Russia default:

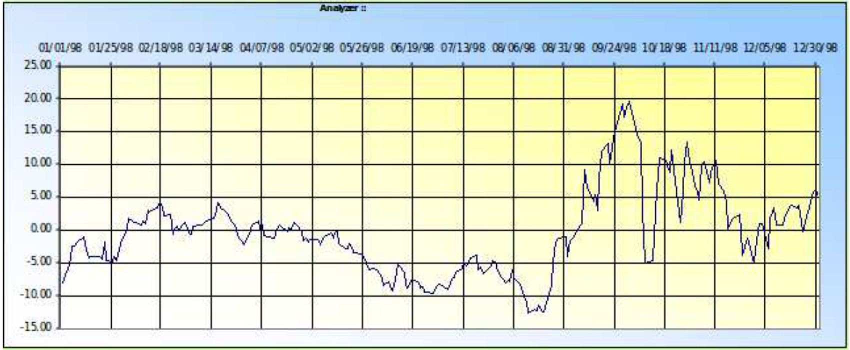
- Bond markets rallied anticipating Fed to cut rate.
- 2-10 steepened: rate cut would have more impact on 2-yr rate.
- Macro traders active in 2-yr and mortgages hedges active in 10-yr – clientele effect.
- Pension funds are natural players in 30-yr but they are not "traders" but are "portfolio rebalancers" who rebalance their portfolio periodically – clientele effect/institutional rigidity.
- Life Insurance companies active in the 30-yr as well. They typically are rate-targeted buyers and as market rallied, they back away from buying – clientele effect/institutional rigidity.

The LTCM Crisis



Model Performance in LTCM

All the above led to "cheapening" of the 30-yr sector – market rates did not rally as much as the model said they should.



Then There was 2008: