Class 11: Time-Varying Volatility

Financial Markets, Fall 2020, SAIF

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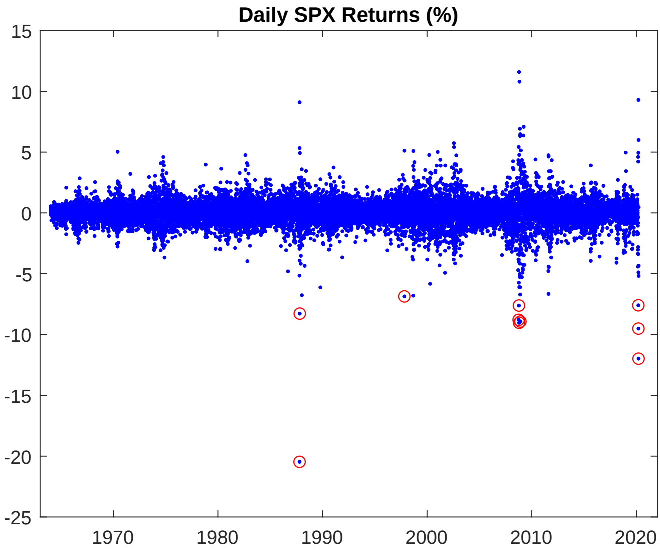
Outline

- The importance of measuring market volatility:
  - Portfolio managers performing optimal asset allocation.
  - Risk managers assessing portfolio risk (e.g., Value-at-Risk).
  - Derivatives investors trading non-linear contracts.
- Volatility σ can be better measured than expected returns μ:
  - Backward looking: estimate volatility using historical data.
  - Forward looking: from derivatives prices.
- Estimating volatility using financial time series:
  - SMA: simple moving average model (traditional approach).
  - EWMA: exponentially weighted moving average model (RiskMetrics).
  - ARCH and GARCH models (Nobel Prize).
- EWMA for covariances and correlations.

Daily Returns on the S&P 500 Index



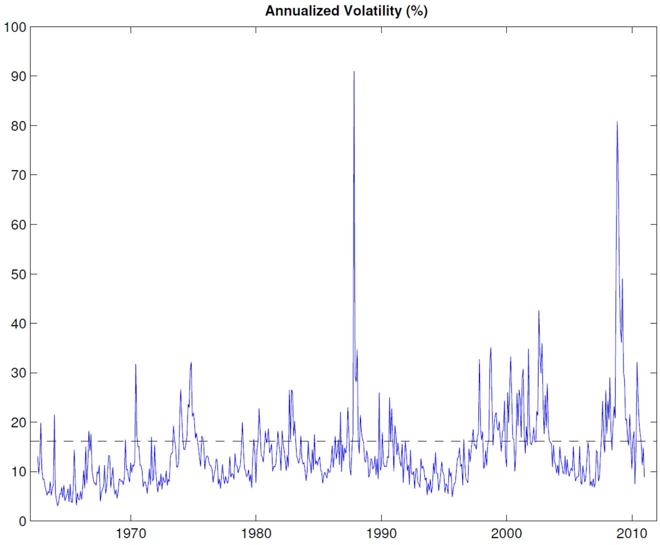
The Simple Moving Average Model

- Unlike expected returns, volatility can be measured with better precision using higher frequency data. So let's use daily data.
- Some have gone into higher frequency by using intra-day data. But micro-structure noises such as bid/ask bounce start to dominate in the intra-day domain. So let's not go there in this class.
- Suppose in month t, there are N trading days, with R_n denoting n-th day return. The simple moving average (SMA) model:

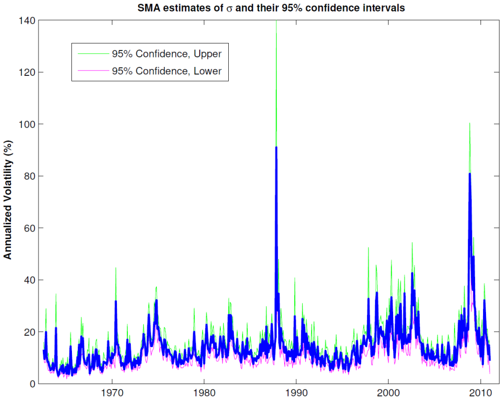
$$\sigma = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(R_n)^2}$$

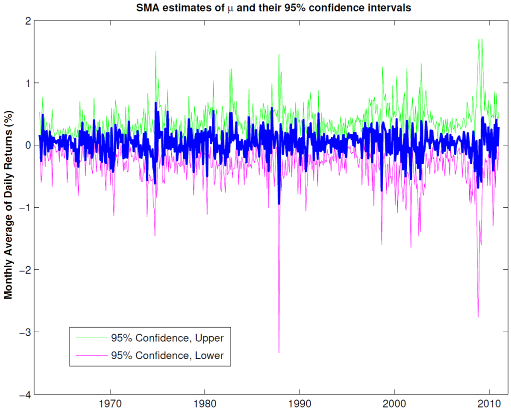
- To get an annualized number: $\sigma \times \sqrt{252}$. (252 trading days per year).

The Monthly SMA Volatility Estimates

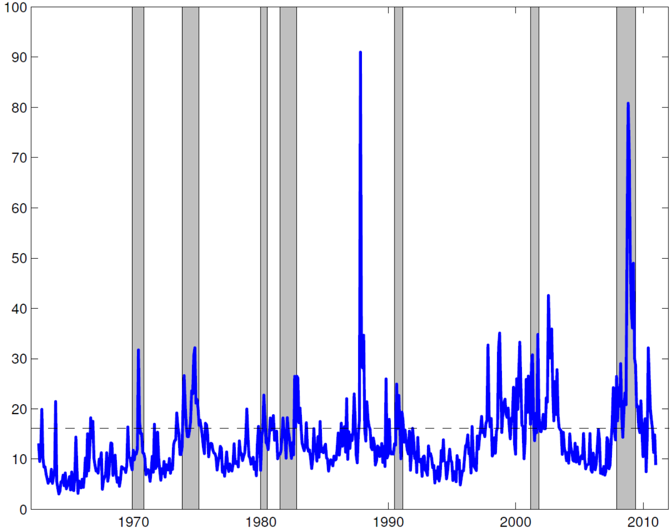


The Precision of the SMA Estimates

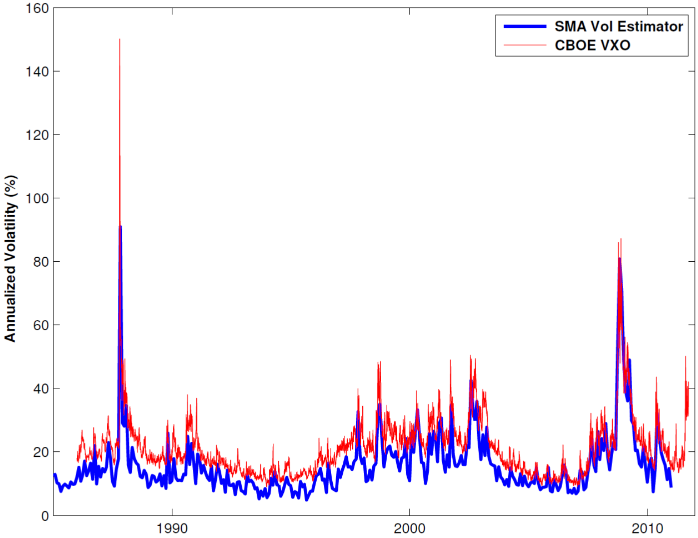




Time-Varying Volatility and Business Cycles



SMA Volatility and Option-Implied Volatility

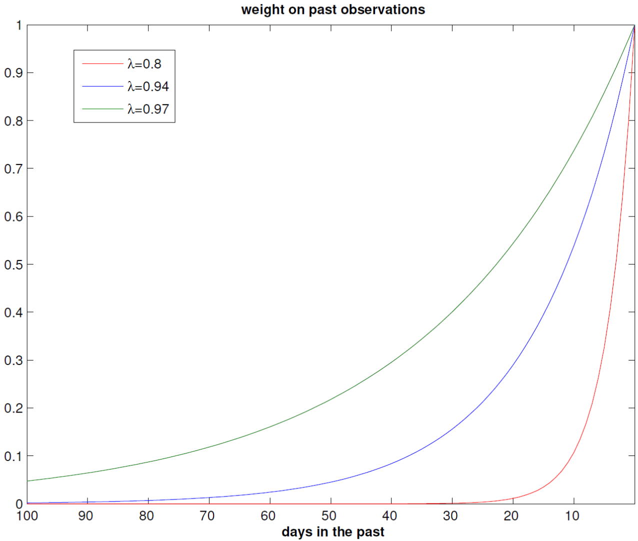


Exponentially Weighted Moving Average Model

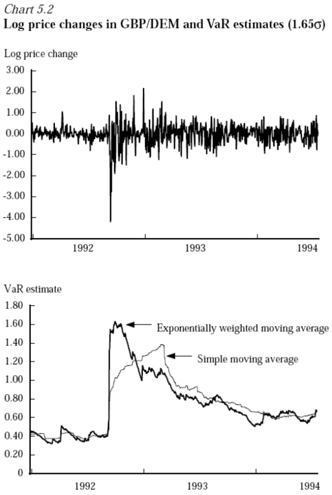
- The simple moving average (SMA) model fixes a time window and applies equal weight to all observations within the window.
- In the exponentially weighted moving average (EWMA) model, the more recent observation carries a higher weight in the volatility estimate.
- The relative weight is controlled by a decay factor λ.
- Suppose R_t is today's realized return, R_{t-1} is yesterday's, and R_{t-n} is the daily return realized n days ago. Volatility estimate σ:

Equally Weighted	**Exponentially Weighted**
$\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}\left(R_{t-n}\right)^2}$	$\sqrt{(1-\lambda)\sum_{n=0}^{N-1}\lambda^n\left(R_{t-n}\right)^2}$

EWMA Weighting Scheme



SMA and EWMA Estimates after a Crash



Source: J.P.Morgan/Reuters RiskMetrics — Technical Document, 1996

Computing EWMA recursively

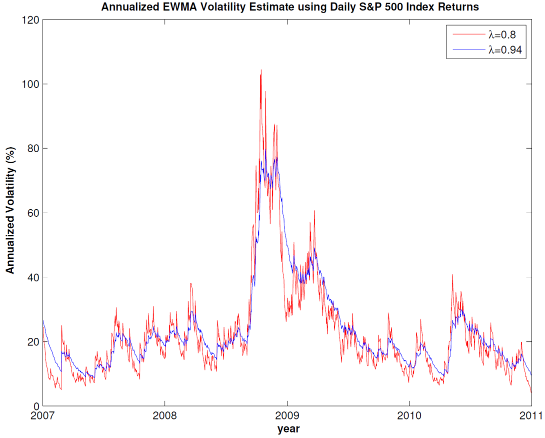
- One attractive feature of the exponentially weighted estimator is that it can be computed recursively.
- Let σ_t be the EWMA volatility estimator using all the information available on day $t-1$ for the purpose of forecasting the volatility on day t.
- Moving one day forward, it's now day t. After the day is over, we observe the realized return R_t.
- We now need to update our EWMA volatility estimator σ_{t+1} using the newly arrived information (i.e. R_t). It turns out that we can do so by

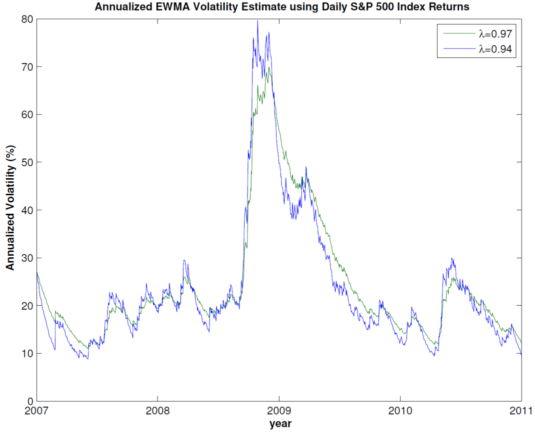
$$\sigma_{t+1}{}^2 = \lambda\,\sigma_t^2 + (1-\lambda)\,R_t{}^2$$

Decay factor, Strong or Weak?

- A strong decay factor (that is, small λ) underweights the far away events more strongly, making the effective sample size smaller.
- A strong decay factor improves on the timeliness of the volatility estimate, but that estimate could be noisy and suffers in precision.
- On the other hand, a weak decay factor improves on the smoothness and precision, but that estimate could be sluggish and slow in response to changing market conditions.
- So there is a tradeoff.

Fast, Medium, and Slow Decay





ARCH and GARCH models

- The ARCH model, autoregressive conditional heteroskedasticity, was proposed by Professor Robert Engle in 1982. The GARCH model is a generalized version of ARCH.
- ARCH and GARCH are statistical models that capture the time-varying volatility:

$$\sigma_{t+1}^2 = a_0 + a_1\, R_t^2 + a_2\, \sigma_t^2$$

- As you can see, it is very similar to the EWMA model. In fact, if we set $a_0 = 0$, $a_2 = \lambda$, and $a_1 = 1 - \lambda$, we are doing the EWMA model.
- This model has three parameters while the EWMA has only one. So it offers more flexibility (e.g., allows for mean reversion and better captures volatility clustering).

The Nobel-Prize Winning Model

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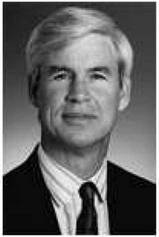
THE ROYAL SWEDISH ACADEMY OF SCIENCES

English
French
German
Swedish



Robert F. Engle III

1/2 of the prize

USA

New York University
New York, NY, USA

b. 1942

Press Release: The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2003

8 October 2003

The Royal Swedish Academy of Sciences has decided that the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, 2003, is to be shared between

Robert F. Engle
New York University, USA

"for methods of analyzing economic time series with time-varying volatility (ARCH)"

and

Clive W. J. Granger
University of California at San Diego, USA

"for methods of analyzing economic time series with common trends (cointegration)".

EWMA Covariances and Correlations

- Our goal is to create the variance-covariance matrix for the key risk factors influencing our portfolio.
- For the moment, let's suppose that there are only two risk factors affecting our portfolio.
- Let R_t^A and R_t^B be the day-t realized returns of these two risk factors. The covariance between A and B:

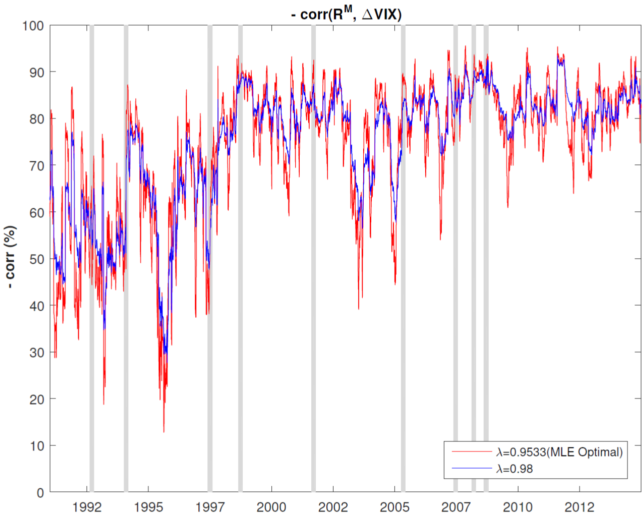
$$\mathsf{cov}_{t+1} = \lambda \, \mathsf{cov}_t + (1-\lambda) \, R_t^A \times R_t^B$$

- And their correlation:

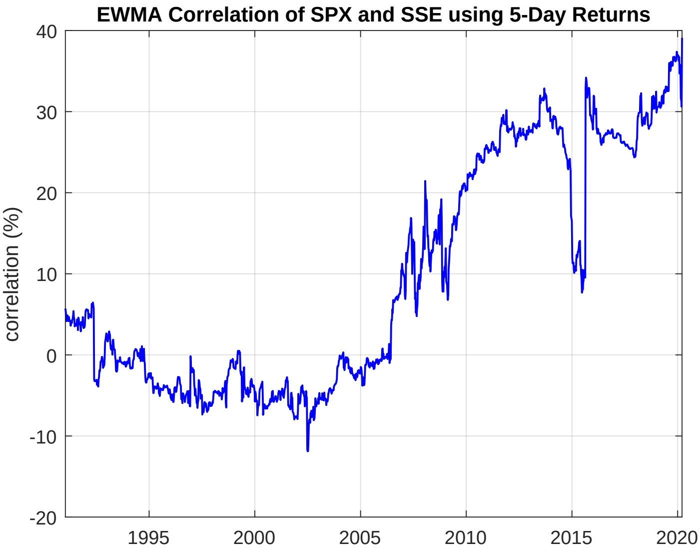
$$\mathsf{corr}_{t+1} = \frac{\mathsf{cov}_{t+1}}{\sigma_{t+1}^A \sigma_{t+1}^B} \, ,$$

where σ_{t+1}^A and σ_{t+1}^B are the EWMA volatility estimates.

Negative Correlation between R^M and ΔVIX



Correlation between SPX and SSE



From Volatility Estimates to VaR

from Goldman Sachs 2010 10-K form

Average Daily VaR

in millions Risk Categories	Year Ended December 2010	December 2009	November 2008
Interest rates	$ 93	$176	$ 142
Equity prices	68	66	72
Currency rates	32	36	30
Commodity prices	33	36	44
Diversification effect [1]	(92)	(96)	(108)
Total	**$134**	**$218**	**$ 180**

1. Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.

The Main Takeaways