Class 3: Optimal Risk-Taking in Theory

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Outline

- Mean-variance analysis:
  - Abstracting from reality, we model investors with utility functions.
  - A mean-variance investor likes expected returns μ and dislikes risk σ.
- Optimal risk and return tradeoff:
  - We ask the investor to make the optimal decision in choosing
    - risky asset with random return $\mathbf{R_t^M}$,
    - and riskfree asset with constant return r_f.
  - Key intuition: the optimal risk and return tradeoff.
- Diversification and the optimal risky portfolio:
  - We introduce multiple risky assets: $\mathbf{R_t^1}$ and $\mathbf{R_t^2}$.
  - Key intuition: the power of diversification.
  - The optimal mix of $\mathbf{R_t^1}$ and $\mathbf{R_t^2}$ becomes the optimal risky portfolio, which has the highest Sharpe ratio.

Policy Portfolio, Harvard Management Company, 2002

	Min	Policy	Max	Benchmark
Domestic equities	10	15	25	80% S&P 500, 8% S&P 400, 12% Russell 2000
Foreign equities	5	10	15	93% EAFE, 7% MSCI Small Cap ex US ex EAFE
Emerging markets	2	5	8	80% MSCI EM Investable, 20% MSCI EM Inv + 5%
Private equities	8	13	18	Cambridge Associates Weighted Composite
Total	30	43	60	
Absolute return	8	12	16	20% equity composite, 20% LIBOR+5%, 60% funds of funds
High-yield	2	5	8	60% Sal. High-Yield/Bankrupt Weighted Composite, 40% EMBI+
Commodities	8	13	18	23% GSCI and 77% NCREIF Timberland Index
Real estate	6	10	14	50% CPI+6, 25% NCREIF, 25% REIT. Leverage adjusted
Total	25	40	50	
Domestic bonds	6	11	21	Lehman 5+ year Treasury Index
Foreign bonds	0	5	10	J.P. Morgan Non U.S.
Inflation-indexed	0	6	15	Salomon 5+ year TIPS
Cash	-10	-5	10	One-month LIBOR

One Risky and One Riskfree

- Mean-variance investor:

$$\text{Utility} = \text{mean} - \frac{1}{2} \times \text{risk aversion} \times \text{variance}\,.$$

- Portfolio weights:
  - Invest y in the risky portfolio $\mathbf{R_t^M}$
  - Leave $1-y$ in riskfree r_f
- Portfolio return:

$$R_t^y = y\,\mathbf{R_t^M} + (1-y)\,r_f\,.$$

- The optimal portfolio weight:

$$y^* = \frac{\text{risk premium}}{\text{variance} \times \text{risk aversion}} = \frac{E\left(\mathbf{R_t^M}\right) - r_f}{\text{var}\left(\mathbf{R_t^M}\right) \times \text{risk aversion}}$$

Two Risky and One Riskfree

- Portfolio weights: invest w_1 in risky asset one $\mathbf{R_t^1}$ and w_2 in risky asset two $\mathbf{R_t^2}$

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

- Portfolio return:

$$R_t^w = w_1\,\mathbf{R_t^1} + w_2\,\mathbf{R_t^2} + (1 - w_1 - w_2)\; r_f$$

- Risk premium:

$$\text{risk premium} = \begin{pmatrix} E\left(\mathbf{R_t^1}\right) - r_f \\ E\left(\mathbf{R_t^2}\right) - r_f \end{pmatrix} = \begin{pmatrix} \mu_1 - r_f \\ \mu_2 - r_f \end{pmatrix}$$

- Variance-Covariance:

$$\Sigma = \begin{pmatrix} \text{variance 1} & \text{covariance} \\ \text{covariance} & \text{variance 2} \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix}$$

The Optimal Risky Portfolio

- The Optimal Portfolio Weights:

$$w^* = \frac{1}{\text{risk aversion}} \times \Sigma^{-1} \times \text{risk premium}$$

- The Optimal Risky Portfolio Weights:

$$\frac{1}{\sum_{i=1}^{N}(w_i^*)} \begin{pmatrix} w_1^* \\ w_2^* \\ . \\ w_i^* \\ . \\ w_N^* \end{pmatrix}$$

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- Investors with different risk aversion hold the same optimal risky portfolio, differing only on their relative weight on the risky portfolio.
- It is also the tangent portfolio, the portfolio with the highest Sharpe ratio.

Matrix Operations

Some useful tips for matrix operation in *Excel*:

- the command for summation is still "$+$"
- the command for multiplication is "mmult"
- the command for inverse, say Σ^{-1}, is "minverse"

Some useful tips for matrix operation in *Matlab*:

- the command for summation is still "$+$"
- the command for multiplication is still "$*$"
- the command for inverse, say Σ^{-1}, is "inv(Σ)"

Group Assignment and Presentation

Download the data and report the estimates:

- Obtain monthly returns of China's stock market from my website under Chinese Data and report μ, σ, and standard error of your estimate of μ.
- Repeat the same for the US. Use the monthly returns data I downloaded from Prof Ken French's website: US Data. The monthly return will be the column of Mkt-RF plus the column of RF. The two other factors will be useful for us in later classes.
- The above data also contains the monthly US riskfree return r_f, which is the column RF. Since in our setting, RF is a constant. So pick your sample period and use the average of the RF. We will do this exercise from the US investor's perspective. As such, we will use the US riskfree rate.
- For simplicity, let's ignore the currency risk in this exercise.
- Estimate the correlation ρ between the monthly returns of US and China. Since the correlation varies over time, please indicate the correlation you would like to use.

Group Assignment and Presentation

- Risk and return tradeoff:
  - Let's assume that the global investor's risk aversion coefficient is 4.
  - Calculate his/her optimal risky portfolio weight y^* for China and US separately.
  - Add any comments and observations you find interesting.
- The optimal risky portfolio:
  - Construct the optimal risky portfolio using US and China as the two risky assets.
  - Report the Sharpe ratio of your optimal risky portfolio.
  - Report the optimal risky portfolio weights.
  - Suppose that the global investor's risk aversion coefficient is 4. Report his/her optimal portfolio weights on US, China, and the riskfree asset.
  - Add any comments and observations you find interesting.

Presentation

- Use the above calculations as the core content of your presentation.
- Add motivations on why this problem is interesting and timely.
- Add discussions and analyses on how sensitive your results are with respect to the parameters, μ, σ, ρ, and risk aversion.
- As a financial adviser to this global investor, write a one-page recommendations advising his/her of the key costs and benefits of investing in China.
- Add any further discussions that you find interesting.