

# Class 7: Financial Models in Quant Investing: CAPM and Beyond

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# Outline

- Quant investing uses quantitative signals to form portfolios:
  - ▶ Size: small-cap stocks minus big-cap stocks.
  - ▶ Value: high book-to-market stocks minus low book-to-market.
  - ▶ Momentum: past winners minus past losers.
- The key insight of the equity quant strategy:
  - ▶ Quant signals: separate the cross-section into high- and low-alpha stocks.
  - ▶ Factor investing: diversify away the unwanted idiosyncratic risk.
  - ▶ Long/short: take out the unwanted systematic.
- The economic interpretations:
  - ▶ The CAPM.
  - ▶ Market efficiency.
  - ▶ Behavioral finance.

# The Fama and French Factors

- Small Minus Big:

$$R_t^{\text{SMB}} = R_t^{\text{small}} - R_t^{\text{big}}$$

- High Minus Low:

$$R_t^{\text{HML}} = R_t^{\text{value}} - R_t^{\text{growth}}$$

	Median ME	
70th BE/ME PERCENTILE	SMALL VALUE	BIG VALUE
30th BE/ME PERCENTILE	SMALL NEUTRAL	BIG NEUTRAL
	SMALL GROWTH	BIG GROWTH

# The Fama-French Three-Factor Alpha and Beta's

$$R_t^i - r_f = \alpha_i + \beta_i (R_t^M - r_f) + s_i R_t^{\text{SMB}} + h_i R_t^{\text{HML}} + \epsilon_t^i$$

- $\beta_i$ : the market beta.
- $s_i$ : the size beta.
- $h_i$ : the value beta.
- $\alpha_i$ : the Fama-French three-factor alpha.

# Factor Exposures

$$R_t^i - r_f = \alpha_i + \beta_i (R_t^M - r_f) + s_i R_t^{\text{SMB}} + h_i R_t^{\text{HML}} + \epsilon_t^i$$

**SMB beta  $s$**

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>A</b>	1.38	1.30	1.10	1.03	1.09
<b>B</b>	0.99	0.87	0.77	0.73	0.87
<b>C</b>	0.73	0.53	0.44	0.40	0.55
<b>D</b>	0.38	0.22	0.18	0.22	0.25
<b>E</b>	-0.24	-0.22	-0.23	-0.20	-0.08

**HML beta  $h$**

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>A</b>	-0.29	0.04	0.28	0.46	0.70
<b>B</b>	-0.39	0.13	0.39	0.56	0.81
<b>C</b>	-0.44	0.18	0.44	0.62	0.77
<b>D</b>	-0.42	0.21	0.45	0.57	0.81
<b>E</b>	-0.36	0.09	0.30	0.60	0.76

# The Explanatory Power of the Factors

- One Factor:

$$R_t^i - r_f = \alpha_i + \beta_i (R_t^M - r_f) + \epsilon_t^i$$

- Three Factors:

$$R_t^i - r_f = \alpha_i + \beta_i (R_t^M - r_f) + s_i R_t^{\text{SMB}} + h_i R_t^{\text{HML}} + \epsilon_t^i$$

**R2 (%) in one-factor**

	1	2	3	4	5
A	63	64	67	64	62
B	75	76	75	73	68
C	80	83	79	75	70
D	85	87	82	78	71
E	89	88	80	72	63

**R2 (%) in three-factor**

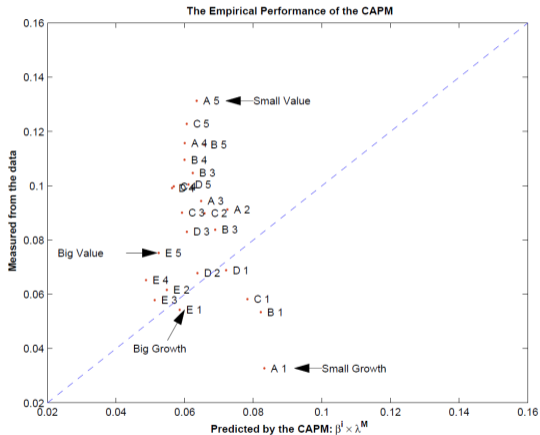
	1	2	3	4	5
A	91	94	95	94	95
B	95	94	94	94	95
C	95	91	90	90	90
D	94	89	88	89	87
E	94	90	86	89	80

# The Pricing Relation

$$E(R_t^i) - r_f = \beta_i (E(R_t^M) - r_f) + s_i E(R_t^{\text{SMB}}) + h_i E(R_t^{\text{HML}})$$

	$E(R_t^M - r_f)$	$E(R_t^{\text{SMB}})$	$E(R_t^{\text{HML}})$
1962-2014	6.46% [2.64]	3.20% [1.68]	5.15% [2.78]
1927-2014	8.40% [3.81]	3.40% [2.28]	5.00% [3.33]

# Fama-French 25 Portfolios and the Three-Factor Model





## Peter Lynch and the Magellan Fund

$$R_t - r_f = \alpha + \beta (R_t^M - r_f) + s R_t^{SMB} + h R_t^{HML} + \epsilon_t$$

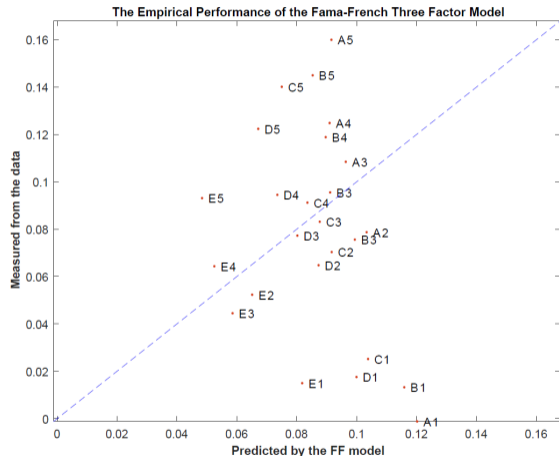
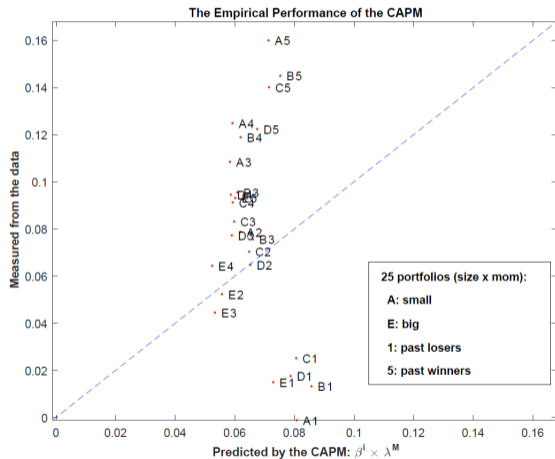
	Overall Period 76/6 – 98/12		Peter Lynch 76/6 – 90/5		Post-Lynch 90/6 – 98/12	
	estimate	s.e.	estimate	s.e.	estimate	s.e.
$\alpha$	0.51	0.11	0.75	0.13	0.07	0.14
$\beta$	1.12	0.03	1.13	0.03	1.04	0.04
$s$	0.34	0.04	0.55	0.05	0.05	0.05
$h$	0.02	0.05	-0.01	0.06	0.005	0.06
$R^2$	0.91		0.94		0.90	

## Warren Buffett and Berkshire Hathaway

Monthly returns of BRK.A from November 1976 through December 2008. The sample mean is 1.69% and the standard deviation is 7.29%.

	Full Sample 197611-200812		First Half 197611-199212		Second Half 199301-200812	
alpha	1.36%	1.11%	1.83%	1.49%	0.84%	0.69%
	[4.04]	[3.38]	[3.69]	[2.99]	[1.91]	[1.74]
Market beta	0.71	0.93	0.93	1.04	0.46	0.70
	[9.50]	[11.60]	[8.70]	[8.38]	[4.53]	[7.16]
SMB beta		-0.26		0.31		-0.57
		[-2.42]		[1.54]		[-4.83]
HML beta		0.58		0.58		0.44
		[4.67]		[2.64]		[3.18]
$R^2$	19.10%	26.33%	28.28%	31.68%	9.72%	29.81%

# Momentum Portfolios



# The Performance of Momentum Strategy in the CAPM

Annualized CAPM Alpha (in %) with t-stat's

	1	2	3	4	5
A	<b>-8.19</b> [-3.31]	1.68 [1.00]	<b>5.01</b> [3.33]	<b>6.57</b> [4.36]	<b>8.87</b> [4.64]
B	<b>-7.25</b> [-3.44]	0.95 [0.65]	<b>3.47</b> [2.82]	<b>5.69</b> [4.54]	<b>6.97</b> [4.16]
C	<b>-5.54</b> [-2.78]	0.55 [0.46]	<b>2.34</b> [2.18]	<b>3.19</b> [3.08]	<b>6.87</b> [4.58]
D	<b>-6.11</b> [-3.08]	-0.05 [-0.04]	1.83 [1.98]	<b>3.59</b> [4.26]	<b>5.49</b> [4.03]
E	<b>-5.79</b> [-3.07]	-0.33 [-0.28]	-0.88 [-1.08]	1.20 [1.46]	<b>3.30</b> [2.70]

Monthly data from January 1962 through July 2015.

# The Performance of Momentum Strategy in the FF3 Model

Annualized FF3 Alpha (in %) with t-stat's

	1	2	3	4	5
A	<b>-12.14</b> [-6.75]	<b>-2.46</b> [-2.66]	1.21 [1.56]	<b>3.39</b> [4.32]	<b>6.84</b> [6.20]
B	<b>-10.27</b> [-6.18]	<b>-2.38</b> [-2.47]	0.44 [0.60]	<b>2.92</b> [4.34]	<b>5.97</b> [5.82]
C	<b>-7.86</b> [-4.33]	<b>-2.13</b> [-2.19]	-0.45 [-0.59]	0.77 [0.97]	<b>6.51</b> [5.80]
D	<b>-8.24</b> [-4.24]	<b>-2.25</b> [-2.06]	-0.29 [-0.36]	<b>2.10</b> [2.69]	<b>5.52</b> [4.55]
E	<b>-6.68</b> [-3.54]	-1.28 [-1.12]	-1.41 [-1.90]	1.19 [1.57]	<b>4.47</b> [3.69]

Monthly data from January 1962 through July 2015.

# Momentum Profits around the World

Portfolio	Mean	Std. Dev.	t(mean)
Panel A: Country-Neutral Momentum Strategies			
All stocks (country-neutral)	0.0093	0.0239	5.36
By country:			
Austria	0.0080	0.0498	2.23
Belgium	0.0110	0.0444	3.42
Denmark	0.0109	0.0478	3.16
France	0.0097	0.0496	2.72
Germany	0.0072	0.0395	2.52
Italy	0.0093	0.0508	2.53
Netherlands	0.0126	0.0497	3.51
Norway	0.0099	0.0658	2.09
Spain	0.0132	0.0801	2.28
Sweden	0.0016	0.0632	0.36
Switzerland	0.0064	0.0428	2.08
United Kingdom	0.0089	0.0408	3.02

“International Momentum Strategies” by Rouwenhorst, *The Journal of Finance*, 1998.

# The Four-Factor Model

Add MOM to the Fama-French three-factor model:

$$E(R_t^i) - r_f = \beta_i (E(R_t^M) - r_f) + s_i E(R_t^{\text{SMB}}) + h_i E(R_t^{\text{HML}}) + w_i E(R_t^{\text{MOM}})$$

where the market beta, size beta, value beta, and momentum beta can be estimated by the following regression:

$$R_t^i - r_f = \alpha_i + \beta_i (R_t^M - r_f) + s_i R_t^{\text{SMB}} + h_i R_t^{\text{HML}} + w_i R_t^{\text{MOM}} + \epsilon_t^i$$

# The Factor Premiums and Volatility

From 1964 to 2014

	$E(R^M - r_f)$	$E(R^{\text{SMB}})$	$E(R^{\text{HML}})$	$E(R^{\text{MOM}})$
yearly	6.46% [2.64]	3.20% [1.68]	5.15% [2.78]	8.63% [3.47]
monthly	0.49% [2.79]	0.22% [1.79]	0.36% [3.23]	0.71% [4.27]
monthly	$\sigma^M$ 4.46%	$\sigma^{\text{SMB}}$ 3.08%	$\sigma^{\text{HML}}$ 2.84%	$\sigma^{\text{MOM}}$ 4.21%



# US and Chinese Stocks

	SPX	CN All	CN LG	CN Med	CN SM
Monthly Returns 1993-2018					
$\mu$	0.81 [3.51]	1.16 [1.85]	0.99 [1.65]	1.41 [2.00]	2.02 [2.60]
$\sigma$	4.10	11.05	10.56	12.49	13.74
Monthly Returns 2000-2018					
$\mu$	0.49 [1.75]	0.86 [1.60]	0.80 [1.52]	1.02 [1.61]	1.43 [2.08]
$\sigma$	4.20	8.16	7.96	9.60	10.42
Monthly Returns 2010-2018					
$\mu$	0.99 [2.89]	0.28 [0.44]	0.21 [0.34]	0.40 [0.48]	0.99 [1.07]
$\sigma$	3.57	6.59	6.38	8.60	9.59

# Chinese Mutual Funds

Panel A. Size of Mutual Funds, by Year

Year	Equity				Mixed				Bond			
	#Funds	TNA(B)	Ret(%)	StdRet(%)	#Funds	TNA(B)	Ret(%)	StdRet(%)	#Funds	TNA(B)	Ret(%)	StdRet(%)
2007	55	323.9	12.60	18.01	80	468.1	4.95	25.08	10	23.1	1.83	3.98
2008	72	376.5	-20.86	10.38	97	488.0	-15.88	8.29	16	50.7	0.44	2.45
2009	111	723.3	13.29	6.52	121	692.7	11.72	6.15	20	32.1	-0.06	2.12
2010	143	810.4	-0.23	5.63	134	690.8	0.07	6.37	40	59.0	-0.08	2.55
2011	184	729.1	-7.64	4.39	156	601.4	-6.53	4.51	72	68.4	-1.49	2.42
2012	220	636.3	1.26	3.90	167	529.6	0.78	3.44	85	91.0	1.19	1.82
2013	270	668.6	3.57	5.98	187	531.4	2.77	5.01	125	132.5	-0.59	2.40
2014	326	616.6	5.62	7.05	210	477.0	4.38	6.37	187	135.3	4.37	5.71
2015	186	357.2	12.40	11.32	431	760.2	8.42	11.39	304	320.6	1.29	5.02
2016	42	35.8	-3.06	6.19	712	905.7	-4.78	8.07	397	632.4	-1.20	3.92
2017	123	159.5	3.21	5.94	1,020	1,300.8	2.24	5.50	456	518.2	-0.11	2.54
2018	177	171.9	-7.24	5.09	1,414	1,237.6	-4.93	5.33	639	715.1	0.28	2.70

# Chinese Mutual Fund Alpha

	Stock funds		Hybrid funds		Bond funds		Money funds	
<b>Period</b>	200212-201812		200110-201812		200211-201812		200401-201812	
<b>Alpha (%)</b>	0.22	0.24	0.43	0.47	0.23	0.16	0.07	0.07
	(1.28)	(1.36)	(2.84)	(2.97)	(3.69)	(2.87)	(5.73)	(5.25)
<b>Mkt Beta</b>	0.79	0.79	0.61	0.61	0.07	0.08	0.00	0.00
	(39.8)	(37.66)	(33.72)	(31.99)	(9.44)	(11.66)	(-0.37)	(-0.34)
<b>Gov't Bond Beta</b>		-0.01		-0.10		0.08		0.01
		(-0.06)		(-0.68)		(1.53)		(0.70)
<b>Corp Bond Beta</b>		-0.10		-0.04		0.22		-0.01
		(-0.62)		(-0.24)		(4.21)		(-0.50)
<b>R-square (%)</b>	89.26	89.20	84.72	84.76	31.70	46.98	0.08	0.37
<b># Month</b>	193	185	207	199	194	186	180	172

# Chinese Stock Mutual Fund, Alpha (%)

	普通股票型	被动指数型	增强指数型
Period	200305-201812	200304-201812	200212-201812
Alpha	0.33 (1.62)	-0.15 (-1.08)	0.16 (0.90)
Mkt Beta	0.78 (32.08)	0.90 (55.92)	0.87 (41.64)
R-square	84.69%	94.36%	90.08%
# Month	188	189	193

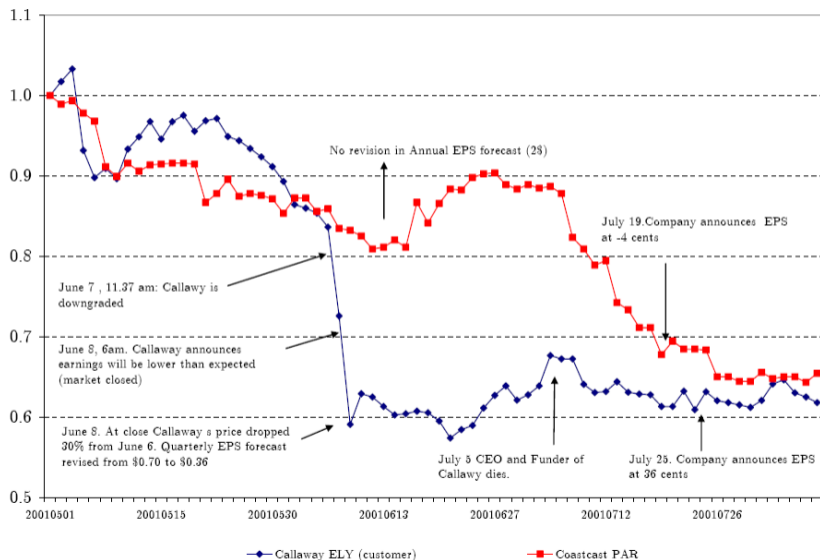
	偏股混合型	平衡混合型	偏债混合型	灵活配置型
Period	200110-201812	200110-201812	200211-201812	200307-201812
Alpha	0.37 (1.99)	0.40 (2.72)	0.42 (4.10)	0.45 (2.64)
Mkt Beta	0.72 (32.29)	0.58 (32.46)	0.25 (20.35)	0.62 (30.98)
R-square	83.57%	83.72%	68.21%	83.92%
# Month	207	207	195	186

# The Performance of Fidelity Magellan

Fidelity Magellan, monthly returns

manager	tenure	mean excess	alpha	market beta	SMB beta	HML beta	MOM beta
Stansky	96-05	0.37 [0.74]	0.03 [0.35]	0.99 [50.41]	-0.14 [-7.72]	-0.04 [-1.46]	-0.01 [-0.50]
Vinik	92-96	0.95 [2.26]	-0.31 [-1.19]	1.00 [9.21]	0.12 [0.88]	0.07 [0.55]	0.29 [2.37]
Smith	90-92	0.80 [0.77]	0.26 [2.09]	1.14 [36.69]	0.01 [0.30]	-0.01 [-0.21]	-0.03 [-0.82]
Lynch	76-90	1.59 [3.45]	0.64 [5.01]	1.12 [36.38]	0.49 [9.67]	0.03 [0.59]	0.16 [4.08]
Habermann	72-76	-0.83 [-0.68]	0.42 [0.64]	1.00 [7.85]	0.79 [3.52]	-0.44 [-2.25]	0.07 [0.38]
Johnson	63-72	2.45 [3.32]	0.83 [2.60]	1.10 [11.67]	1.20 [10.07]	0.13 [0.90]	0.75 [7.36]

# The Economic Link between Customers and Suppliers



# Portfolio Returns of Suppliers, Sorted by Past Returns of Their Customers

Value weights	xret	alpha	MKT	SMB	HML	UMD	R <sup>2</sup>
Q1 (low)	-0.596 [-1.42]	-0.821 [-2.93]	<b>0.989</b> [14.31]	<b>0.384</b> [4.47]	-0.318 [-3.10]	-0.235 [-3.88]	0.626
Q2	-0.157 [-0.41]	-0.741 [-3.28]	<b>1.057</b> [17.57]	<b>0.307</b> [4.10]	-0.115 [-1.28]	-0.022 [-0.42]	0.658
Q3	0.125 [0.32]	-0.488 [-1.89]	<b>1.063</b> [16.81]	<b>0.309</b> [3.92]	-0.09 [-0.96]	-0.029 [-0.52]	0.633
Q4	0.313 [0.79]	-0.193 [-0.72]	<b>1.039</b> [14.43]	<b>0.217</b> [2.42]	-0.15 [-1.40]	-0.076 [-1.20]	0.564
Q5 (high)	<b>0.982</b> [2.14]	<b>0.556</b> [1.99]	<b>0.982</b> [13.80]	<b>0.681</b> [7.69]	<b>-0.363</b> [-3.43]	-0.056 [-0.90]	0.650
L/S	<b>1.578</b> [3.79]	<b>1.376</b> [3.13]	-0.007 [-0.07]	0.296 [1.26]	-0.045 [-0.28]	0.179 [1.93]	0.041

“Economic links and predictable returns” by Cohen and Frazzini, *Journal of Finance*, 2008.

## Sorting Stocks by their Options Trading Volume (Put/Call)

	day relative to portfolio formation										
	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10
Panel A: average daily returns of PC-ranked portfolios (in basis points)											
low PC	31.4	25.0	15.5	12.1	11.4	10.2	9.3	6.9	8.7	7.2	7.8
PC 2	28.6	27.2	12.1	8.3	6.8	6.1	7.3	3.7	4.2	4.6	3.9
PC 3	15.5	12.5	7.1	6.1	5.4	5.6	4.6	4.6	5.2	6.4	3.6
PC 4	13.0	-0.3	3.1	2.1	6.4	4.7	5.2	6.4	6.1	5.1	7.2
high PC	-5.9	-14.6	-6.1	-0.8	-0.7	1.4	3.2	4.3	4.0	4.3	3.7
Panel B: average daily returns of low-PC minus high-PC (in basis points)											
	37.4	39.6	21.6	12.9	12.1	8.8	6.2	2.6	4.7	2.9	4.1
t-stats	19.77	23.79	13.11	8.18	7.77	5.50	3.86	1.67	2.94	1.80	2.62

"The information in option volume for future stock prices" by Pan and Poteshman, *Review of Financial Studies*, 2006.



# Main Takeaways