

# Class 15: The Yield Curve

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- Risk factors affecting the bond market:
  - ▶ Level: Parallel shifts of the yield curve; duration hedging.
  - ▶ Slope: Differential movements between the long- and short-term interest rates.
- Principal component analysis (PCA):
  - ▶ Identify common risk components in the data.
  - ▶ Quantify the risk factors in terms of their importance.
- PCA Applications: financial markets with common risk factors.
  - ▶ Bond markets: across maturities.
  - ▶ Domestic equity markets: SMB, HML, momentum, etc.
  - ▶ International equity markets: across countries.
  - ▶ Currency markets: across currencies.
  - ▶ Commodity futures: across maturities.

## An Illustrative Example

- To simplify our analysis, let's start with zero-coupon bonds.
- Coupon bonds: weighted sums of the zero-coupon bond prices.
- Initially, the zero rates are:

	2yr	5yr	10yr	30yr
zero rate	3.5%	4.5%	5%	5.5%
maturity	2	5	10	30
modified duration	$\frac{2}{1+3.5\%}$	$\frac{5}{1+4.5\%}$	$\frac{10}{1+5\%}$	$\frac{30}{1+5.5\%}$
	1.93	4.78	9.52	28.44

- Using the 2yr zero as the reference bond,
  - ▶ 5-to-2 spread: 100 bps.
  - ▶ 10-to-2 spread: 150 bps.
  - ▶ 30-to-2 spread: 200 bps.

## Directional Trade to Bet on a Parallel Shift in Level

- The yield on the 2yr zero subsequently increases by 10 bps.
- All spreads remain the same.

	notional (\$ million)	market value (\$ million)			
		initial	later	change	approximation
2yr	$(1 + 3.5\%)^2$	1	$\frac{(1+3.5\%)^2}{(1+3.5\%+10\text{bps})^2}$	-19.30 bps	$-1.93 \times 10 \text{ bps}$
5yr	$(1 + 4.5\%)^5$	1	$\frac{(1+4.5\%)^5}{(1+4.5\%+10\text{bps})^5}$	-47.71 bps	$-4.78 \times 10 \text{ bps}$
10yr	$(1 + 5.0\%)^{10}$	1	$\frac{(1+5.0\%)^{10}}{(1+5.0\%+10\text{bps})^{10}}$	-94.74 bps	$-9.52 \times 10 \text{ bps}$
30yr	$(1 + 5.5\%)^{30}$	1	$\frac{(1+5.5\%)^{30}}{(1+5.5\%+10\text{bps})^{30}}$	-280.22 bps	$-28.44 \times 10 \text{ bps}$

# Steepener

- Your view:
  - ▶ The 10-to-2 spread will increase.
  - ▶ No view on the overall direction of the interest rate.
- Your strategy: steepener.
  - ▶ Long \$4.9286M of 2yr zero.
  - ▶ Short \$1M of 10yr zero.

	initial market value (\$ million)	notional amount (\$ million)
2yr	4.9286	$4.9286 \times (1 + 3.5\%)^2$
10yr	-1	$-(1 + 5.0\%)^{10}$

- Why 4.9286? Because  $4.9286 = 9.52 / 1.93$ .

## Two Scenarios:

- ① A parallel shift of +10 bps:

	initial (\$ Million)	later (\$ Million)	change (\$ Million )	approximation (\$ Million)
2yr	4.9286	$\frac{4.9286 \times (1+3.5\%)^2}{(1+3.5\%+10\text{bps})^2}$	-95.10 bps	$-1.93 \times 10 \text{ bps} \times 4.9286$
10yr	-1	$-\frac{(1+5.0\%)^{10}}{(1+5.0\%+10\text{bps})^{10}}$	94.74 bps	$9.52 \times 10 \text{ bps}$
Total			<b>-0.36 bps</b>	

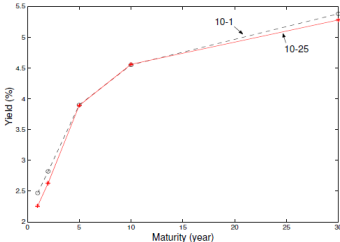
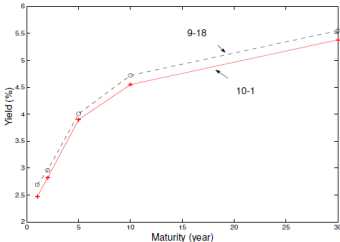
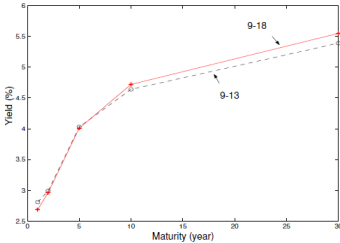
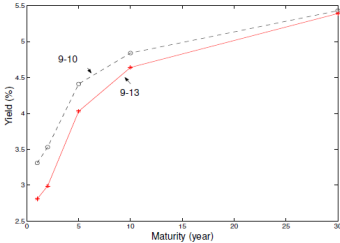
- ② The 2yr zero rate decreases by 15 bps and the 10yr zero rate increases by 5 bps.

	initial (\$ Million)	later (\$ Million)	change (\$ Million )	approximation (\$ Million)
2yr	4.9286	$\frac{4.9286 \times (1+3.5\%)^2}{(1+3.5\%-15\text{bps})^2}$	143.17 bps	$1.93 \times 15 \text{ bps} \times 4.9286$
10yr	-1	$-\frac{(1+5.0\%)^{10}}{(1+5.0\%+5\text{bps})^{10}}$	47.49 bps	$9.52 \times 5 \text{ bps}$
Total			<b>190.66 bps</b>	

## Curvature (or Butterfly):

- Your view: the 2yr and 30yr zeros will move in the same direction while the 10yr zero will move in opposite direction. Not sure about the overall directional of the interest rate, nor about the slope of the yield curve.
- Your strategy: butterfly trade.
- Long 30yr zeros, short 10yr zeros (to hedge against parallel shifts), and long 2yr zeros (to hedge against slope steepening or flattening).

# Yield Curve Movement after 911

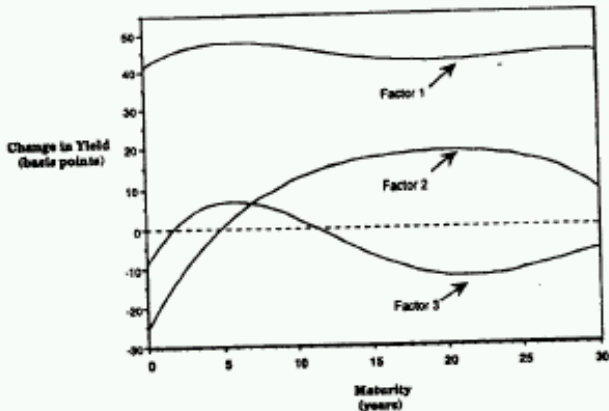




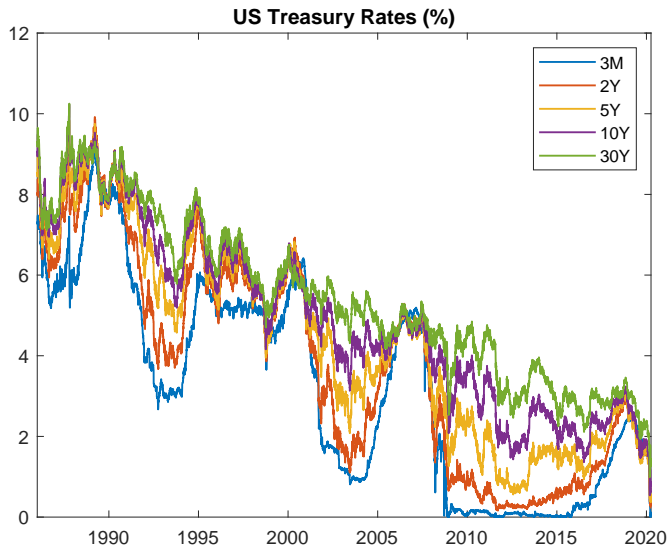
## Three Major Factors in the Fixed-Income Market:

Closely related to the three trading strategies are the three major risk factors reported by Litterman and Scheinkman for the fixed income market:

**FIGURE 2 ■ Yield Curve Impact**



# Treasury Yields



## Variance-Covariance Matrix

$\text{cov}(\Delta y)$  (unit:  $\text{bps}^2$ , or equivalently,  $\times 10^{-8}$ )

	3M	1Y	2Y	5Y	10Y	30Y
3M	64.9300	40.4601	32.1459	27.1468	22.3368	17.8525
1Y	40.4601	48.3104	42.5293	39.0606	33.7733	27.6243
2Y	32.1459	42.5293	48.4517	45.1757	39.4565	31.9805
5Y	27.1468	39.0606	45.1757	51.6705	46.6458	38.8411
10Y	22.3368	33.7733	39.4565	46.6458	47.5872	40.7433
30Y	17.8525	27.6243	31.9805	38.8411	40.7433	39.7206

$\text{corr}(\Delta y)$

	3M	1Y	2Y	5Y	10Y	30Y
3M	1.0000	0.7224	0.5731	0.4687	0.4018	0.3515
1Y	0.7224	1.0000	0.8790	0.7818	0.7044	0.6306
2Y	0.5731	0.8790	1.0000	0.9029	0.8217	0.7290
5Y	0.4687	0.7818	0.9029	1.0000	0.9407	0.8574
10Y	0.4018	0.7044	0.8217	0.9407	1.0000	0.9371
30Y	0.3515	0.6306	0.7290	0.8574	0.9371	1.0000

# Principal Component Analysis

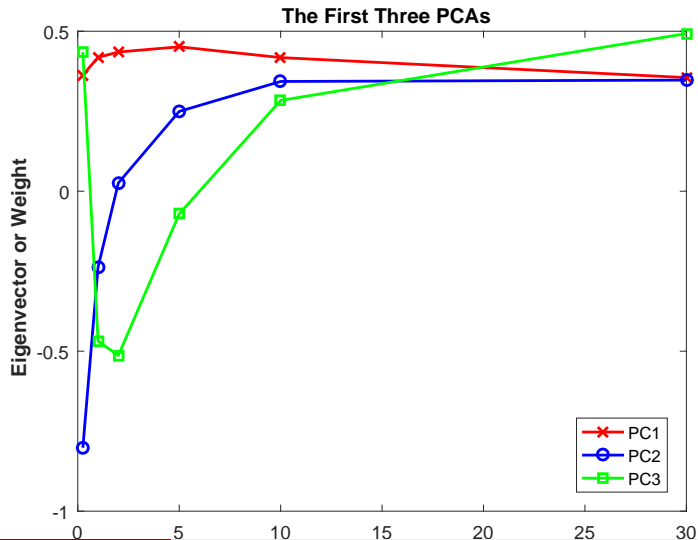
## Eigenvalues E

E	PC1	PC2	PC3	PC4	PC5	PC6
E (bps <sup>2</sup> )	226.99	50.14	13.77	5.45	2.86	1.47
E/sum(E) (%)	75.49	16.68	4.58	1.81	0.95	0.49

## Eigenvectors D

D	PC1	PC2	PC3	PC4	PC5	PC6
3M	0.3630	-0.8017	0.4347	0.1876	-0.0365	0.0006
1Y	0.4182	-0.2371	-0.4682	-0.6806	0.2939	0.0016
2Y	0.4351	0.0257	-0.5134	0.3309	-0.6505	0.1176
5Y	0.4513	0.2493	-0.0709	0.4572	0.5076	-0.5124
10Y	0.4176	0.3430	0.2837	0.0418	0.2271	0.7577
30Y	0.3550	0.3472	0.4926	-0.4258	-0.4242	-0.3866

# The First Three PCAs



## Level, Slope, and Curvature

- They name the first factor the *level* factor as it has a similar impact of the yield curve as a parallel shift of the yield curve.
- They call the second factor *steepness*, since shock from this factor lowers the yields of zeros up to five years, and raises the yields for zeros of longer maturities.
- They call the third factor *curvature*, since it increases curvature of the yield curve in the range of maturities below twenty years.

## The Relative Importance of the Three Factors:

**Table 2 • Implied Zeros: Relative Importance of Factors • (Percent)**

<u>Maturity</u>	<u>Total Variance Explained</u>	<u>Proportion of Total Explained Variance Accounted for by</u>		
		<u>Factor 1</u>	<u>Factor 2</u>	<u>Factor 3</u>
6 months	99.5	79.5	17.2	3.3
1 year	99.4	89.7	10.1	0.2
2 years	98.2	93.4	2.4	4.2
5 years	98.8	98.2	1.1	0.7
8 years	98.7	95.4	4.6	0.0
10 years	98.8	92.9	6.9	0.2
14 years	98.4	86.2	11.5	2.2
18 years	95.3	80.5	14.3	5.2
<b>Average</b>	<b>98.4</b>	<b>89.5</b>	<b>8.5</b>	<b>2.0</b>

The first factor is by far the most important, supporting the idea that “first factor” hedging – or its close cousin, duration hedging – takes care of most of the return risk.

$$\Delta y_t = a + \beta^{PC1} PC1_t + \beta^{PC2} PC2_t + \beta^{PC3} PC3_t + \epsilon_t.$$

	PC1 $\beta$	PC2 $\beta$	PC3 $\beta$	PC1 R2 (%)	PC2 R2 (%)	PC3 R2 (%)	Total R2 (%)
3M	0.3630	-0.8017	0.4347	46.06	49.63	4.01	99.70
1Y	0.4182	-0.2371	-0.4682	82.18	5.83	6.25	94.26
2Y	0.4351	0.0257	-0.5134	88.67	0.07	7.49	96.23
5Y	0.4513	0.2493	-0.0709	89.46	6.03	0.13	95.62
10Y	0.4176	0.3430	0.2837	83.17	12.39	2.33	97.89
30Y	0.3550	0.3472	0.4926	72.04	15.22	8.41	95.66



## Daily Currency Returns from 2000 through 2016

std (%)	GBP	EUR	AUD	CAD	CNY	INR	JPY	CHF	THB	RUB
	0.60	0.63	0.82	0.59	0.10	0.38	0.64	0.70	0.36	0.77
corr (%)	GBP	EUR	AUD	CAD	CNY	INR	JPY	CHF	THB	RUB
GBP		64.4	53.1	45.3	12.9	25.2	12.3	49.7	18.6	23.9
EUR	64.4		55.3	46.0	10.4	23.3	27.5	78.2	21.1	25.7
AUD	53.1	55.3		62.3	12.8	33.9	2.9	39.2	23.4	32.8
CAD	45.3	46.0	62.3		9.4	27.0	1.0	32.7	19.9	33.3
CNY	12.9	10.4	12.8	9.4		16.8	5.6	8.8	16.1	10.7
INR	25.2	23.3	33.9	27.0	16.8		-4.0	15.1	25.5	26.6
JPY	12.3	27.5	2.9	1.0	5.6	-4.0		37.5	17.6	-1.1
CHF	49.7	78.2	39.2	32.7	8.8	15.1	37.5		17.7	19.3
THB	18.6	21.1	23.4	19.9	16.1	25.5	17.6	17.7		15.9
RUB	23.9	25.7	32.8	33.3	10.7	26.6	-1.1	19.3	15.9	

## The First Three PCs

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The first three PC's of currency returns

	PC 1	PC 2	PC3
E/sum(E)	44.32%	17.07%	12.27%
GBP	0.3502	0.0715	0.1382
EUR	0.4191	0.2531	0.0037
AUD	0.5410	-0.2512	0.4398
CAD	0.3259	-0.1870	0.1890
CNY	0.0122	-0.0032	-0.0046
INR	0.1154	-0.1104	0.0113
JPY	0.1226	0.5569	-0.3908
CHF	0.4100	0.4488	-0.1497
THB	0.0882	0.0084	-0.0390
RUB	0.3138	-0.5558	-0.7582

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## The First Three PCs

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The first three PC's of currency returns

	PC 1	PC 2	PC3
E/sum(E)	54.03%	20.25%	10.10%
GBP	0.3748	-0.0312	0.2595
EUR	0.4538	0.1878	0.2925
AUD	0.5621	-0.4869	-0.4163
CAD	0.3324	-0.2946	-0.1879
JPY	0.1499	0.6699	-0.7030
CHF	0.4524	0.4371	0.3799

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Fama-French 25 Portfolios			
	PC 1	PC 2	PC3
E/sum(E)	83.84%	4.39%	3.19%
A1	0.2874	-0.5850	-0.6434
A2	0.2456	-0.2909	0.4001
A3	0.2378	-0.1526	0.0693
A4	0.2225	-0.1552	0.1332
A5	0.2468	-0.1586	0.0062
B1	0.2048	-0.1685	0.2221
B2	0.2013	-0.0520	0.2160
B3	0.1981	-0.0093	0.1173
B4	0.2033	0.0110	0.0517
B5	0.2353	-0.0044	-0.0768
C1	0.1938	-0.0751	0.1897
C2	0.1732	0.0673	0.1030
C3	0.1765	0.1135	0.0537
C4	0.1873	0.1154	0.0207
C5	0.2283	0.1122	-0.1330
D1	0.1555	0.0565	0.1347
D2	0.1606	0.1471	0.0868
D3	0.1707	0.1824	0.0360
D4	0.1810	0.1743	-0.0947
D5	0.2289	0.2343	-0.1251
E1	0.1264	0.1461	0.0572
E2	0.1298	0.2066	0.0048
E3	0.1380	0.2427	-0.0676
E4	0.1662	0.2704	-0.1142
E5	0.2050	0.2846	-0.3807

Momentum 25 Portfolios			
	PC 1	PC 2	PC 3
E/sum(E)	82.40%	5.53%	4.06%
A1	0.2791	0.0183	0.4184
A2	0.2498	-0.0775	0.3119
A3	0.2311	-0.1137	0.2692
A4	0.2267	-0.2113	0.1814
A5	0.2150	-0.3085	0.0656
B1	0.2629	0.1010	0.1670
B2	0.2212	-0.0021	0.1058
B3	0.1945	-0.0576	0.0161
B4	0.1906	-0.1614	-0.0098
B5	0.1963	-0.2603	-0.1152
C1	0.2480	0.2102	0.0695
C2	0.2057	0.0630	-0.0203
C3	0.1844	-0.0147	-0.0639
C4	0.1633	-0.0974	-0.1426
C5	0.1654	-0.2323	-0.2459
D1	0.2399	0.2932	-0.0217
D2	0.1898	0.1263	-0.1283
D3	0.1665	0.0267	-0.1472
D4	0.1569	-0.0562	-0.2211
D5	0.1499	-0.2015	-0.3146
E1	0.2049	0.6397	-0.1310
E2	0.1573	0.1926	-0.1621
E3	0.1430	0.0898	-0.2025
E4	0.1228	-0.0192	-0.2724
E5	0.1243	-0.1436	-0.3605

# Main Takeaways