The risk that matters.

Running regressions to estimate the CAPM alpha and beta.
The Risk that Matters

- So far, we’ve focused on one time series.
- This time series turns out to be a very important risk factor.
- According to the CAPM, investors are only rewarded for bearing *systematic risk*, the type of risk that *cannot* be diversified away.
- They should not be rewarded for bearing *idiosyncratic risk*, since this uncertainty can be mitigated through appropriate diversification.
- The U.S. aggregate stock market has been commonly adopted as a proxy for the systematic risk.
- With this time series serving as an anchor, we can now talk about the pricing of individual stocks or other portfolios.
- Let’s start by referring to it as $R^M$. 
The CAPM

- The CAPM identifies one single portfolio, the market portfolio $R^M$, to be the only source of risk that matters.
- The market risk premium $= E(R^M) - r_f$, where $r_f$ is the riskfree rate.
- So far, our estimate for $E(R^M)$ is around 12% per year (and very noisy). The riskfree rate is on average 4% per year. So a good estimate for the market risk premium is around 8%.
- The risk of each individual stock, say GE, is measured *not by its own volatility* but by its *exposure* to the market risk:

\[ \beta^{GE} = \frac{\text{covariance} \left( R^{GE}, R^M \right)}{\text{variance} \left( R^M \right)} \]

- The reward is proportional to the risk:

\[ E(R^{GE}) - r_f = \beta^{GE} \times \left( E(R^M) - r_f \right) \]
Running Regression to Estimate the CAPM $\beta$:

- Identify an index as the market portfolio. Typical choice: the CRSP value-weighted index (e.g., the academics), the S&P 500 index (e.g., Merrill Lynch’s beta book), and the NYSE index (e.g., Value-line).
- Identify the stock or portfolio of interest.
- Collect time-series of returns:
  - For the market portfolio: $R_t^M$, $t = 1, 2, 3, \ldots T$.
  - For the test portfolio: $R_t^{GE}$, $t = 1, 2, 3, \ldots T$.
- Run the following regression (typically monthly data over a five-year rolling window):

$$R_t^{GE} - r_f = \alpha + \beta \left( R_t^M - r_f \right) + \epsilon_t$$
Two Sources of Uncertainty in a Stock

- By running this regression, we break the total uncertainty in a stock into two components:

$$R_{t}^{GE} - r_f = \alpha + \beta \left( R_{t}^{M} - r_f \right) + \epsilon_t$$

  - One is due to its exposure to the market portfolio: $\beta \left( R_{t}^{M} - r_f \right)$.
  - The other is idiosyncratic, as captured by the regression residual $\epsilon_t$.

- By construction, the residual of a regression is uncorrelated with the explanatory variable: $\text{cov}(R_{t}^{M}, \epsilon_t) = 0$.

- The R-squared tells us how much of GE’s variance can be explained by the variance in the market portfolio:

$$\text{R-squared} = \frac{\beta^2 \text{var}(R_{t}^{M})}{\text{var}(R_{t}^{GE})} = \frac{\beta^2 \text{var}(R_{t}^{M})}{\beta^2 \text{var}(R_{t}^{M}) + \text{var}(\epsilon)}$$
The CAPM $\alpha$

- If you re-arrange that regression equation, you get

$$\alpha = R_t^{GE} - r_f - \beta \left( R_t^M - r_f \right) - \epsilon_t .$$

- Taking expectations on both sides, we have

$$\alpha = E \left( R_t^{GE} - r_f \right) - \beta E \left( R_t^M - r_f \right) ,$$

$\alpha$ is the *expected* excess stock return, after taking out the reward associated with the systematic component.

- So testing the CAPM pricing formula is the same as testing whether or not $\alpha$ is zero.

- Conversely, if we can construct many portfolios with positive and statistically significant $\alpha$’s, then the CAPM pricing formula is under a severe challenge.
Using t-stat

\[ t\text{-stat} = \frac{\text{estimate}}{\text{s.e.}} \]

- In finance, we often use historical data to estimate financial models. The model parameters (e.g., \( \alpha \) and \( \beta \)) are always estimated with noise.
- The standard errors and t-stat inform us on the precision. We can then decide whether or not to take the estimates seriously.
- As a rule of thumb, we take an estimate seriously if the absolute value of its t-stat is larger than 1.96:

\[ |t\text{-stat}| \geq 1.96 \]

- If it is less than 1.96, then we don’t take it as seriously: statistically insignificant from zero.
## Alpha, Beta, and R-Squared

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All time series end on 201112
### Alpha, Beta, and R-Squared

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All time series start on 200409 and end on 201112
Searching for investment opportunities with positive alpha is the goal of every active fund manager.

One thing for sure, without taking on idiosyncratic risk $\epsilon$, a portfolio manager’s alpha is always zero. So effectively, he is hoping to get $\alpha$ as a reward for holding $\epsilon_t$.

In the world of the CAPM, this is impossible.

So do active fund managers actually provide positive $\alpha$? We need to look into the data to find out.
Alpha of a Mutual Fund

Alpha (\%), Fidelity Magellan

Financial Markets, Day 1, Class 3
Alpha, Beta and the CAPM
Jun Pan