Options, Part 1

15.433 Financial Markets

October 12 & 17, 2017
Outline

• An overview of the options market:
  – Why options?
  – History, trading volume and market size
  – Options exchanges and market participants

• The Black-Scholes option pricing model:
  – The Black-Scholes model
  – Risk-neutral pricing
  – The Black-Scholes formula

• Using the Black-Scholes formula:
  – The pricing of at-the-money options
  – Black-Scholes option implied volatility
Modern Finance

- Portfolio Theory (Markowitz)
- Option Pricing Theory (Black, Scholes, Merton)
- Two-Fund Separation (Torbin)
- Investments and Capital Structure (Modigliani and Miller)
- Behavior of Securities Prices (Samuelson)
- First Major Study of Mutual Funds (Jensen)
- Efficient Markets Hypothesis (Fama)
- CAPM (Sharpe)
- First US Options Exchange, CBOE
- Birth of Index Funds (McQuown)
- Index Funds
- Rise of Junk Bonds
- Mortgage-Backed Securities
- Index Futures
- OTC Derivatives (e.g. interest-rate swaps)
- Credit Derivatives (e.g. CDS)
- Large Derivatives Losses
- S&L Bailout
- Crash
- Enron
- .com Peak
- .com Bottom
- Fall 2001
- 2002
- 2003
- 2004
- 2005
- 2006
- 2007
- 2008
- 2009
- 1994
- 1995
- 1996
- 1997
- 1998
- 1999
- 2000
- 2001
- 2002
- 2003
- 2004
- 2005
- 2006
- 2007
- 2008
- 2009
- Fall 2017
Sample the Right Tail: Call Options with Increasing Strikes

Stock Price at the Time of Expiration ($)

Probability Density Function

Option Payoff ($)

- Strike at 100
- Strike at 110
- Strike at 120
Sample the Left Tail: Put Options with Decreasing Strikes

- Strike at 100
- Strike at 90
- Strike at 80

Stock Price at the Time of Expiration ($)

Option Payoff ($)
A Brief History

- 1973: CBOE founded as the first US options exchange, and 911 contracts were traded on 16 underlying stocks on first day of trading.
- 1975: The Black-Scholes model was adopted for pricing options.
- 1983: On March 11, index option (OEX) trading begins; On July 1, options trading on the S&P 500 index (SPX) was launched.
- 1993: Introduces CBOE Volatility Index (VIX).
- 2004: CBOE Launches futures on VIX.
### OCC Monthly Report for September 2015

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>ETF</th>
<th>Index</th>
<th>SPX</th>
<th>VIX</th>
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<td>avg daily contract (million)</td>
<td>7.57</td>
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<td>avg daily premium ($ billion)</td>
<td>1.58</td>
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<td>2.05</td>
<td>1.43</td>
<td>1.48</td>
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</table>

- For Sept 2015, the average daily trading in options is 16.94 million contracts and $6.30 billion; the average daily trading in stocks is 7.92 billion shares and $321 billion.

- End of Sept 2015, the open interest for equity and ETF options is 292 million contracts, and 23.7 million contracts for index options. The overall market size: about $95.7 billion.
Returns to an At-the-Money Call Option (%)

- Underlying Stock Returns (%)
- Returns to an At-the-Money Call Option (%)
  - one-month ($\sigma=20\%$)
  - three-month ($\sigma=20\%$)
Returns to an At-the-Money Put Option (%)

- Underlying Stock Returns (%)
- Returns to an At-the-Money Put Option (%)

- one-month ($\sigma=20\%$)
- three-month ($\sigma=20\%$)
SPX: Avg Daily Volume and Put/Call Ratio

SPX Volume (left axis)

SPX Put/Call Ratio (right axis)

Fall 2017
Jun Pan, MIT Sloan

–11–
Options Exchanges

- After 6 years of litigation, CBOE in 2012 was able to retain its exclusive licenses on options on the S&P 500 index. As a result, CBOE remains its dominance in index options with over 98% of the market share.

- The trading of equity and ETF options, however, is spread over 12 options exchanges:

<table>
<thead>
<tr>
<th>OCC Monthly Report for September 2015</th>
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</thead>
<tbody>
<tr>
<td>CBOE</td>
</tr>
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<td>---</td>
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<tr>
<td>Equity (%)</td>
</tr>
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<td>ETF (%)</td>
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</table>

- This level of decentralized trading is not an option-only phenomenon. By now, US stocks are regularly traded on 11 exchanges (lit market) and many alternative platforms (dark pools).
Market Participants

• By types,
  – Designated market makers: facilitate trades and provide liquidity.
  – Customers from full service brokerage firms: e.g., hedge funds.
  – Customers from discount brokerage firms: e.g., retail investors.

• By their activities against the market makers:
  – open buy: buy options to open a new position.
  – open sell: sell/write options to open a new position.
  – close buy: buy options to close an existing position.
  – close sell: sell/write options to close an existing position.
<table>
<thead>
<tr>
<th></th>
<th>open buy</th>
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<th>open sell</th>
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<td>put</td>
<td>call</td>
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<td>call</td>
<td>put</td>
<td>call</td>
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<td>call</td>
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<td><strong>Small Stocks</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>avg volume</td>
<td>16</td>
<td>53</td>
<td>18</td>
<td>49</td>
<td>8</td>
<td>18</td>
<td>9</td>
<td>26</td>
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<tr>
<td>% Firm Proprietary</td>
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<td>4.46</td>
<td>5.42</td>
<td>4.09</td>
<td>4.42</td>
<td>4.84</td>
<td>3.83</td>
<td>3.75</td>
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<td>9.96</td>
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<td>7.81</td>
<td>11.14</td>
<td>6.74</td>
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<td>% Full-Service Broker</td>
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<td>71.73</td>
<td>75.84</td>
<td>73.66</td>
<td>77.90</td>
<td>72.09</td>
<td>75.96</td>
<td>71.60</td>
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<td><strong>Medium Stocks</strong></td>
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<td>avg volume</td>
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<td>89</td>
<td>17</td>
<td>39</td>
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<td>57</td>
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<tr>
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<td>8.49</td>
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<td>9.38</td>
<td>9.97</td>
<td>8.67</td>
<td>9.34</td>
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<tr>
<td>avg volume</td>
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<td>314</td>
<td>66</td>
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<td>% Discount Broker</td>
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<td>7.55</td>
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<td>68.72</td>
<td>69.95</td>
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<td>65.84</td>
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<td><strong>S&amp;P 500 (SPX)</strong></td>
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<tr>
<td>avg volume</td>
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<td>% Firm Proprietary</td>
<td>23.51</td>
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<td>35.71</td>
<td>25.51</td>
<td>32.51</td>
<td>20.05</td>
<td>20.10</td>
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<tr>
<td>% Discount Broker</td>
<td>4.22</td>
<td>4.19</td>
<td>1.38</td>
<td>1.59</td>
<td>1.48</td>
<td>1.72</td>
<td>4.45</td>
<td>4.78</td>
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<tr>
<td>% Full-Service Broker</td>
<td>58.24</td>
<td>48.16</td>
<td>48.81</td>
<td>59.45</td>
<td>49.75</td>
<td>63.79</td>
<td>59.58</td>
<td>51.72</td>
</tr>
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</table>

The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 1997

*for a new method to determine the value of derivatives*

Robert C. Merton

- 1/2 of the prize
- USA
- Harvard University
- Cambridge, MA, USA
- b. 1944

Myron S. Scholes

- 1/2 of the prize
- USA
- Long Term Capital Management
- Greenwich, CT, USA
- b. 1941
  (in Timmins, ON, Canada)
The Black-Scholes Model

- **The Model:** Let $S_t$ be the time-$t$ stock price, ex dividend. Prof. Black, Merton, and Scholes use a geometric Brownian motion to model $S_t$:

$$dS_t = (\mu - q) S_t \, dt + \sigma S_t \, dB_t.$$

- **Drift:** $(\mu - q) S_t \, dt$ is the deterministic component of the stock price. The stock price, ex dividend, grows at the rate of $\mu - q$ per year:
  - $\mu$: expected stock return (continuously compounded), around 12% per year for the S&P 500 index.
  - $q$: dividend yield, round 2% per year for the S&P 500 index.

- **Diffusion:** $\sigma S_t \, dB_t$ is the random component, with $B_t$ as a Brownian motion. $\sigma$ is the stock return volatility, around 20% per year for the S&P 500 index.
Brownian Motion

- **Independence of increments**: For all $0 = t_0 < t_1 < \ldots < t_m$, the increments are independent:

  $$B(t_1) - B(t_0), B(t_2) - B(t_1), \ldots, B(t_m) - B(t_{m-1})$$

  *Translating to Finance:* stock returns are independently distributed. No predictability and zero auto-correlation $\rho = 0$.

- **Stationary normal increments**: $B_t - B_s$ is normally distributed with zero mean and variance $t - s$.

  *Translating to Finance:* stock returns are normally distributed. Over a fixed horizon of $T$, return volatility is scaled by $\sqrt{T}$.

- **Continuity of paths**: $B(t), t \geq 0$ are continuous functions of $t$.

  *Translating to Finance:* stock prices move in a continuous fashion. There are no jumps or discontinuities.
The Model in $R_T$

- It is more convenient to work in the log-return space:
  \[ R_T = \ln S_T - \ln S_0 , \text{ or equivalently, } S_T = S_0 e^{R_T} \]

- Using the model for $S_T$, we get
  \[ R_T = \left( \mu - q - \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} \epsilon_T , \]

- Most of the terms are familiar to us:
  - $(\mu - q)T$ is the expected growth rate, ex dividend, over time $T$.
  - $\sigma \sqrt{T}$ is the stock return volatility over time $T$.
  - $\epsilon_T$ is a standard normal (inherited from the Brownian motion).

- The extra term of $-\frac{1}{2} \sigma^2 T$ is called the Ito’s term. It needs to be there because the transformation from $S_T$ to $R_T$ involves taking a log, which is a non-linear (concave) function, of the random variable $S_T$. 
The Payoff of a Call Option: 

\[(S_T - K) \mathbb{1}_{S_T > K}\]

\[\mathbb{1}_{S_T > K} = 1 \text{ if } S_T > K \text{ and zero otherwise.}\]
Valuation in Finance

- By now, we have a dynamic model for the stock price $S_T$. We also know the payoff function of a call option: $(S_T - K) 1_{S_T > K}$.

- We are now ready to calculate the market value, $C_t$, of this call option at any given time $t$ before the expiration date $T$.

- We do so by arbitrage pricing. Recall that in obtaining the CAPM pricing relation, we used **equilibrium pricing**, where mean-variance investors optimize their utility functions, and the equity and bond markets clear.

- In Finance, when it comes to valuation, there are just two approaches: equilibrium pricing and arbitrage pricing.
The Insight of Arbitrage Pricing

- The key insight of arbitrage pricing is very simple: replication.

- A security offers me a stream of random payoffs:
  - If I can replicate that cash flow (no matter how random they might be), then the price tag equates the cost of replication.
  - Simple? In reality, it is difficult to find such exact replications.
  - This makes sense: Why do we need a security that can be replicated?

- An option offers a random payoff at the time of expiration $T$:
  - The most important insight: dynamic replication.
  - The limitation: the replication is done under the Black-Scholes model.
  - The pricing formula is valid if the assumptions of the model are true.
Risk-Neutral Pricing

• Risk-neutral pricing is a widely adopted tool in arbitrage pricing.

• Our model in the return space:

  P-measure: \[ R_T = \left( \mu - q - \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} \epsilon_T. \]

• In risk-neutral pricing, we bend the reality by making the stock grow instead at the riskfree rate \( r \):

  Q-measure: \[ R_T = \left( r - q - \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} \epsilon^Q_T. \]

• Risk-neutral pricing: cash flows are discounted by the riskfree rate \( r \) and expectations are done under the Q-measure:

\[ C_0 = E^Q \left( e^{-rT} (S_T - K) 1_{S_T > K} \right) \]
Pricing a Stock

• Consider the S&P 500 index and assume zero dividend $q = 0$. The index’s final payoff is $S_T$. How much are you willing to pay for it today? Of course, $S_0$.

• Under P-measure:

$$e^{-\mu T} E^P(S_T) = e^{-\mu T} S_0 e^{\mu T} = S_0$$

• Under Q-measure:

$$e^{-r T} E^Q(S_T) = e^{-r T} S_0 e^{r T} = S_0$$

• Pricing using a Risk-neutral investor:

$$e^{-r T} E^P(S_T) = e^{-r T} S_0 e^{\mu T} = S_0 e^{(\mu-r)T}$$

• Risk-neutral pricing does not mean pricing using a risk-neutral investor.
Pricing a Call Option

- Let $C_0$ be the present value of a European-style call option on $S_T$ with strike price $K$. Using risk-neutral pricing:

$$
C_0 = E^Q \left( e^{-rT} (S_T - K) 1_{S_T > K} \right)
$$

$$
= e^{-rT} E^Q (S_T 1_{S_T > K}) \quad \text{– blue box}
$$

$$
- e^{-rT} K E^Q (1_{S_T > K}) \quad \text{– blue box}
$$

- Let’s go directly to the solution (again assume $q = 0$ for simplicity):

$$
C_0 = S_0 N(d_1) \quad \text{– green box}
$$

$$
- e^{-rT} K N(d_2) \quad \text{– blue box}
$$

where $N(d)$ is the cumulative distribution function of a standard normal.

- Comparing the terms in blue, we have $N(d_2) = E^Q (1_{S_T > K})$, which is $\text{Prob}^Q (S_T > K)$, the probability that the option expires in the money under the Q-measure.

- Comparing the terms in green: $N(d_1) = e^{-rT} E^Q \left( \frac{S_T}{S_0} 1_{S_T > K} \right)$.
Understanding $d_2$ and $d_1$:

$$d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + (r + \sigma^2/2) T}{\sigma \sqrt{T}}; \quad d_2 = \frac{\ln \left( \frac{S_0}{K} \right) + (r - \sigma^2/2) T}{\sigma \sqrt{T}}$$

- The model for $S_T$ under Q-measure is $S_T = S_0 e^{R_T}$ with

  $$R_T = \left( r - \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} \epsilon_T^Q$$

- We can verify that $N(d_2)$ indeed gives us $\text{Prob}^Q(S_T > K)$: the probability that the option expires in the money under the Q-measure.

- What about $N(d_1)$? With $E(S_T 1_{S_T > K})$, it calculates the expectation of $S_T$ only when $S_T > K$. This calculation is not required for exams.

- If you like, you can think of $N(d_1)$ as $\text{Prob}^{QQ}(S_T > K)$,

  $$R_T = \left( r + \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} \epsilon_T^{QQ}$$
The Black-Scholes Formula:

- The Black-Scholes formula for a call option (bring dividend back),

\[ C_0 = e^{-qT} S_0 N(d_1) - e^{-rT} K N(d_2) \]

\[ d_1 = \frac{\ln (S_0/K) + (r - q + \sigma^2/2) T}{\sigma \sqrt{T}} \]
\[ d_2 = \frac{\ln (S_0/K) + (r - q - \sigma^2/2) T}{\sigma \sqrt{T}} \]

- Put/call parity is model free. Holds even if the Black-Scholes model fails,

\[ C_0 - P_0 = e^{-qT} S_0 - e^{-rT} K. \]

Empirically, this relation holds well in the data and is similar in spirit to the arbitrage activity between the futures and cash markets.

- Using put/call parity, the Black-Scholes pricing formula for a put option is:

\[ P_0 = -e^{-qT} S_0 (1 - N(d_1)) + e^{-rT} K (1 - N(d_2)) \]
\[ = -e^{-qT} S_0 N(-d_1) + e^{-rT} K N(-d_2) \]
**At-the-Money Options** (assume $q = 0$)

- For an at-the-money option, whose strike price is $K = S_0 e^{rT}$

\[
C_0 = P_0 = S_0 \left[ N \left( \frac{1}{2} \sigma \sqrt{T} \right) - N \left( -\frac{1}{2} \sigma \sqrt{T} \right) \right]
\]

- Recall that $N(d)$ is the cdf of a standard normal,

\[
N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx
\]

- So the pricing formula can be further simplified to

\[
\frac{C_0}{S_0} = \frac{P_0}{S_0} = \int_{-\frac{1}{2} \sigma \sqrt{T}}^{\frac{1}{2} \sigma \sqrt{T}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx \approx \frac{1}{\sqrt{2\pi}} \sigma \sqrt{T},
\]

which works well for small $\sigma \sqrt{T}$. For large $\sigma \sqrt{T}$ (volatile markets or long-dated options), non-linearity becomes important and this approximation is imprecise.
ATM Options: \( d_1 = \frac{1}{2} \sigma \sqrt{T} \) and \( d_2 = -\frac{1}{2} \sigma \sqrt{T} \)
ATM Options as a Linear Contract on $\sigma \sqrt{T}$

Approximation: $\frac{C_0}{S_0} = \frac{P_0}{S_0} \approx \frac{\sigma \sqrt{T}}{\sqrt{2\pi}}$
The Black-Scholes Option Implied Volatility:

- At time 0, a call option struck at $K$ and expiring on date $T$ is traded at $C_0$. At the same time, the underlying stock price is traded at $S_0$, and the riskfree rate is $r$.

- If we know the market volatility $\sigma$ at time 0, we can apply the Black-Scholes formula:

$$C_{0\text{Model}} = \text{BS}(S_0, K, T, \sigma, r, q)$$

- Volatility is something that we don’t observe directly. But using the market-observed price $C_{0\text{Market}}$, we can back it out:

$$C_{0\text{Market}} = C_{0\text{Model}} = \text{BS}(S_0, K, T, \sigma^I, r, q).$$

- If the Black-Scholes model is the correct model, then the Option Implied Volatility $\sigma^I$ should be exactly the same as the true volatility $\sigma$. 
Example 1: Option Quotes from CBOE (Oct 2011):

The current index level is at 1200.86, down -23.72 (or 1.94%) from the day before.

The near-term and near-to-the-money call option is the “SPX1122J1200-E” contract, which is traded at $13.40 bid and $16.90 ask. The mid-quote price is $15.15. (The bid/ask spread is $3.5 and the percentage bid/ask spread is 23%).

The last transaction price is $15.00, down $16.70 (or 52.68%) from the day before.

There were 22,631 such contracts traded and the open interest (or number of option contracts outstanding) is 130,393.
Back out the option implied volatility:

\[
\begin{array}{cccccc}
S_0 & K & T & r & q & \sigma \\
1200.86 & 1200 & 5/365 & 0.0024 & 0.02 & ?
\end{array}
\]

![Graph showing the relationship between implied volatility and call option price.](image_url)
**Try a Few Other Contracts:**

<table>
<thead>
<tr>
<th></th>
<th>$C_0$ or $P_0$</th>
<th>$S_0$</th>
<th>$K$</th>
<th>$T$</th>
<th>$r$</th>
<th>$q$</th>
<th>$\sigma$ (%)</th>
</tr>
</thead>
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<td>Call</td>
<td>15.00</td>
<td>1200.86</td>
<td>1200</td>
<td>5/365</td>
<td>0.0024</td>
<td>0.02</td>
<td>26.25</td>
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<td>Put</td>
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<td>1200</td>
<td>5/365</td>
<td>0.0024</td>
<td>0.02</td>
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<td>1200</td>
<td>34/365</td>
<td>0.0024</td>
<td>0.02</td>
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<td>1200</td>
<td>34/365</td>
<td>0.0024</td>
<td>0.02</td>
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</table>
Data Synchronization Issues

• Notice that in our calculation of the Black-Scholes implied volatility, we assume that the index options are traded at exactly the same time as the underlying index (which was marked at 1200.86).

• In practice, they are marked as “Last Sale,” and we actually do not know when the last sale took place for each option.

• So to be precise, we need to have the time stamp of each option transaction and then use the time stamp to retrieve the underlying index level at exactly the same time. CBOE offer such transaction level data, but it is expensive.

• The trading hours for SPX options are 8:30am-3:15pm Chicago time, while the NYSE hours are 9:30am-4:00pm New York time.
• Notice also that put/call parity is violated by close to 4 dollars for the November 1200 contracts. In this market, put/call parity holds pretty well. In other words, it has to be a data issue.

• If we are confident that the paired call/put options (the same time to expiration and the same strike price) are traded at exactly the same time, then we can actually use put/call parity to back out the underlying index level (instead of relying on the underlying market).

• According to CBOE, the VIX index measured at close is 33.39 and the VXO index measured at close is 31.40.
Example 2: Option Quotes from CBOE (Oct 2005):

- The current index level is at 1195.90, up 4.41 from the day before.
- The *near-term* and *near-to-the-money* call option is the “05 Oct 1195” contract, which is traded at $13.10 bid and $14.30 ask. The mid-quote price is $13.70.
- The last transaction price is $13.50, up $1 from the day before.
- There were 3524 such contracts traded and the open interest (or number of option contracts outstanding) is 2212.
- The total dollar trading volume for this Oct contract is $13.7 \times 100 \times 3524 = \$4.8m.$
Back out the option implied volatility:

<table>
<thead>
<tr>
<th>$S_0$</th>
<th>$K$</th>
<th>$T$</th>
<th>$r$</th>
<th>$q$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1195.90</td>
<td>1195</td>
<td>15/365</td>
<td>0.04</td>
<td>0.02</td>
<td>?</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between Call Option Price ($) and Sigma (%). The graph is a linear function with a positive slope, indicating that as Sigma (%) increases, the Call Option Price ($) also increases.]
Some Details:

- October contracts expire on Oct. 22, which is the Saturday after the third Friday of the month. The option trading took price on Oct. 7, which is 15 calendar days away from the expiration date.

- The assumptions of $r = 4\%$ and $q = 2\%$ are approximations. For longer-dated options, assumptions on the riskfree rate and dividend yield become more important.

- In Excel, you can back out the implied volatility using a solver.

- We can perform the same exercise on put options to obtain put option implied volatility.
Bring the Black-Scholes Model to the Data

- The key assumptions of the model: constant volatility, continuous price movements, and log-normal distribution.
- The data: S&P 500 index options of different levels of moneyness and time to expiration.
- The basic tool: the Black-Scholes Option Implied Volatility.