Relating Equity and Credit Markets through Structural Models: Evidence from Volatilities

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Abstract

This paper examines the connection between the return volatilities of credit market securities, equities, and Treasuries using a Merton model with stochastic interest rates. Focusing primarily on monthly bond and CDS returns, we find that the credit market exhibits volatility in excess of what the equity market and the Merton model suggest. In conjunction with the evidence in Schaefer and Strebulaev (2008), this suggests that while the co-movement of returns in the credit and equity markets can be characterized correctly on average by a Merton model, the value in credit markets sometimes deviates from fundamentals. Furthermore, we find that the excess volatility in credit markets is associated with less liquid issues and issues with poorer ratings, but does not appear to be worse at the height of the Financial Crisis.

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1 Introduction

Research in structural models of default has largely found that these models fail in explaining the level of debt prices. Huang and Huang (2003) find that a number of structural models with different mechanisms underpredict corporate bond yield spreads, reflecting a generally pessimistic view of the applicability of structural models.[1] In contrast, Schaefer and Strebulaev (2008) find that the Merton (1974) model is successful in explaining the sensitivities of debt values to changes in asset value. Specifically, Schaefer and Strebulaev (2008) find that level of bond returns can be explained by contemporaneous equity returns and a Merton model-implied sensitivity of bond returns to equity returns. Implicitly, debt and equity returns are linked through asset returns. The results in Schaefer and Strebulaev (2008) are intriguing and provide researchers with a direction where structural models may be more fruitfully used.[2]

Using the Schaefer and Strebulaev (2008) results as a starting point, we aim to further understand the co-movement of securities in different markets. We examine whether a Merton model with stochastic interest rates can successfully relate the contemporaneously realized empirical volatilities of corporate bonds and equities of the same firm, finding that the empirical volatility of monthly corporate bond returns exceed model-implied volatility estimates by 2.19 percentage points. In the CDS market, we find that empirical volatilities exceed model-implied volatilities by an average of 1.92 percentage points and 2.84 percentage points when daily and monthly returns are used, respectively. Empirical volatilities for corporate bonds are calculated from returns using transaction size-weighted prices to avoid volatility arising solely from effective bid-ask spreads and empirical CDS volatilities are based on consensus mid prices. Very importantly, we use monthly bond returns rather than higher frequency returns to avoid a direct effect of liquidity on the estimated volatilities. As Bao, Pan, and Wang (2011) show, the autocovariance of returns in the corporate

[1]Collin-Dufresne, Goldstein, and Martin (2001) find that changes in corporate bond yield spreads are difficult to explain in a reduced-form framework.

[2]Also pursuing this direction, Bhamra, Kuehn, and Strebulaev (2009) examine the credit spread and equity premium puzzles in a unified framework.

bond market is quite high and negative. A negative autocovariance is symptomatic of a large effective bid-ask spread. At short horizons, empirical volatilities that use transaction prices are dominated by volatilities from this spread.[3] Model volatilities are robust to different ways of implementing the model. Thus, while Schaefer and Strebulaev (2008) find that, *on average*, bond returns can be explained by equity returns and Merton model hedge ratios, we find that bond returns exhibit additional noise. We emphasize that our results are a further characterization of the relative realized returns in the two markets and are complementary, rather than contradictory, to the results in Schaefer and Strebulaev (2008).

The excess volatility in the corporate bond and CDS markets along with the Schaefer and Strebulaev (2008) result that the relative returns in the two markets are correct on average are consistent with time-varying illiquidity in credit markets. If the only effect of illiquidity is to generate a constant level of excess yield spreads, we would not expect to see excess volatility. Instead, the results are consistent with price pressure in the OTC credit markets temporarily driving prices away from fundamentals as described theoretically by Duffie (2010) and examined in the bond market by Feldhutter (2012).[4] This price pressure, which can create time-varying prices even in the absence of changes in firm fundamentals, may contribute to the additional volatility in the credit market. To the extent that less liquid securities in OTC markets are more likely to have prices temporarily driven away from fundamentals, this explanation implies that excess volatility should be correlated with proxies for liquidity.

Next, we examine empirical and model volatility in the time-series and in the cross-section to determine if there is a systematic pattern to excess volatilities. In the time-series, we examine the volatility of CDS, calculated each month using daily returns. We find that period-by-period average model-implied volatilities are typically lower than average empirical

[3]In an earlier draft of this paper, we found that mean annualized empirical volatilities were 21.77% when daily returns from transaction prices were used as compared to 8.10% when monthly returns were used. See also Corwin and Schultz (2012) who note that fundamental volatility increases proportionally with the length of the trading period while volatility due to bid-ask spreads does not.

[4]See also Bongaerts, de Jong, and Driessen (2011b) who show in an equilibrium model that assets in an illiquid market can have lower *or higher* prices than in a liquid market. This result is particularly relevant for markets with zero net supply such as CDS markets.

volatilities, but that there is strong co-movement between the two series. Interestingly, the model does well late in 2008, during the height of the Financial Crisis. At the individual bond and CDS level, we also find that empirical spreads tend to be high when model spreads are high, suggesting that while the Merton model cannot match the levels of empirical volatilities, it can characterize the time-series variation in volatilities. In panel regressions with firm fixed-effects, we find little evidence that excess volatility can be explained by changing macroeconomic conditions. The one variable that is associated with excess volatility is contemporaneous volatility in the bid-ask spread of CDS.

In the cross-section, we examine the correlation between excess volatility and both firm-level characteristics and security-level liquidity. Most firm-level characteristics are unimportant. The most significant drivers of excess volatility in the cross-section are a firm's credit quality and the liquidity of the corporate bond or CDS. As structural models are not designed to measure liquidity, the fact that most of the variables that are correlated with excess volatility are liquidity variables bodes well for the Merton model. Overall, we view our results as largely supportive of the Schaefer and Strebulaev (2008) conclusion that structural models of default are useful.

Our paper is mostly closely tied to two strands of literature on corporate bonds, structural models of default and liquidity. In addition to Huang and Huang (2003), Jones, Mason, and Rosenfeld (1984) and Eom, Helwege, and Huang (2004) also focus on whether structural models can generate the correct levels of bond prices. Focusing on the Merton model and a sample of 27 firms from 1975 to 1981, Jones, Mason, and Rosenfeld (1984) find that model prices are higher than empirical prices. Eom, Helwege, and Huang (2004) use a sample of 182 data points, finding that the Merton model underpredicts empirical yield spreads, but other models actually overpredict yield spreads.[5] In contrast to these papers, we focus on volatilities of returns rather than the levels of bond prices as Schaefer and Strebulaev (2008) have shown that structural models characterize returns better than prices.

[5] A number of other papers have related evaluations of the Merton model, including Crosbie and Bohn (2003), Leland (2004), and Bharath and Shumway (2008).

In the literature on illiquidity in the corporate bond market, Edwards, Harris, and Piwowar (2007) find that the effective bid-ask spread of corporate bonds is quite large, particularly for trades of small sizes. A series of other authors, including Chen, Lesmond, and Wei (2007), Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhutter, and Lando (2012), and Bongaerts, de Jong, and Driessen (2011a) find evidence that liquidity characteristics are priced. Arguing that the CDS market is significantly more liquid than the corporate bond market, Longstaff, Mithal, and Neis (2005) show that liquidity is important in the corporate bond market by using CDS as a control for credit risk. However, Tang and Yan (2007) find significant liquidity effects in the CDS market. Bongaerts, de Jong, and Driessen (2011b) confirm that there are liquidity effects in the CDS market, but argue that these effects are economically small.

Finally, our paper is related to Vassalou and Xing (2004), Campbell, Hilscher, and Szilagyi (2007), and Ang, Hodrick, Xing, and Zhang (2006). Vassalou and Xing (2004) use a Merton model to estimate a distance-to-default and find some evidence of a positive default risk premium in the equity market. Campbell, Hilscher, and Szilagyi (2007) use a logit-based model, finding lower returns for high default likelihood firms. Ang, Hodrick, Xing, and Zhang (2006) examine volatility, but of idiosyncratic equity returns and find that high idiosyncratic equity volatility portfolios have lower future equity returns. We note that an important difference between our paper and these papers is that these papers look to predict relative future returns. Instead, we aim to explain contemporaneously observed asset pricing moments in the credit and equity markets.

The rest of the paper is organized as follows. Section 2 outlines the empirical specification. Section 3 summarizes the data and the sample. Section 4 documents the volatility estimates. Section 5 discusses alternative specifications, the co-movement of empirical and model volatilities, and possible sources of the disconnect in volatilities. Section 6 concludes.

2 Empirical Specification

2.1 The Merton Model

We use the Merton (1974) model to connect the equity and corporate bonds of the same firm.

Let V be the total firm value, whose risk-neutral dynamics are assumed to be

$$\frac{dV_t}{V_t} = (r_t - \delta)\, dt + \sigma_v\, dW_t^Q\,, \tag{1}$$

where W is a standard Brownian motion, and where the payout rate δ and the asset volatility σ_v are assumed to be constant.

We adopt a simple extension of the Merton model to allow for a stochastic interest rate.[6] This is important for our purposes because a large component of the corporate bond volatility comes from the Treasury market. Specifically, we model the risk-free rate using the Vasicek (1977) model:

$$dr_t = \kappa\,(\theta - r_t)\; dt + \sigma_r\, dZ_t^Q\,, \tag{2}$$

where Z is a standard Brownian motion independent of W, and where the mean-reversion rate κ, long-run mean θ and the diffusion coefficient σ_r are assumed to be constant.

Following Merton (1974), let us assume for the moment that the firm has, in addition to its equity, a single homogeneous class of debt, and promises to pay a total of K dollars to the bondholders on the pre-specified date T. Equity then becomes a call option on V:

$$E_t = V_t\, e^{-\delta\tau}\, N(d_1) - K\, e^{a(\tau)+b(\tau)\, r_t}\, N(d_2), \tag{3}$$

where $\tau = T - t$, $N(\cdot)$ is the cumulative distribution function for a standard normal, $d_1 = d_2 + \sqrt{\Sigma}$,

$$d_2 = \frac{\ln(V/K) - a(\tau) - b(\tau) r_t - \delta\tau - \frac{1}{2}\Sigma}{\sqrt{\Sigma}}, \tag{4}$$

[6] See Shimko, Tejima, and van Deventer (1993).

$$\Sigma = \tau(\sigma_v^2 + \frac{\sigma_r^2}{\kappa^2}) + \frac{2\sigma_r^2}{\kappa^3}(e^{-\kappa\tau} - 1) - \frac{\sigma_r^2}{2\kappa^3}(e^{-2\kappa\tau} - 1), \tag{5}$$

and where $a(\tau)$ and $b(\tau)$ are the exponents of the discount function of the Vasicek model:

$$b(\tau) = \frac{e^{-\kappa\tau} - 1}{\kappa}; \quad a(\tau) = \theta\left(\frac{1 - e^{-\kappa\tau}}{\kappa} - \tau\right) + \frac{\sigma^2}{2\kappa^2}\left(\frac{1 - e^{-2\kappa\tau}}{2\kappa} - 2\frac{1 - e^{-\kappa\tau}}{\kappa} + \tau\right). \tag{6}$$

Note that a Merton model extended to have Vasicek interest rates simply has $e^{-r\tau}$ replaced by $e^{a(\tau)+b(\tau)r_t}$ and $\sigma_v^2\tau$ replaced by Σ.

2.2 From Equity Volatility to Asset Volatility

We first use the Merton model to link the firm's asset volatility to its equity volatility. Let σ_E be the volatility of instantaneous equity returns. In the model, the equity volatility is affected by two sources of random fluctuations:[7]

$$\sigma_E^2 = \left(\frac{\partial \ln E_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln E_t}{\partial r_t}\right)^2 \sigma_r^2 . \tag{7}$$

Using equation (3), we can calculate the sensitivities of equity returns to the random shocks in asset returns and risk-free rates:

$$\frac{\partial \ln E_t}{\partial \ln V_t} = \frac{1}{1 - \mathcal{L}} \quad \text{and} \quad \frac{\partial \ln E_t}{\partial r_t} = \frac{-b(\tau)\,\mathcal{L}}{1 - \mathcal{L}},$$

where

$$\mathcal{L} = \frac{K}{V}\frac{N(d_2)}{N(d_1)} \exp\left(\delta\,\tau + a(\tau) + b(\tau)\,r_t\right) . \tag{8}$$

Combining the above equations, we have

$$\sigma_E^2 = \left(\frac{1}{1 - \mathcal{L}}\right)^2 \sigma_v^2 + \left(\frac{\mathcal{L}}{1 - \mathcal{L}}\right)^2 b(\tau)^2\,\sigma_r^2 . \tag{9}$$

[7]This relation between equity and asset volatility requires parameters for interest rate dynamics. We discuss interest rate calibrations in Appendix A.

As expected, the firm's equity volatility σ_E is closely related to its asset volatility σ_v. In addition, it is also affected by the Treasury volatility σ_r through the firm's borrowing activity in the bond market. This is reflected in the second term of equation (9), with $-b(T)\,\sigma_r$ being the volatility of instantaneous returns on a zero-coupon risk-free bond of the same maturity T. The actual impact of these two random shocks is further amplified through $\mathcal{L}$, which, for lack of a better expression, we refer to as the "modified leverage." Specifically, for a firm with a higher $\mathcal{L}$, a one unit shock to its asset return is translated to a larger shock to its equity return – this is the standard leverage effect. Moreover, as shown in the second term of equation (9), for such a highly "levered" firm, its equity return also bears more interest rate risk. Conversely, for an all-equity firm, $\mathcal{L} = 0$, and the interest-rate component diminishes to zero.

As is true in many empirical studies before us, a structural model such as the Merton model plays a crucial role in connecting the asset value of a firm to its equity value. Ours is not the first empirical exercise to back out asset volatility using observations from the equity market.[8] In the existing literature, there are at least two alternative ways to approximate K/V. In the approach pioneered and popularized by Moody's KMV, the Merton model is used to calculate $\partial E/\partial V$ as well as to infer the firm value V through equation (3). By contrast, we use the Merton model to derive the entire piece of the sensitivity or elasticity function $\partial \ln E/\partial \ln V$, as opposed to using only $\partial E/\partial V$ from the model and then plugging in the market observed equity value E for the scaling component. At a conceptual level, we believe that taking the entire piece of the sensitivity function from the Merton model is a more consistent approach. At a practical level, while the Merton model might have its limitations in the exact valuation of bonds and equities, it is still valuable in providing insights on how a percentage change in asset value propagates to percentage changes in equity value for a levered firm.[9]

[8]See, for example, Crosbie and Bohn (2003), Eom, Helwege, and Huang (2004), Bharath and Shumway (2008), and Vassalou and Xing (2004).

[9]Particularly in light of the results by Schaefer and Strebulaev (2008) that a Merton model does well in relating corporate bond and equity returns and the Huang and Huang (2003) results that the levels of corporate bond yield spreads are too low, we feel that using the Merton model to provide model elasticities

In this respect, our reliance on the Merton model centers on the sensitivity measure. To the extent the Merton model is important in our empirical implementation, it is in deriving the analytical expressions that enter equation (9). In particular, we rely on the Merton model to tell us how the sensitivities or elasticities vary as functions of the key parameters of the model including leverage K/V, asset volatility σ_v, payout rate δ, and debt maturity T. When it comes to the actual calculations of these key parameters, we deviate from the Merton model as follows.

The key parameter that enters equation (9) is the ratio K/V, where K is the book value of debt and V is the market value of the firm. We calculate the book debt K using the sum of long-term debt and debt in current liabilities from Compustat, and approximate the firm value V by its definition $V = S + D$, where S is the market value of equity and D is the market value of debt. To estimate the market value of debt D, we start with the book value of debt K. To further improve on this approximation, we collect, for each firm, all of its bonds in TRACE, calculate an issuance weighted market-to-book ratio, and multiply K by this ratio.

Implicit in our estimation of the firm value V is the acknowledgment that firms do not issue discount bonds as prescribed by the Merton model. In particular, we deviate from the zero-coupon structure of the Merton model in order to take into account the fact that firms typically issue bonds at par. By adopting this empirical implementation, however, we do have to live with one internal inconsistency with respect to the relation between K and D, and central to this inconsistency is the problem of applying a model designed for zero-coupon bonds to coupon bonds.

The main implication of our choice of V is on the ratio of K/V, which in turn, affects the firm's actual leverage. We can therefore gauge the impact of our implementation strategy by comparing the market leverage implied by the Merton model with the empirically estimated market leverage. Our results show that with our choice of K/V, the two market leverage numbers, model implied vs. empirically estimated, are actually very close for the sample of

alone rather than model prices is the best use of the model.

firms considered in this paper. Closely related to this comparison is the alternative estimation strategy that infers K/V by matching the two market leverage ratios: model-implied and empirically estimated.[10] From our analysis, we expect this approach to yield K/V ratios that are close to ours.

Finally, two other parameters that enter equation (9) are the firm-level debt maturity T and the firm's payout ratio δ. Taking into account the actual maturity structure of the firm, we collect, for each firm, all of its bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. To calculate the payout ratio δ, we first take a firm's average coupon payment times its face value K and add this to its equity dividends from Compustat. We then scale this sum by firm value V, with the details of calculating V summarized above. Estimating the asset volatility, σ_v, then relies on using the variables described in this section (K, V, δ, T), interest rate parameters described in Appendix A $(\kappa, \theta, \sigma_r, r)$, equity volatility, and equation (9) to calculate an implied asset volatility.[11]

2.3 Model-Implied Bond Volatility

The second step of our empirical implementation is to calculate, bond-by-bond, the volatility of its instantaneous returns, taking the inferred asset volatility σ_v from the first step as a key input. These model-implied bond volatilities can then be compared to empirically observed bond volatilities. Again, we have to make a simplification to the Merton model to accommodate the bonds of varying maturities issued by the same firm. Specifically, we rely on the Merton model to tell us, for any given time τ, the value of payments at τ contingent

[10] We thank Hayne Leland for pointing this out and for extensive discussions on this issue.

[11] Conceptually, this is related to using the Black-Scholes model to calculate an implied volatility. The main differences are that the volatility of equity returns is used as an input rather than the value of equity and that the implied asset volatility is contemporaneous to the equity volatility used in the calculation.

on $V_\tau > K$. Compared to taking the Merton model literally, which would imply no default between time 0 and the maturity date T, we find this to be a more realistic adoption of the model.

Equipped with the term structure of default probabilities implied by the Merton model, we can now price defaultable bonds issued by each firm. Consider a τ-year bond paying semi-annual coupons with an annual rate of c. Assuming a face value of \$1, the time-t price of the bond is

$$\begin{aligned} B_t = & \sum_{i=1}^{2\tau} \frac{c}{2} E_t^Q \left[\exp\left(-\int_t^{t+i/2} r_s ds \right) \mathbf{1}_{\{V_{t+i/2}>K\}} \right] + E_t^Q \left[\exp\left(-\int_t^{T} r_s ds \right) \mathbf{1}_{\{V_T>K\}} \right] \\ & + \sum_{i=1}^{2\tau} \mathcal{R} \left\{ E_t^Q \left[\exp\left(-\int_t^{t+i/2} r_s ds \right) \mathbf{1}_{\{V_{t+(i-1)/2}>K\}} \right] - E_t^Q \left[\exp\left(-\int_t^{t+i/2} r_s ds \right) \mathbf{1}_{\{V_{t+i/2}>K\}} \right] \right\} \end{aligned} \quad (10)$$

where $\mathcal{R}$ is the risk-neutral expected recovery rate of the bond upon default.[12] The first two terms in equation (10) collect the coupon and the principal payments, taking into account the probabilities of survival up to each payment. The third term collects the recovery of the bond taking into account the probability of default happening exactly within each six-month period. The solutions to these expectations and the full bond pricing formula are given in Appendix B.

Let σ_D^{Merton} be the volatility of the instantaneous returns of the defaultable bond. The model-implied bond volatility can be calculated as

$$\left(\sigma_D^{\text{Merton}}\right)^2 = \left(\frac{\partial \ln B_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln B_t}{\partial r_t}\right)^2 \sigma_r^2 . \quad (11)$$

The sensitivities in equation (11) can be calculated based on partial derivatives of equation (10). The asset-sensitivity, $\partial \ln B_t / \partial \ln V_t$, arises from the sequence of (present value adjusted) risk-neutral default probabilities while the Treasury-sensitivity, $\partial \ln B_t / \partial r_t$, arises both explicitly from the sequence of Vasicek discount functions and implicitly from the se-

[12] We use a recovery of 50%. Huang and Huang (2003) use a recovery rate of 51.31%.

quence of risk-neutral default probabilities. The model-implied CDS volatility can also be calculated using equation (11). Details of the model prices of CDS are given in Appendix C. The main distinction to note between calculating corporate bond and CDS model volatilities is that CDS are significantly less sensitive to interest rates than corporate bonds, but maintain a similar level of sensitivity to asset value. Thus, CDS data not only provides us with both an additional sample to examine volatilities, but one which is less reliant on the interest rate model used.

It might be instructive to consider a τ-year zero-coupon bond, since its calculation can be further simplified to

$$\frac{\partial \ln B_t}{\partial \ln V_t} = \frac{n(d_2)\,(1-\mathcal{R})}{N(d_2) + (1 - N(d_2))\,\mathcal{R}}\,\frac{1}{\sqrt{\Sigma}} \quad \text{and} \quad \frac{\partial \ln B_t}{\partial r_t} = b(\tau)\left(1 - \frac{\partial \ln B_t}{\partial \ln V_t}\right),$$

where $n(\cdot)$ is the probability distribution function of a standard normal. As expected, with full recovery upon default, $\mathcal{R} = 1$, the bond is equivalent to a treasury bond and its asset-sensitivity is zero and its Treasury-sensitivity becomes $b(\tau)$. The asset-sensitivity becomes more important with increasing loss given default, $1 - \mathcal{R}$, as well as with increasing firm leverage K/V. From this example, we can also see the importance of allowing for a stochastic risk-free rate, as the Treasury volatility is an important component in the defaultable bond volatility.

It is also useful to use the zero-coupon bond to illustrate that model corporate bond volatility is not necessarily increasing in the riskiness of the firm. Consider a safe corporate bond with low K/V. For this bond, $\partial \ln B_t / \partial \ln V_t$ approaches 0 and $\partial \ln B_t / \partial r_t$ approaches $b(\tau)$. A bond with higher K/V is riskier and has a higher sensitivity to firm value ($\partial \ln B_t / \partial \ln V_t$), but a lower sensitivity to interest rates ($\partial \ln B_t / \partial r_t$) in magnitude. Define $x \equiv \partial \ln B_t / \partial \ln V_t$. Then,

$$\left(\sigma_D^{Merton}\right)^2 = x^2 \sigma_v^2 + b(\tau)^2 (1-x)^2 \sigma_r^2$$

for a zero-coupon bond. It can be shown that,

$$\frac{\partial\left(\sigma_D^{Merton}\right)^2}{\partial x} = 2x\left(\sigma_v^2 + b\left(\tau\right)^2\sigma_r^2\right) - 2b\left(\tau\right)^2\sigma_r^2$$
$$\frac{\partial\left(\sigma_D^{Merton}\right)^2}{\partial x} < 0 \text{ if } x < \frac{b(\tau)^2\sigma_r^2}{\sigma_v^2 + b(\tau)^2\sigma_r^2}$$

That is, for low values of $\partial \ln B_t/\partial \ln V_t$, model variance is decreasing in $\partial \ln B_t/\partial \ln V_t$.[13] The intuition for this result is that for a Treasury bond, the sensitivity to interest rates is strongly negative, whereas for a defaultable bond, there are two effects. While a higher discount rate decreases the value of debt through a discounting channel, it also increases the value of debt as the larger risk-neutral drift for firm value decreases the likelihood of bankruptcy. The two countervailing effects tend to make a defaultable bond less sensitive to interest rates than a risk-free bond. For small values of $\partial \ln B_t/\partial \ln V_t$, this decreased sensitivity to interest rates along with an increased sensitivity to firm value actually leads to a decreased model bond volatility as the former effect dominates the latter. For a CDS, which is less sensitive to interest rates, this effect is less relevant.

In calculating the model-implied bond volatility, we take advantage of the model-implied term structure of survival probabilities but avoid treating the defaultable bond as one large piece of zero-coupon bond with face value of K and maturity of T. This calculation is similar to the reduced-form approach of Duffie and Singleton (1999), except for the fact that our term structure of survival probabilities come from a structural model while theirs derives from a stochastic default intensity.

[13] It can similarly be shown that σ_D is decreasing in σ_v for safe firms.

3 Data

3.1 Data Sources

The bond pricing data for this paper are obtained from FINRA's TRACE (Transaction Reporting and Compliance Engine). This data set is a result of recent regulatory initiatives to increase the price transparency in the secondary corporate bond markets. FINRA, formerly NASD, is responsible for operating the reporting and dissemination facility for over-the-counter corporate trades. Trade reports are time-stamped and include information on the clean price and par value traded, although the par value traded is top-coded at \$1 million for speculative grade bonds and at \$5 million for investment grade bonds.

The cross-sections of bonds in our sample vary with the expansion of coverage by TRACE. On July 1, 2002, the NASD began Phase I of bond transaction reporting, requiring that transaction information be disseminated for investment grade securities with an initial issue of \$1 billion or greater. At the end of 2002, the NASD was disseminating information on approximately 520 bonds. Phase II, implemented on April 14, 2003, expanded reporting requirements, bringing the number of bonds to approximately 4,650. Phase III, implemented on February 7, 2005, required reporting on approximately 99% of all public transactions.

The CDS data for this paper are obtained from Datastream. Prior to 2007, Datastream's sole source of CDS data was CMA Datavision. Mayordomo, Pena, and Schwartz (2010) find that the CMA database leads the price discovery process in comparison with a number of CDS databases including Markit. In 2007, Datastream began reporting CDS data from Thomson Reuters and eventually ceased its coverage of the CMA data in September 2010. Given the evidence that the CMA data is of high quality and the uncertainty regarding the quality of the Thomson data, we focus on the CMA data, which covers the period from January 2004 to September 2010, and use 5-year credit default swaps as they are the most liquid. Over this period of time, the CMA data in Datastream covers 695 names for 5-year senior CDS, though many names are only covered for a short subset of the period. This data consists of bid, ask, and mid consensus prices.

3.2 Sample Description

We use transaction-level data from TRACE to construct bond return volatilities for non-financial firms. First, we construct monthly bond returns as follows. For a bond in month t, we take all trades from the 21st of the month and later. We calculate the clean price for the end of the month as the transaction size-weighted average of these trades.[14] Returns are then calculated as:

$$R_t = \ln\left(\frac{P_t + AI_t + C_t}{P_{t-1} + AI_{t-1}}\right)$$

where P_t is the transaction size-weighted average clean price, AI_t is the accrued interest, and C_t is the coupon paid in month t. Bond-level information is obtained from FISD for coupon rates and maturities. Accrued interest is calculated using the standard 30/360 convention and returns are only calculated for month t if we have a transaction price for both month t and month $t-1$.[15] We do not calculate daily returns in for the corporate bond sample. At short horizons, small components of the bid-ask spread that are not fully eliminated can significantly contribute to volatility. In the CDS sample, we consider both daily and monthly returns, using consensus mid prices. For each bond-year and CDS-year, we then calculate the volatility of monthly returns in a year if there are at least 10 returns available and annualize. For each CDS-month, we calculate the volatility of daily returns and annualize.[16]

Table 1 summarizes the corporate bonds in our sample and Table 2 summarizes the firms corresponding to the corporate bonds and CDS in our sample. As Panel A of Table 1 shows,

[14] Bessembinder, Kahle, Maxwell, and Xu (2009) recommend calculating prices as the transaction-weighted average of prices. This minimizes the effects of bid-ask spreads in prices. As shown in Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011), these effects are largest for small trades. Our choice of considering trades on the 21st or later is based on obtaining a balance between prices that reflect month-end prices and maintaining a reasonable number of trades to calculate average prices.

[15] An alternative treatment would be to use the last trade in a month regardless of what day the trade occurred and to treat clean prices as unchanged if no trades occurred. However, this would lead to returns in the bond market that do not necessarily reflect changes in asset value during the month, breaking the link between equities and corporate bonds.

[16] The full procedure for calculating returns and volatilities for CDS is described in Appendix C.

there are 1,021 distinct bonds in our sample and 2,883 bond-years. Similar to most studies using TRACE, our sample is limited simply because many bonds do not trade frequently. Imposing the restriction that prices must be from the 21st of the month or later and that there must be at least 10 returns in a year to calculate a volatility, there are close to 28,000 bond-years and 10,000 distinct bonds. The sample is further reduced to about 24,000 observations when we impose the restriction that the bond-year must match to ordinary equity in CRSP. About one-third of the remaining observations are Financials, which are dropped. Additional filters that decrease the sample size include filtering out putables, convertibles, and callables along with dropping bonds issued by firms with insufficient information in Compustat. The primary reason for the decrease in sample size at this stage is due to the fact that most corporate bonds, particularly those issued by non-financials, are callable.[17]

Due to the fact that large issues tend to trade more frequently, the bonds in our sample are larger issues than the typical bonds in FISD, with an average face value of $585mm compared to $184mm for the full FISD sample. The bonds in our sample also tend to be older, but are of similar ratings on average (7=A3). The average number of trades in a year for the bonds in our sample is approximately 1,500, which is frequent in the corporate bond market. By contrast, Edwards, Harris, and Piwowar (2007) report that the average bond in their sample trades 2.4 times a day and the median bond 1.1 times a day.

In Table 2, we present summary statistics for the firms represented in our corporate bond (Panel A) and CDS (Panel B) samples. There are 735 firm-years in our corporate bond sample or an average of 92 firms per year. These firms are relatively large, averaging $40 billion in equity market capitalization and representing an average of $3.7 trillion in total equity market capitalization and $4.3 trillion in total book assets per year. The firms represented in our CDS sample are broader, with an average of 303 firms per year. These firms are also large, with an average market capitalization of $22.59 billion. This implies that the firms in the CDS sample cover an average of $6.8 trillion in total equity market

[17] Note that the number of bonds in Dick-Nielsen, Feldhutter, and Lando (2012) is 2,224 (Table 2 of their paper) and the number of bonds in Bao, Pan, and Wang (2011) is 1,035. Both of these papers include Financials, but also have different filtering criteria due to their different questions.

capitalization each year. As a comparison, the total market capitalization for non-financial ordinary shares in CRSP was \$9.3 trillion in 2008. In addition to being large, the average firm in our sample is healthy as the average firm is profitable and has a coverage ratio close to 10.

4 Volatility Estimates

4.1 Empirical Bond Return Volatility $\hat{\sigma}_D$

In the first column of Table 3, we report the empirical bond and CDS volatilities. Empirical bond volatilities using monthly bond returns are presented in Panel A. We find that the average annualized volatility for the full sample is 6.86% and that there is an interesting pattern to the average bond volatility each year. From 2003 to 2007, the average bond volatility decreases each year, despite the fact that FINRA introduced coverage of additional issues, which were believed to be less liquid. During the Financial Crisis in 2008 and 2009, empirical bond volatility spikes, before returning to levels closer to those observed pre-crisis in 2010. There are two sources to this pattern. First, we show in Appendix A that Treasury volatility decreased during the early part of our sample. Second, volatility in markets, including the equity market, increased during the Financial Crisis. As corporate bonds and equities are both sensitive to underlying firm conditions, we would typically expect corporate bond volatilities to be high when equity volatilities are high.

To better understand the empirically estimated bond volatilities, we sort bonds into quartiles *each year* by bond- or firm-level characteristics and report the average contemporaneous empirical bond volatility in Panel A of Table 4. We find that less liquid bonds (lower amount outstanding, greater proportion of zero trading days, higher Amihud measure, and higher Implied Round-trip Cost), poorer rated bonds, and longer maturity bonds tend to have higher empirical volatilities. Firm characteristics are also important as firms with higher equity volatility, K/V, and payout ratios also tend to have higher volatilities. These results are generally robust to both the first and second half of our sample, though the spread in empirical bond volatility across quartiles tends to be larger in the second half of the sample.

We report estimates of empirical CDS volatility in Panels B (daily returns used to calculate volatility each month) and C (monthly returns used to calculate volatility each year) of Table 3. We find that the average empirical volatilities are 4.87% and 5.56%, respectively. Both estimates are lower than in the corporate bond market, as CDS are much less sensitive to interest rates. Similar to corporate bonds, we find that CDS volatility spikes during around the Financial Crisis.

In the bottom half of Panel A in Table 4, we examine the relation between CDS volatility (calculated using monthly returns) and characteristics by performing similar year-by-year sorts as for corporate bonds. Many of our conclusions are similar to those for corporate bonds. Lower credit quality and more illiquid CDS have higher average empirical volatilities. The results hold for both the first and second half of our sample, though the spread is again wider during the second half.

4.2 Equity Return Volatility $\hat{\sigma}_E$

The equity return volatility, from which the asset volatility of a firm can be backed out, is one key input to the structural model. Equity volatility is calculated each year using monthly returns when matched to bond or CDS volatilities from monthly returns. When matched to the sample using CDS volatilities calculated each month using daily returns, we calculate equity volatilities each month using daily returns. In Table 3, we summarize equity volatility for the issuers of corporate bonds and reference entities for CDS in our sample. For the firms represented in our corporate bond sample, we find a similar pattern of equity volatility as we did for bond volatility in Section 4.1. Just prior to the crisis, equity volatilities were low and during the crisis, they spiked. The mean of equity volatility for the full corporate bond sample is 27.59%, as compared to 6.86% for corporate bond volatility. However, without implementing a structural model, it is difficult to determine if these relative magnitudes are reasonable.

For the firms in our CDS sample, we also see a similar pattern for equity volatility over time, as equity volatility is particularly high around the Financial Crisis. Generally, the equity volatility for firms in our CDS sample is slightly higher on average as compared to firms in our corporate bond sample at 34.55% and 31.46% when daily and monthly returns are used, respectively. Given that our CDS sample includes a broader set of firms, many of which are smaller, this seems reasonable.

4.3 Model-Implied Volatilities

For each firm in our sample, we back out its asset volatility, σ_v^{Merton} via equation (9). Details of the calculation are described in detail in Section 2.2, but the basic methodology is that for each bond i in year t, we use leverage K/V, payout ratio δ, firm T, and interest rate parameters in equation (9) and find the asset volatility, σ_v^{Merton}, such that the model equity volatility given in equation (9) matches empirically observed equity volatility for the corresponding firm in year t. We note that there are some cases where asset volatility cannot be backed out from equation (9). For highly levered firms in our sample, even a low asset volatility implies a high equity volatility. This is due to the fact that for highly levered firms, a low asset volatility implies a very low value of equity. With a very low value of equity, both $\partial \ln E/\partial \ln V$ and $\partial \ln E/\partial r$ are large. If the empirically observed equity volatility is low, there is no asset volatility that can satisfy equation (9). In about 18% of our initial bond-year sample and 5% of our CDS sample, this occurs.[18] An alternative methodology for implementation of the Merton model that we consider in section 5.1.2 mitigates this problem.

With asset volatility σ_v^{Merton} estimated, we can then calculate model-implied bond volatility, σ_D^{Merton} following the methodology described in Section 2.3. In the last column, of Table 3, we summarize our model-implied bond volatility estimates. For our corporate bond,

[18] Such observations are not included in our main sample and are not included in the summary statistics or volatilities reported in Tables 1 to 7.

CDS using daily returns, and CDS using monthly returns samples, the mean model-implied volatilities are 4.66%, 2.95%, and 2.72%, respectively. As equity volatility is one of our main inputs into the calculation of asset volatility and then model bond and CDS volatility, our model-implied bond and CDS volatilities exhibit similar patterns to equity volatility. They are lower during the early part of our sample, but show a pronounced increase during the Financial Crisis. However, we also note that the mean model-implied bond and CDS volatilities are smaller than the empirical bond and CDS volatilities also reported in Table 3.

We further examine the characteristics of our model-implied volatilities in Panel B of Table 4. Sorting on different security- and firm-level characteristics each year as in Sections 4.1 and 4.2, we find that the model-implied volatilities appear to be related to both variables that proxy for risk and also liquidity variables. While the former is predicted by the model, the latter result is suggestive of a correlation between liquidity variables and fundamental firm characteristics. Longer maturity bonds, bonds issued by firms with poorer ratings, and bonds issued by firms with higher equity volatility have higher model-implied bond volatility. However, we note that the relation between model volatility and rating and equity volatility is largely driven by the second half of our sample. The explanation for this lies in the fact that model-implied bond volatility is not monotonic in asset volatility and credit risk. As noted in Section 2.3, a riskier bond has a higher sensitivity to asset value, but a lower sensitivity to interest rates than a very safe bond. At low levels of riskiness, the increase in model-implied volatility from the increase in sensitivity to asset value is more than off-set by the decrease in model-implied volatility from the decrease in interest rate sensitivity. At high levels of riskiness, which are more common in the second half of the period, the higher sensitivity to asset value dominates and model-implied volatilities are particularly high for the fourth quartile of rating and equity volatility. By contrast, the model-implied CDS volatility is higher for firms with poorer credit ratings, higher CDS spreads, and greater equity volatility for both halves of our sample. This is due to the fact that CDS have little sensitivity to interest rates. Thus, for a CDS, the increase in model-implied volatility from an increase in sensitivity to asset value dominates the decrease in model-implied volatility form a decrease in sensitivity to interest rates even at low levels of credit risk.

4.4 Empirical vs. Model Return Volatilities

In Tables 5 and 6, we report the differences between empirically estimated and model implied volatilities for corporate bonds and CDS. For corporate bonds, the excess volatility is 2.19% on average, with a t-stat of 2.74.[19] The median excess volatility is 0.58% and the 25th percentile is 0.18%. As the distribution of excess volatility is positively skewed, we also winsorize excess volatility to decrease the effects of extreme observations. When we winsorize 1% of each tail, we find a mean excess volatility of 2.02% with a t-stat of 2.92. At 2.5% winsorization, we find a mean of 1.95% and a t-stat of 3.04. Thus, while winsorization decreases the mean excess volatility since the data is positively skewed, it also decreases the standard errors, making the results more statistically significant. In Table 5, we also find that excess volatility is more severe for bonds with poorer ratings and also longer maturity bonds. However, whether this shows that the model fails to capture fundamentals is unclear as longer maturity bonds and bonds with poorer ratings also tend to be less liquid.

We also consider callable bonds in Table 5. For all of the other analysis, we have omitted callable bonds because the Merton model does not deal with callability. However, as most bonds issued by non-financials are callable, we report results for callable bonds here in an effort to provide some guidance as to whether our results generalize to the broader bond market. Callable bonds have an average excess volatility of 2.71% and a t-stat of 2.29. Thus, we conclude that our results are similar for callable bonds.

Excess CDS volatilities are reported in Table 6 and our conclusions are similar. When daily CDS returns are used to calculate volatilities each month, the mean excess volatility is 1.92% (t = 7.23). When monthly CDS returns are used, the mean excess volatility is 2.84% (t = 3.77). The distribution of excess volatility is positively skewed, as with corporate

[19]Standard errors are clusterd by firm and time as discussed by Cameron, Gelbach, and Miller (2011). In addition, bootstrapped standard errors are discussed in Appendix D.

bonds and thus, we also calculate the mean excess volatility with 1% and 2.5% of each tail winsorized. For daily returns, we find excess volatility of 1.25% (t = 8.50) and 1.02% (t = 7.19) for the two levels of winsorization. For monthly returns, we find excess volatility of 2.54% (t = 4.22) and 2.02% (t = 6.29) for the two levels of winsorization. Thus, while excess volatility for CDS is positively skewed, it does not appear to be driven solely by the tails.

Finally, we consider an overlapping sample for corporate bonds and CDS. For most of our analysis, we have maintained both a corporate bond sample and CDS sample in an effort to maintain as comprehensive a sample as possible. In Table 7, we restrict the corporate bond and CDS (using monthly returns) samples to firm-years for which we have both a CDS and at least one bond in order to facilitate comparison. We find that for this overlapping sample, the mean excess volatility for corporate bonds is 2.72% and the mean excess volatility for CDS is 2.52%. Overall, it appears that the volatility in the credit market is higher than can be explained by equity markets and the Merton model. The source of this difference is examined in the following sections.

5 Further Examination

In this section, we aim to better understand why empirically observed volatilities in the credit market are higher than volatilities implied by the Merton model and equity markets. First, we consider different implementations of the model. Next, we examine the co-movement of empirical and model volatilities and whether excess volatility is related to firm-level accounting ratios, liquidity variables, or macroeconomic variables.

5.1 Modeling Asset Volatility

One limitation of our modeling approach is the tension between the short-run observed volatility and the long-run volatility relevant for pricing securities with long-dated maturities.[20] This issue can be cast in two ways that are conceptually different, but numerically similar. First, suppose that the true model does, in fact, have a constant asset volatility σ_v. However, the realized volatility each period may be different than this constant, long-run asset volatility. Thus, there is a distinction between the true, long-run asset volatility, σ_v, which is relevant for the sensitivity coefficients in equations (7) and (11) and the asset volatility realized during a specific period, which is closely tied to realized equity and bond volatilities.

Second, asset volatility may be time-varying. Thus, the realized volatility during a period of time only reflects the conditional volatility. The typical method for modeling this is to model asset variance as a mean-reverting process. Intuitively, the current asset variance should be related to the current equity variance, but the long-run mean of asset variance is particularly relevant for pricing and the sensitivity of both equity and debt to firm value in equations (7) and (11) as long as mean-reversion is fast. Numerically, the long-run mean variance in the stochastic volatility model will be similar to the long-run variance, σ_v^2, in the first model, which distinguishes between short-run realized and long-run asset volatility.

The main calibrations in our paper lean heavily on period-by-period volatility estimates. In particular, for much of our analysis, we estimate equity volatility each year by using monthly returns during the year. This equity volatility is then used in equation (7) to estimate a single asset volatility, σ_v. This has the implicit implication that if the current volatility is low, it will stay low for the entire life of the firm and if it is high, it will stay high. Our data includes both a lower volatility period (pre-crisis) and a higher volatility period (the Financial Crisis), so we have periods where the estimated volatility might be lower than the long-run volatility and also periods where the estimated volatility may be higher. However, we further examine the robustness of our results in this section. Here, we consider three different calibration strategies to address these issues. First, for our analysis on corporate bonds, we consider a "hybrid" approach where a long-run (unconditional) asset volatility is

[20]Another potential modeling complication is that firms may have mean-reverting leverage ratios. We consider the sensitivity of model volatility to mean-reverting leverage ratios in Appendix G.

inferred from equity volatility calculated from a longer sample of returns. Second, using CDS volatilities calculated each month (using daily returns), we estimate both short-run realized and a long-run asset volatility for each firm under the assumption that the true long-run volatility is constant. Finally, we estimate a stochastic volatility model.

5.1.1 Hybrid Approach

The starting point of the hybrid approach is equation (7), which we reproduce here:

$$\sigma_E^2 = \left(\frac{\partial \ln E_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln E_t}{\partial r_t}\right)^2 \sigma_r^2$$

First, we obtain unconditional estimates of equity and Treasury bond volatilities using monthly equity and Treasury bond returns going as far back in history as possible.[21] We can then use these unconditional equity volatilities in (7) along with other firm-level parameters to obtain unconditional asset volatilities for each firm. The mean and median estimates of the unconditional volatility in our sample are 20.14% and 19.35%, respectively. This is higher than the estimates in our base estimation, which are 16.68% and 13.50%, respectively. As discussed in Section 2.3, a higher asset volatility does not necessarily imply a higher model bond volatility as this simultaneously decreases the sensitivity to interest rates while increasing the sensitivity to asset value. With the unconditional asset volatility, we can then use equation (7) to estimate conditional asset volatilites, plugging in the unconditional asset volatility into the sensitivities ($\partial \ln E/\partial \ln V$ and $\partial \ln E/\partial r$) and the other appropriate firm-level parameters into equation (7). Specifically, we use the equation

$$\sigma_{E,t}^2 = \left(\frac{\partial \ln E}{\partial \ln V}\right) \sigma_{v,t}^2 + \left(\frac{\partial \ln E}{\partial r}\right) \sigma_{r,t}^2$$

where we use t subscripts to emphasize that we are relating the conditional equity volatility to conditional asset and interest rate volatilities. The sensitivities, however, are function of

[21] As robustness, we also consider estimating equity volatility using a GARCH(1,1) model and find similar results.

the unconditional asset and interest rate volatilities.

Similarly, model-implied bond volatilities can be calculated by applying the unconditional asset volatility to the sensitivities ($\partial \ln B/\partial \ln V$ and $\partial \ln B/\partial r$), but the conditional volatilities otherwise in equation (11). For this estimation, methodology, we find results similar to our previous results. The mean difference between empirical and model bond volatilities is 2.36 percentage points with a t-stat of 2.96.

5.1.2 Realized vs. Long-run Volatility

While the previous section uses a long-run unconditional equity volatility to calculate a long-run asset volatility, a more precise estimate would be to take period-by-period realized equity volatility to infer period-by-period realized asset volatility and the long-run asset volatility. In a constant asset volatility model, equity volatility varies with the leverage of a firm. Over a short horizon, leverage is typically stable enough to treat equity volatility as constant. As we will need to estimate equity volatility at a short horizon, we focus on volatilities calculated each month using daily returns. Thus, our focus will be on the CDS sample where daily returns are used. Since our focus is on CDS, which have little sensitivity to interest rates, we turn off stochastic interest rates in the model so that the interpretation of our results is more straightforward. The firm value process is:

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sigma_v dZ_t^Q \tag{12}$$

and the relation between realized equity volatility and asset volatility is then given by:

$$\sigma_{E,t}^2 = \left(\frac{\partial \ln E_t}{\partial \ln V_t}\right)^2 \sigma_{v,t}^2 \tag{13}$$

where $\sigma_{E,t}$ and $\sigma_{v,t}$ are realized volatilities and $\partial \ln E/\partial \ln V$ relies on the long-run asset volatility, σ_v.

Since log returns follow a normal distribution with variance σ_v^2, $\frac{(n-1)\sigma_{v,t}^2}{\sigma_v^2} \sim \chi_{n-1}^2$ and the expectation of the realized variance $\sigma_{v,t}^2$ is equal to the true long-run variance σ_v^2. Thus, we use:

$$\sigma_v^2 = \frac{1}{T}\sum \sigma_{v,t}^2 = \frac{1}{T}\sum \frac{\sigma_{E,t}^2}{\left(\frac{\partial \ln E_t}{\partial \ln V_t}\right)^2} \tag{14}$$

Using equation (14), we determine the long-run asset volatility, σ_v, and using equation (13), we infer the realized asset volatility for each period, $\sigma_{v,t}$. From these asset volatility estimates, we can then estimate model bond volatility using:

$$\sigma_{D,t}^2 = \left(\frac{\partial \ln B_t}{\partial \ln V_t}\right)^2 \sigma_{v,t}^2 \tag{15}$$

A benefit of implementing the Merton model in this way is that it avoids the missing asset volatility problem described in Section 4.3.[22] The intuition is the following. Suppose we start with equation (13), but do not make a distinction between the realized asset volatility, $\sigma_{v,t}$, and the long-run asset volatility, σ_v (which is in the sensitivity). If the realized equity volatility, $\sigma_{E,t}$ is low, we might expect that a low asset volatility, σ_v, would be consistent with equation (13). However, for high K/V and payout ratio firms, a low σ_v implies a large $\partial \ln E/\partial \ln V$ because of the low value of equity. The outcome is a large model equity volatility. Larger values of σ_v imply lower values of $\partial \ln E/\partial \ln V$, but the hedge ratio then scales a larger σ_v in (13). Thus, often times, there is no σ_v that can generate a low enough model equity volatility.

Here, we use a distinction between the realized volatility $\sigma_{v,t}$ and the long-run volatility, which is relevant for $\partial \ln E/\partial \ln V$. Equation (14) allows for low and high realized volatility periods to be averaged out over time when determining the long-run asset volatility. Thus, a few low realizations of equity volatility will not imply that the long-run asset volatility cannot be calculated. With the long-run asset volatility fixed in the sensitivity coefficient

[22]The disadvantage is that we require a reasonable number of estimates for equity volatility to calculate long-run asset volatility in equation (14).

$\partial \ln E / \partial \ln V$ in (13), a low realized equity volatility implies a low realized asset volatility, $\sigma_{v,t}$.

To implement this model, we first take firms with CDS data and limit our sample to firms that have at least 10 years of firm data.[23] We use firm-level data from 1991 to 2010 whenever possible. We then follow the methodology described above to calculate model CDS volatilities. In addition, we also calculate model CDS volatilities using our base case methodology as described in Section 2 and Appendix C, but with stochastic interest rates turned off. This facilitates a comparison between the methodology described above and our base case. Panel B of Table 8 provides the results to the model described in this section while Panel A describes the base case. The excess volatility for CDS in both models is positive and statistically significant, but the excess volatility in the base case (1.85%) is higher than in the implementation described in this section (1.35%). Upon further examination, this is due to the fact that average equity variance over the last 20 years is higher than the average equity variance in our sample period. This leads to estimates of the long-run asset volatility, σ_v, that are higher than the asset volatility in the base case. In turn, this leads to greater values of bond sensitivity to firm value ($\partial \ln B / \partial \ln V$) and larger model CDS volatility.[24]

5.1.3 Stochastic Volatility

In this section, we consider the use of a stochastic volatility model. We model the firm value and asset variance processes as:

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sqrt{H_t}\left(\rho dZ_t^{(1)Q} + \sqrt{1-\rho^2}dZ_t^{(2)Q}\right) \tag{16}$$

$$dH_t = \kappa_H(\theta_H - H_t)dt + \sigma_H\sqrt{H_t}dZ_t^{(1)Q} \tag{17}$$

where H_t is the instantaneous asset variance.

[23] We do include those firm-years that were omitted in Section 4.3 in order to illustrate that there is still excess volatility when those firm-years are included.

[24] With interest rate sensitivity turned off, there is no counteracting decrease in volatility due to a decrease in interest rate sensitivity. However, even if we had maintained stochastic interest rates in this section, the effect would be minimal due to the low interest rate sensitivity of CDS.

Full details of the calibration methodology are discussed in Appendix F and here, we limit our discussion to the empirical results. The data used to implement this model are the same as in Section 5.1.2. In Table 8, Panel C, we report the results of the stochastic volatility model. Across the full sample, the mean difference between empirically estimated CDS volatility and stochastic volatility model-implied CDS volatility is 1.28%, with a t-stat of 5.13. Thus, for the overall sample, there remains evidence of excess volatility. However, the result is insignificant for both 2009 and 2010. The results for the stochastic volatility model are extremely similar to the results in Section 5.1.2. The reason is that θ_H, the long run mean variance in the stochastic volatility model, plays a similar role to σ_v^2, the long-run variance in the model in 5.1.2. The two are very highly correlated. The current asset variance, H_t plays a similar role to $\sigma_{v,t}^2$, the realized asset variance in the previous section and the two are very highly correlated. Thus, the two models generate very similar model-implied CDS volatilities and excess volatility estimates.

5.2 Volatility in the Time-Series

5.2.1 Empirical vs. Model Volatilities

To examine the performance of the Merton model in explaining the time-series of volatility, we start by focusing on volatilities calculated from daily returns each month in the CDS market. Using these monthly estimates of volatility allows us to assess the performance of the model during the changing market conditions in our sample. For model CDS volatilities, we consider both the base case calibration as described in Section 4.3 and the short-run realized volatility as described in Section 5.1.2. We take the mean of empirical volatility, model volatility, and the difference between empirical and model volatility each month. We then plot these series for both calibrations considered in this section in Figure 1. While Figure 1 shows that the typical levels of model volatility are lower than empirical volatility, the plots do provide some encouraging evidence for the Merton model. The correlation between the mean empirical volatility and the mean model volatility is 0.9547 and 0.9533 for the base case and short-run realized volatility calibrations, respectively. In addition, the model performs reasonably well around the Financial Crisis, particularly for the short-run realized volatility calibration case. It may seem surprising that the empirical volatility is not much higher than model volatility during the Financial Crisis, as this was a particularly illiquid period for credit markets. There are two explanations for this. First, during the Financial Crisis, the variance of fundamentals also increased. Second, price pressures during normal times are more likely to be transitory and contribute to empirical volatility estimates whereas price pressures during the Financial Crisis are more likely to be persistent due to persistent strong selling pressures as in Feldhutter (2012).[25]

Though the plots in Figure 1 already suggest that average empirical CDS volatilities move with average Merton model CDS volatilities, we further examine this relation by running panel regressions of empirical volatilities on model volatilities with bond or CDS fixed-effects. By including fixed-effects, we acknowledge that the level of empirical volatilities for most bonds (and CDS) are higher than those implied by the Merton model. This higher level may be due to a variety of issues including liquidity and is absorbed into the fixed-effects, allowing us to gauge whether empirical and model volatilities line-up over time when the level difference is accounted for.

The results in Table 9 are promising for the Merton model. When regressing empirical bond volatility on model bond volatility and bond fixed-effects, we find a coefficient of 1.14. Though a t-test of whether this coefficient equals 1 yield a t-statistic of 2.60 when standard errors are clustered by time, accounting for the small number of clusters by using a wild cluster bootstrap-t as in Cameron, Gelbach, and Miller (2008) suggests that the cutoff for a 5% rejection is 2.64. Running a similar regression using annual volatilities calculated from monthly CDS returns, we find a coefficient of 1.38 and a t-statistic of 1.52 when testing against a null of 1. When we use volatilities calculated from daily CDS returns and regress empirical volatilities on model volatilities with CDS fixed-effects, we find coefficients close to

[25] In credit markets, bid-ask spreads were particularly large during the Financial Crisis and using transaction-level prices would lead to large volatility estimates. However, we avoid this issue by using consensus mid prices.

0.8 regardless of whether the base methodology or the methodology in Section 5.1.2 is used. The within-group R^2 of our four specifications ranges from 33.67% to 46.28%, suggesting that a substantial proportion of the variation in empirical volatilities is explained by the model.

Overall, we find that empirical bond and CDS volatilities tend to be higher than average exactly when model-implied volatilities are higher than average. To the extent that equity returns reflect fundamentals, this suggests that a Merton model transformation of fundamental volatility does well in explaining the empirical variation of volatility in credit markets.

5.2.2 Excess Volatility in the Time-Series

In the prior section, we find that empirical and model CDS volatilities move well together. Bond and CDS fixed-effects were used to take into account the difference in levels between empirical and model volatilities. Here, we consider what might explain the variation of this difference in levels across time. We use the excess volatility of CDS from daily returns as our left-hand side variable. As explanatory variables, we include the changes to a number of macroeconomic variables and also market returns. We also include CDS- and firm-level variables to proxy for market conditions of the firm. Our regressions also include CDS fixed-effects as we aim to determine the types of conditions over time that are associated with higher than usual excess volatility.

Macroeconomic Conditions: In incorporating macroeconomic variables, our goal is to determine whether the CDS in our sample have higher or lower than average excess volatility as market conditions change. For example, the VIX index is known as the “fear gauge” of the market. We aim to determine whether excess volatility is higher or lower in months when the fear gauge of the market increases. In line with our use of VIX, we also consider the University of Michigan Consumer Sentiment Index and the National Association of Purchasing Management’s Business Conditions Index, both of which are based on surveys. Both are used here as proxies to help determine whether excess volatility is higher or lower in periods where investor confidence decreases. We also include the 3-month Repo rate, the 3-month LIBOR rate, and the term spread as existing evidence has shown that interest rates change with macroeconomic conditions. The S&P 500 return and Barclay's US Investment Grade Corporate Bond Index returns are also included as measures of how the aggregate markets performed over the course of the month.

CDS- and Firm-level Variables: We consider the contemporaneous stock return of the underlying firm and the change in CDS spread during the month to proxy for changing firm-level financial conditions. We also control for changing liquidity conditions by using the change in CDS bid-ask spread and for the variation of liquidity conditions with the volatility of bid-ask spread.

The results in Table 10 suggest that there is little evidence that excess volatility is related to changing macroeconomic conditions. Across the set of variables that control for changes in aggregate fear/investor sentiment, changes in interest rates, and aggregate market returns, only the bond market return is significant at a 10% level and this is only true in one specification. The evidence related to changing firm-level financial conditions is inconclusive as stock returns and changes in CDS spreads are only significant in some specifications. However, we find that the volatility of bid-ask spread is related to contemporaneous excess volatility. This suggests that in periods with large variation in CDS liquidity, there is also large volatility in the price of CDS.

Overall, the prognosis for the Merton model in the time-series is good. Though the model cannot perfectly match the levels of empirically observed volatilities, it strongly co-moves with empirically observed volatilities. In regressions, we find little evidence that excess volatility for CDS is related to changing macroeconomic conditions or changing firm fundamentals. The absence of clear correlations between excess volatility and changing macroeconomic conditions and firm-level fundamentals is a good sign for the Merton model as it should impound this information from equity volatility into model-implied CDS volatility.

5.3 Volatility in the Cross-Section

To provide guidance as to what types of firms and bonds have excess volatility, we examine the relation between excess volatility and firm- and security-level variables in panel regressions with time-fixed effects. Thus, the cross-sectional average of excess volatility and each explanatory variable is taken out. We include accounting ratios, security characteristics, and liquidity variables in order to provide some suggestive evidence as to what types of characteristics may be driving excess volatility and to also provide some guidance as to what important dynamics the Merton model might be missing.

Accounting Variables: We choose a number of accounting variables to proxy for firm conditions. EBIT/Assets, Sales/Assets, and Retained Earnings/Assets are motivated by their inclusion in the Altman (1968) Z-score to predict bankruptcy. Net Income/Assets and ln(Total Assets) are motivated by the logit default prediction model in Campbell, Hilscher, and Szilagyi (2007). Coverage Ratio is also included as it reflects the ability of a firm to cover interest expenses from earnings.

Other Non-Liquidity Characteristics: For our corporate bond sample, we also include the Moody's rating as an additional control for firm conditions and the time to maturity as it was shown in Table 5 that excess volatility is most severe in magnitude for bonds with a longer time to maturity. For CDS, we include the CDS spread as an additional control for firm conditions, but cannot control for maturity as our sample is limited to 5-year CDS.

Liquidity Controls: At the bond-level, we include a number of controls for liquidity. Houweling, Mentink, and Vorst (2005) cite a number of studies that have used age and amount outstanding of a bond as proxies for bond liquidity. Thus, we include these. We use the bid-ask spread from Bloomberg as a control for liquidity, following Chen, Lesmond, and Wei (2007) and also the standard deviation of the bid-ask spread. As the magnitude of these bid-ask spreads have been shown to be small compared to effective bid-ask spreads, we also include a number of other proxies for bond liquidity in the literature. Specifically, we include five proxies that are used in Dick-Nielsen, Feldhutter, and Lando (2012), Bond

Zeros, the Amihud measure, the Implied Round-trip Cost (IRC), the volatility of the Amihud measure, and the volatility of IRC.[26] The latter four form the core liquidity measure used in Dick-Nielsen, Feldhutter, and Lando (2012). Our calculations of these variables follow the description given in the Appendix of Dick-Nielsen, Feldhutter, and Lando (2012).

We report the results for corporate bonds in Table 11. Our regressions are split into four groups, controls for accounting variables, controls for bond-level characteristics, controls for a subset of the liquidity variables, and controls for the four most important liquidity variables from Dick-Nielsen, Feldhutter, and Lando (2012). We then compare the importance of accounting variables, bond-level characteristics, and liquidity variables. When firm-level accounting ratios are the only explanatory variables, a number of the explanatory variables are significant, though their signs give conflicting interpretations. A high coverage ratio is associated with greater excess volatility, suggesting that it is safer firms that have excess volatility. However, the coefficient on EBIT/Assets, Retained Earnings/Assets, and ln(Total Assets) are all significantly negative. To the extent that profitable and larger firms are safer, this suggests that safer firms have lower excess volatility.

When only bond-level characteristics are used, the coefficients on ratings and time to maturity are positive and the coefficient on ln(Amount Outstanding) is negative. The results for ratings and time to maturity are consistent with those in Table 5 which suggested that excess volatility is most severe for longer time to maturity, poorly rated bonds. Liquidity variables are also important, as we find age, the bid-ask spread, and the standard deviations of bid-ask spread to all be positive and significant, consistent with a positive correlation between illiquidity and excess volatility. Of the four liquidity measures used in the core Dick-Nielsen, Feldhutter, and Lando (2012) liquidity measure, IRC and the volatility of the Amihud measure are positive and significant, also consistent with a positive correlation between excess volatility and illiquidity. Both the Amihud measure and the volatility of IRC are insignificant.

[26] We do not include the Roll (1984) measure as precise estimates of the Roll measure require higher frequency data.

When accounting variables, bond-level variables, and liquidity controls are included simultaneously, our main conclusions about bond-level variables and liquidity are unchanged. Excess volatility is most severe for longer time to maturity bonds and bonds with poorer ratings. A change in the rating of three notches is associated with an additional excess volatility of slightly more than one percentage point while a longer time to maturity of five years is associated with an additional excess volatility of approximately 0.6 percentage points. Illiquidity still plays a role as more illiquid bonds have higher excess volatility. An Implied Round-trip Cost (IRC) that is greater by 0.28 (one standard deviation) is associated with an excess volatility that is approximately 0.8 percentage points higher. However, most of the accounting variables are insignificant, with the exception of Coverage Ratio, which surprisingly is positively related to excess volatility. These results provide two conclusions for the use of the Merton model. First, if there are dimensions where the Merton model might have shortcomings in explaining bond return dynamics, it is in the realm of higher credit risk firms and longer maturity bonds. Second, even with modifications made in these dimensions, we should expect that the bond market will still exhibit some excess volatility on average due to the inherent illiquidity of the corporate bond market. As noted by Duffie (2010) and Feldhutter (2012), price pressures in OTC markets temporarily can drive prices away from fundamentals. To the extent that this is more severe for more illiquid securities, we would expect to see more severe excess volatility for particularly illiquid bonds.

In Table 12, we examine the relation between excess volatility in the CDS market and accounting variables, the CDS spread, and CDS liquidity variables. We consider CDS volatilities calculated from daily returns in both our base case and the case where we explicitly model realized short-run and constant long-run volatilities (Section 5.1.2) in addition to CDS volatilities calculated from monthly returns. Similar to our corporate bond sample, we find most of accounting ratios to be insignificant. The two exceptions are EBIT/Assets, which are positive only if the base case with volatilities from daily returns are used and the log of total assets. Both results are surprising as this suggests that if anything, larger firms and firms with higher earnings have greater excess volatility. CDS with greater standard deviation of bid-ask spread have greater excess volatility, consistent with part of the excess volatility being associated with illiquidity. The CDS spread is significantly positive in some, but not all specifications. Its importance is weaker when CDS liquidity variables are included, suggesting that perhaps the relation between credit quality and excess volatility may be related to the fact that securities with poorer credit ratings are also less liquid.

Overall, we find that there is little clear relation between firm-level accounting characteristics and excess volatility even though these accounting ratios have been found to be important for explaining the likelihood of default. Liquidity variables, however, are important in explaining which bonds and CDS have excess volatility. The Merton model is not designed to explain liquidity and thus, our results are complementary to both Huang and Huang (2003) who argue that the levels of prices may be different from those implied by models because of a non-default component and also Schaefer and Strebulaev (2008) who find that the Merton model does well in explaining the average relative returns of corporate bonds and equities. Our results are also consistent with Duffie (2010) and Feldhutter (2012) on prices deviating from fundamentals in OTC markets. If there are dimensions where the Merton model might be improved in explaining relative return volatilities in the cross-section, it would be in longer time to maturity bonds and also in lower credit rated bonds.

6 Conclusion

In the last 10 years, there has been considerable research in assessing the empirical performance of structural models of default. Huang and Huang (2003) provide a nice summary of the inability of a number of these models to match the empirical level of prices. Much of the empirical literature on corporate bonds has focused on a natural explanation for this failure of structural models to match the level of prices, the illiquidity of corporate bonds. Schaefer and Strebulaev (2008) find the intriguing result that structural models such as the Merton model are useful, not for the levels of prices, but for explaining the relative returns of corporate bonds and equities. Our paper starts from this important result and aims to further characterize the use of structural models in explaining the relative dynamics of corporate bond and equity returns. By using volatilities, we are able to include implications of liquidity in the relative returns. If the effect of liquidity in the bond market is simply a higher yield for corporate bonds than structural models can explain, and thus, a slightly higher average return each period, this should have little effect on volatility. If, however, bond prices can vary significantly due to changing liquidity premia, bond markets are likely to exhibit excess volatility.

Using transaction size-weighted prices to calculate monthly bond returns and consensus CDS prices in order to avoid volatility that can be generated by bid-ask bounce, we calculate the realized volatility of returns in the credit market and compare these volatilities to the credit market volatilities implied by a Merton model and realized equity market volatilities. Our sample is focused mostly on large companies, and thus, our conclusions are about volatilities in the credit market for the largest non-financial US firms. We find that there is excess volatility in credit markets; the empirically calculated volatilities for corporate bonds and CDS in our sample are larger than those implied by the Merton model and equity volatilities. This result is robust to different ways of calculating model-implied bond volatilities. Examining these excess volatilities and their relation to macroeconomic variables, firm-level accounting variables, security-level characteristics, and liquidity variables, we find that a large driver of the excess volatility is liquidity. While there is some evidence that credit quality is related to excess volatility, there is little evidence that macroeconomic conditions and firm conditions (as proxied by accounting variables) are related to excess volatility. Thus, we conclude that the Merton model is fairly successful, though not perfect, in capturing fundamental volatilities.

Tables and Figures

Table 1: **Bond Sample Summary Statistics**

	2003-2006			2007-2010			Full Period		
	mean	med	std	mean	med	std	mean	med	std
Panel A: Our Corporate Bond Sample									
Obs	1,454			1,429			2,883		
Bonds	742			645			1,021		
Maturity	6.50	3.34	7.63	7.27	4.12	9.10	6.88	3.71	8.40
Amt	562	300	736	608	306	829	585	300	784
Rating	6.79	7.00	3.66	7.11	6.00	4.18	6.95	7.00	3.93
Age	6.69	6.52	3.94	8.41	7.86	5.35	7.54	7.04	4.77
Trades	1,282	490	2,376	1,750	741	2,814	1,514	596	2,612
Volume	425	147	767	340	119	620	383	132	699
Turnover	59.07	47.47	46.97	46.77	38.66	36.23	52.97	42.22	42.43
Avg Trd Size	352	272	304	209	144	228	281	196	278
Bond Zero	63.88	71.43	27.88	65.60	75.79	28.66	64.73	73.41	28.28
Amihud	0.92	0.44	1.54	2.08	1.22	2.67	1.49	0.72	2.25
Amihud Vol	1.85	1.45	1.78	3.29	2.43	3.18	2.56	1.79	2.66
IRC	0.24	0.18	0.21	0.40	0.31	0.32	0.32	0.24	0.28
IRC Vol	0.31	0.25	0.26	0.45	0.36	0.36	0.38	0.30	0.32
Panel B: US Corporates in FISD									
Obs	82,402			95,948			178,350		
Bonds	35,586			37,523			53,828		
Maturity	7.62	4.71	8.88	7.82	4.62	8.95	7.73	4.67	8.91
Amt	169	40	338	197	20	498	184	27	432
Rating	7.29	6.00	4.16	7.27	6.00	4.48	7.28	6.00	4.33
Age	4.85	3.38	3.97	5.04	3.96	4.00	4.95	3.74	3.99

Summary statistics for the bonds in our sample (Panel A) and for all US non-Treasury bonds in FISD (Panel B). Observations are reported at the bond-year level. *Bonds* is the number of distinct bonds. *Maturity* is a bond's time to maturity in years. *Amt* is a bond's amount outstanding in $mm of face value. *Rating* is a numerical translation of Moody's rating, where 1=Aaa and 21=C. *Age* is the time since issuance in years. *Trades* is the number of trades in a year for a bond. *Volume* is a bond's trading volume in $mm face value for a year. *Turnover* is Volume/Amount Outstanding for a bond in a year in %. *Avg Trd Size* is the average trade size of a bond in $k of face value. *Bond Zero*, *Amihud*, *Amihud Vol*, *IRC*, and *IRC Vol* are defined and calculated as in Dick-Nielsen, Feldhutter, and Lando (2012). *Bond Zero* is expressed in %. *Amihud*, *Amihud Vol*, *IRC*, and *IRC Vol* are scaled by 100 as compared to Dick-Nielsen, Feldhutter, and Lando (2012).

Table 2: **Firm Summary Statistics**

Panel A: Firms in Our Corporate Bond Sample

	2003-2006			2007-2010			Full Period		
	mean	med	std	mean	med	std	mean	med	std
Firm-Years	376			359			735		
Equity Mktcap	41.48	21.47	56.91	38.63	18.92	55.72	40.09	19.97	56.31
EBIT/Assets	0.10	0.09	0.06	0.09	0.08	0.06	0.10	0.09	0.06
Coverage Ratio	10.21	6.23	12.14	8.73	5.53	9.49	9.49	5.92	10.94
Sales/Assets	0.91	0.76	0.58	0.88	0.75	0.51	0.90	0.76	0.55
RE/Assets	0.21	0.23	0.39	0.21	0.22	0.43	0.21	0.23	0.41
NI/Assets	0.05	0.05	0.06	0.04	0.05	0.07	0.05	0.05	0.06
Assets	45.00	25.13	82.10	49.73	25.12	93.31	47.31	25.12	87.74
Equity B/A	0.18	0.12	0.19	0.09	0.06	0.08	0.13	0.09	0.15

Panel B: Firms in Our CDS Sample

	2004-2006			2007-2009			Full Period		
	mean	med	std	mean	med	std	mean	med	std
Firm-Years	937			882			1,819		
Equity Mktcap	22.69	10.52	41.84	22.49	9.47	41.43	22.59	10.17	41.63
EBIT/Assets	0.10	0.09	0.06	0.09	0.09	0.07	0.10	0.09	0.07
Coverage Ratio	10.69	6.18	13.72	9.12	5.75	11.03	9.93	5.95	12.51
Sales/Assets	0.97	0.84	0.66	0.97	0.81	0.68	0.97	0.82	0.67
RE/Assets	0.21	0.20	0.31	0.21	0.22	0.32	0.21	0.21	0.32
NI/Assets	0.05	0.05	0.06	0.04	0.05	0.08	0.05	0.05	0.07
Assets	22.10	11.45	48.19	24.18	13.12	34.27	23.11	12.42	42.04
Equity B/A	0.25	0.12	1.32	0.13	0.09	0.27	0.19	0.10	0.97

Summary statistics for the firms with bonds (Panel A) or CDS (Panel B) in our sample are reported. *Equity Mktcap* is the equity market capitalization of a firm in \$bn. *EBIT/Assets* is defined using Compustat data as OIADP/AT. *Coverage Ratio* is defined as (OIADP + XINT)/XINT, following Blume, Lim, and MacKinlay (1998). *Sales/Assets* if defined as SALE/AT. *RE/Assets* is the ratio of retained earnings to assets and is defined as RE/AT. *NI/Assets* is the ratio of Net Income to Assets and is defined as NI/AT. *Assets* is total book assets in \$bn. *Equity B/A* is the bid-ask spread of equity in our sample from TAQ at the end of June and December of each year and is expressed as a percentage of stock price.

Table 3: **Volatility Estimates**

Panel A: Corporate Bond Sample

	$\hat{\sigma}_D$			$\hat{\sigma}_E$			σ_D^{Merton}		
	mean	med	sd	mean	med	sd	mean	med	sd
2003	7.20	6.71	4.58	26.95	27.59	9.13	5.60	5.17	2.80
2004	4.52	4.06	3.02	18.27	16.47	7.48	3.91	3.72	2.46
2005	4.60	3.48	4.07	20.17	18.95	10.21	3.19	2.87	2.11
2006	4.08	2.91	3.53	18.76	16.63	8.05	2.26	2.07	1.70
2007	4.00	3.12	3.09	18.92	15.49	8.28	3.47	3.66	2.28
2008	13.09	10.38	10.05	38.86	33.59	16.18	6.21	5.63	4.60
2009	15.98	9.95	18.23	54.19	56.01	27.68	10.67	8.75	7.18
2010	5.57	5.01	4.00	32.64	34.04	10.77	4.67	4.69	2.69
Full	6.86	4.55	8.84	27.59	23.04	18.09	4.66	3.75	4.30

Panel B: CDS Sample (Daily Returns Used)

	$\hat{\sigma}_D$			$\hat{\sigma}_E$			σ_D^{Merton}		
	mean	med	sd	mean	med	sd	mean	med	sd
2004	2.58	1.01	13.68	23.02	20.40	11.34	0.99	0.42	2.13
2005	3.01	1.39	7.45	24.53	21.57	13.86	1.04	0.35	2.51
2006	2.49	1.19	5.30	24.70	21.89	12.88	1.05	0.30	2.47
2007	3.29	1.35	6.02	27.45	24.10	15.00	1.52	0.44	3.33
2008	9.78	3.59	32.13	58.52	44.15	45.08	6.87	3.11	7.99
2009	8.63	3.04	24.65	49.80	39.54	39.27	6.32	2.64	7.82
2010	4.00	2.10	5.53	32.34	28.74	16.83	2.82	0.60	4.22
Full	4.87	1.74	17.18	34.55	26.50	29.10	2.95	0.52	5.53

Panel C: CDS Sample (Monthly Returns Used)

	$\hat{\sigma}_D$			$\hat{\sigma}_E$			σ_D^{Merton}		
	mean	med	sd	mean	med	sd	mean	med	sd
2004	2.97	0.97	8.95	22.81	19.86	11.50	0.98	0.44	2.02
2005	3.45	1.41	8.75	23.90	22.12	12.50	1.17	0.36	2.95
2006	2.23	1.07	2.93	22.77	20.71	11.16	0.84	0.24	1.95
2007	3.54	1.61	3.71	23.27	21.10	10.31	1.12	0.40	2.09
2008	9.68	5.06	16.72	48.62	42.26	27.85	7.00	3.71	8.16
2009	12.06	4.78	22.74	49.53	37.81	33.07	5.79	1.73	7.03
Full	5.56	2.09	13.18	31.46	24.98	23.14	2.72	0.48	5.30

The mean, median, and standard deviation of empirical bond and CDS volatilities ($\hat{\sigma}_D$), empirical equity volatilities ($\hat{\sigma}_E$), and model-implied bond and CDS volatilities (σ_D^{Merton}) are reported in %. Panel A reports annualized volatilities for the corporate bond sample where volatilities are calculated each year using monthly returns. Panel B reports annualized volatilities for the CDS sample where volatilities are calculated each month using daily returns. Panel C reports annualized volatilities for the CDS sample where volatilities are calculated each year using monthly returns. Volatilities using monthly CDS returns are not calculated in 2010 as Datastream ceased coverage of CDS prices from CMA Datavision in September 2010.

Table 4: **Volatility Estimates by Bond or Firm Characteristics**

Panel A: Empirical Volatility

Corporate Bonds	2003-2006				2007-2010				All			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Amt	5.06	4.76	4.47	3.93	10.70	10.17	8.35	7.30	7.97	7.34	6.30	5.68
Maturity	1.45	3.06	5.07	8.79	4.65	7.49	9.41	15.24	3.03	5.28	7.21	11.98
Rating	3.68	4.28	4.15	6.65	5.56	6.00	9.79	17.69	4.57	5.13	7.20	12.11
Equity Vol	3.84	4.15	4.59	5.84	7.17	7.00	6.84	16.67	5.42	5.60	5.77	10.99
Firm K/V	4.00	4.93	4.25	5.19	7.28	9.24	8.02	16.19	5.65	7.08	6.43	8.99
Firm Payout	4.52	3.91	4.64	5.38	7.99	4.75	9.54	14.03	6.44	4.26	7.02	9.81
Bond Zero	4.04	4.18	4.25	5.86	6.86	9.88	10.00	9.84	5.44	6.98	7.08	7.84
Amihud	3.38	4.10	4.74	5.97	7.02	7.02	8.48	14.15	5.17	5.54	6.58	9.99
SD(Amihud)	3.28	3.50	4.49	6.78	5.33	6.71	9.24	15.50	4.29	5.08	6.82	11.07
IRC	2.61	3.55	4.82	7.32	4.13	6.55	10.38	15.45	3.36	5.03	7.57	11.34
SD(IRC)	2.94	3.22	4.76	7.26	3.97	6.54	9.84	16.31	3.45	4.86	7.27	11.72
CDS	2004-2006				2007-2009				All			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Firm Rating	0.73	1.36	2.59	8.45	2.33	4.27	7.29	24.44	1.48	2.76	5.07	16.05
CDS Spread	0.52	0.90	2.23	7.91	1.73	2.94	6.06	23.00	1.11	1.89	4.09	15.25
Equity Vol	0.98	1.52	2.47	6.57	2.60	3.66	6.25	21.22	1.76	2.55	4.30	13.70
Firm K/V	1.20	1.54	2.41	6.39	2.70	4.44	7.44	19.13	1.93	2.94	4.85	12.59
CDS B/A Spread	0.72	1.20	2.11	7.55	2.06	3.66	5.98	22.12	1.38	2.36	3.99	14.64

Panel B: Model Volatility

Corporate Bonds	2003-2006				2007-2010				All			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Amt	3.20	3.27	3.28	3.12	6.23	6.35	6.14	5.78	4.77	4.74	4.63	4.50
Maturity	0.92	2.42	4.04	5.55	3.57	5.43	7.19	8.37	2.23	3.93	5.60	6.95
Rating	3.12	3.30	2.99	3.60	5.27	4.56	5.95	9.63	4.14	3.92	4.59	6.58
Equity Vol	3.18	3.13	3.18	3.41	4.67	4.08	6.26	9.92	3.89	3.61	4.79	6.51
Firm K/V	3.08	3.37	3.01	3.45	4.87	5.91	6.65	7.72	3.98	4.64	5.11	4.93
Firm Payout	3.25	3.11	3.24	3.31	5.93	4.09	6.44	7.74	4.73	3.52	4.80	5.58
Bond Zero	3.05	3.07	3.16	3.62	5.89	6.52	6.01	6.15	4.46	4.77	4.56	4.88
Amihud	2.51	3.07	3.58	3.76	4.54	5.59	6.53	8.08	3.50	4.31	5.03	5.88
SD(Amihud)	2.28	2.66	3.41	4.58	3.95	5.43	6.89	8.76	3.10	4.02	5.12	6.63
IRC	2.05	2.92	3.65	4.29	3.29	5.83	7.07	8.46	2.66	4.36	5.34	6.35
SD(IRC)	2.24	2.59	3.59	4.49	3.29	5.35	7.10	9.04	2.76	3.95	5.33	6.73
CDS	2004-2006				2007-2009				All			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Firm Rating	0.35	0.43	0.63	3.04	1.21	2.82	4.54	12.05	0.75	1.58	2.69	7.32
CDS Spread	0.33	0.36	0.47	2.85	0.94	1.72	4.11	11.48	0.63	1.02	2.23	7.05
Equity Vol	0.33	0.35	0.42	2.91	0.54	1.11	4.32	12.28	0.43	0.72	2.31	7.47
Firm K/V	0.35	0.43	0.73	2.49	0.96	2.09	4.51	10.69	0.64	1.23	2.56	6.48
CDS B/A Spread	0.35	0.39	0.58	2.69	1.17	2.42	3.98	10.75	0.75	1.35	2.23	6.61

All volatilities are annualized and expressed as percentages. Panel A reports empirical volatilities for corporate bonds and CDS. Panel B reports model volatilities for corporate bonds and CDS. Volatilities are calculated each year using monthly returns. The variable given in each row is the variable that is sorted on. Sorts are done each year and the average, contemporaneous volatilities are reported. Note that in the case of a tie in the sorting variable, a bond is put in the lower category. Thus, quartiles typically do not have exactly 25% of the observations. *Amt*, *Maturity*, *Rating*, *Bond Zero*, *Amihud*, *SD(Amihud)*, *IRC*, and *SD(IRC)* are as defined in Table 1. *Equity Vol* is the annualized equity volatility of the underlying firm calculated using monthly returns. *Firm K/V* is the ratio of the face value of debt to the total value of a firm. *Firm Payout* is the payout ratio of a firm. *Firm Rating* is the S&P long-term credit rating of a firm from Compustat where a lower number is a better rating. *CDS Spread* is the mid price for CDS from Datastream (bpm). *CDS B/A Spread* is the difference between the offer price (bpo) and bid price (bpb) for CDS.

Table 5: **Data Estimated vs. Model Implied Bond Volatility**

	$\hat{\sigma}_D - \sigma_D^{Merton}$, Weighted prices used					
	#obs	mean	t-stat	25th	Median	75th
Straight	2,883	2.19	2.74	-0.18	0.58	2.68
Callable	9,322	2.71	2.29	-0.39	0.84	3.44
By Year						
2003	94	1.60	1.59	-0.89	0.35	2.63
2004	362	0.61	4.69	-0.14	0.17	0.77
2005	525	1.41	6.41	-0.20	0.49	2.00
2006	473	1.82	10.23	0.23	0.84	2.62
2007	358	0.52	2.92	-0.54	0.07	0.95
2008	279	6.87	10.04	2.00	4.65	10.71
2009	347	5.32	1.90	-1.99	1.71	8.04
2010	445	0.90	2.79	-0.69	0.28	1.86
By Rating						
Aaa & Aa	723	0.45	0.81	-0.61	0.07	1.05
A	1,038	1.20	2.55	-0.19	0.32	1.80
Baa	621	2.71	2.66	0.01	0.97	3.51
Junk	451	6.41	2.24	1.17	2.89	7.05
By Time to Maturity						
0 - 2	882	0.80	3.40	-0.08	0.31	1.32
2 - 4	627	1.30	2.16	-0.57	0.07	1.65
4 - 6	311	1.27	1.39	-0.64	0.18	1.47
6 - 8	264	1.60	2.70	-0.37	0.55	2.46
> 8	799	5.00	3.08	0.81	2.73	6.29

The difference between empirically estimated bond volatility and model-implied bond volatility. Volatilities are expressed in % and are calculated each year using monthly returns. The main sample uses data from 2003 to 2010 and excludes putable, convertible, and callable bonds. *t-stats* are calculated using standard errors clustered by time and by firm, with the exception of the by-year results which use standard errors clustered by firm.

Table 6: **Data Estimated vs. Model Implied CDS Volatility**

$\hat{\sigma}_D - \sigma_D^{Merton}$

Panel A: Daily Returns

	#obs	mean	t-stat	25th	median	75th
2004	2,546	1.59	5.47	0.17	0.53	1.22
2005	3,954	1.97	6.69	0.37	0.87	2.11
2006	4,190	1.44	8.95	0.28	0.69	1.65
2007	4,129	1.77	6.83	0.18	0.67	1.97
2008	3,852	2.91	2.90	-3.25	0.48	2.23
2009	3,489	2.31	3.12	-2.26	0.57	1.69
2010	2,740	1.18	4.04	0.26	0.88	2.05
Full	24,900	1.92	7.23	0.12	0.68	1.86

Panel B: Monthly Returns

	#obs	mean	t-stat	25th	median	75th
2004	287	1.99	4.38	0.00	0.42	1.71
2005	319	2.28	5.81	0.31	0.99	2.72
2006	331	1.39	10.87	0.25	0.70	1.58
2007	302	2.42	14.15	0.51	1.09	3.70
2008	268	2.69	3.41	-0.96	1.61	2.98
2009	312	6.27	5.78	0.77	1.87	5.12
Full	1,819	2.84	3.77	0.26	1.02	2.82

The difference between empirically estimated CDS volatility and model-implied CDS volatility. Volatilities are expressed in annualized % and are calculated each month using daily returns in Panel A and each year using monthly returns in Panel B. The full sample uses data from 2004 to 2010 in Panel A and 2004 to 2009 in Panel B. *t-stats* are calculated using standard errors clustered by time and by firm, with the exception of the by-year results in Panel B which use White standard errors.

Table 7: **Data Estimated vs. Model Implied Volatility, Overlapping Sample**

	Obs	Mean	t-stat	25th	50th	75th
Corporate Bonds	1,954	2.72	2.41	-0.02	0.72	2.94
CDS	542	2.56	2.98	0.20	0.82	2.37

The difference between empirically estimated bond and CDS volatility and model-implied and CDS bond volatility. Volatilities are in %. Only firm-years that are in both the corporate bond and CDS sample are included. *t-stats* are calculated using standard errors clustered by time and by firm.

Table 8: **Data Estimated vs. Model Implied Volatility for CDS, Alternative Models**

Panel A: Base Case						
	#obs	mean	t-stat	25th	median	75th
2004	2,399	1.79	5.86	0.52	0.85	1.48
2005	3,572	2.15	7.56	0.66	1.17	2.33
2006	3,614	1.49	9.32	0.52	0.89	1.74
2007	3,476	1.83	7.85	0.47	0.97	2.10
2008	3,268	2.37	2.64	-2.34	-0.24	2.02
2009	3,060	1.77	3.28	-2.09	0.41	1.95
2010	2,267	1.40	4.96	0.53	1.27	2.19
Full	21,656	1.85	7.86	-0.28	0.74	1.96
Panel B: Conditional Volatility Case						
	#obs	mean	t-stat	25th	median	75th
2004	2,485	2.09	2.48	-0.12	0.50	0.96
2005	3,638	2.15	3.99	0.40	0.92	1.78
2006	3,637	1.06	7.05	0.35	0.67	1.23
2007	3,496	1.56	6.05	0.49	0.90	1.78
2008	3,317	1.73	2.76	-1.79	1.26	2.85
2009	3,176	0.01	0.02	-2.53	0.12	1.68
2010	2,331	0.73	1.72	-0.42	0.85	1.61
Full	22,080	1.35	5.34	-0.52	0.74	1.70
Panel C: Stochastic Volatility Case						
	#obs	mean	t-stat	25th	median	75th
2004	2,485	2.05	2.43	-0.16	0.45	0.93
2005	3,638	2.08	3.87	0.35	0.87	1.72
2006	3,637	1.00	6.61	0.30	0.62	1.18
2007	3,496	1.48	5.81	0.43	0.85	1.71
2008	3,317	1.53	2.33	-2.01	0.97	2.60
2009	3,176	0.06	0.15	-2.59	0.03	1.56
2010	2,331	0.65	1.58	-0.47	0.78	1.52
Full	22,080	1.28	5.13	-0.60	0.65	1.61

The difference between empirically estimated and model-implied CDS volatility. Volatilities are calculated using daily returns each month, annualized and in %. Panel A reports the difference between empirically estimated and model implied volatilities for the base case. Panel B uses a long run volatility estimate for hedge ratios and estimates a short-run model volatility to compare with the empirical volatility. Details are in Section 5.1.2. Panel C uses a stochastic volatility model to account for time-varying volatility. Details are in Section 5.1.3. *t-stats* are calculated using standard errors clustered by time and by firm.

Table 9: **Time-Series Relation between Empirical and Model Volatilities**

	(1)	(2)	(3)	(4)
	$\hat{\sigma}_D$	$\hat{\sigma}_{CDS}$	$\hat{\sigma}_{CDS}$	$\hat{\sigma}_{CDS}$
σ_D^{Merton}	1.14			
	(2.60)			
σ_{CDS}^{Merton}		1.38	0.78	0.84
		(1.52)	(-8.54)	(-5.77)
R-squared	0.443	0.463	0.337	0.447
Obs	2,883	1,819	24,900	22,080
Return Horizon	Monthly	Monthly	Daily	Daily
Calibration	Base	Base	Base	Short-run

Regressions of empirical volatilities on model volatilities with bond-level fixed effects. The first two columns use volatilities calculated each year from monthly returns of corporate bonds and CDS, respectively. The last two columns use volatilities calculated each month from daily returns on CDS. Column 3 uses the base methodlogy described in 4.3. Column 4 uses the methodology described in 5.1.2. Standard errors are clustered by time and t-stats are reported. Reported R^2 values are within-group R^2's.

Table 10: **Excess CDS Volatility and Macroeconomic Factors and Liquidity**

	(1)	(2)	(3)	(4)	(5)	(6)
	Base	Base	Base	Short-Run	Short-Run	Short-Run
Δ VIX	0.0386		0.0648	0.0180		0.0387
	(0.86)		(1.09)	(0.37)		(0.70)
Δ Consumer Sentiment	-0.0184		-0.0454	-0.0312		-0.0459
	(-0.69)		(-1.27)	(-1.05)		(-1.38)
Δ Business Cond Index	-0.0120		-0.0163	-0.00602		-0.00804
	(-1.40)		(-1.30)	(-0.63)		(-0.71)
Δ Repo Rate	1.091		1.413	-0.263		-0.127
	(1.30)		(1.17)	(-0.34)		(-0.13)
Δ LIBOR	-0.444		0.137	0.293		0.605
	(-0.57)		(0.11)	(0.40)		(0.65)
Δ Term Spread	0.305		0.626	0.0303		0.221
	(0.41)		(0.51)	(0.03)		(0.20)
Stock Market Return	0.0444		0.132	0.00396		0.0377
	(1.08)		(1.81)	(0.07)		(0.61)
Bond Market Return	0.140		0.176	0.0561		0.0850
	(1.69)		(1.38)	(0.59)		(0.73)
Stock Return		0.0372	0.0146		0.0244	0.0236
		(2.08)	(1.37)		(1.62)	(1.94)
Δ CDS Spread		0.00438	0.00711		0.00263	0.00333
		(1.39)	(2.20)		(0.82)	(1.12)
SD(CDS B/A)		0.664	0.686		0.323	0.332
		(22.16)	(18.63)		(7.84)	(8.93)
Δ CDS B/A		-0.0196	-0.0198		-0.00371	-0.00616
		(-1.12)	(-1.13)		(-0.22)	(-0.36)
Observations	24,900	24,681	24,681	22,080	21,895	21,895
R-squared	0.204	0.380	0.394	0.167	0.241	0.247

Reported are panel regressions with CDS fixed-effects. The dependent variable is the difference between empirical CDS volatility and model volatility, expressed in annualized %. Volatilities are calculated each month using daily returns. The first three columns use the base calibration described in Section 4. The last three columns use the calibration described in Section 5.1.2. *VIX* is the CBOE VIX index, expressed in %. *Consumer Sentiment* is the University of Michigan Consumer Sentiment Index. *Business Cond Index* is the National Association of Purchasing Management's Business Conditions Index (from Capital IQ). *Repo Rate* is the 3-month repo rate in %. *LIBOR* is the 3-month LIBOR rate in %. *Term Spread* is the difference between the yield of a 10-year Treasury and a 2-year Treasury expressed in %, where yields are from the Constant Maturity Treasury series. *Stock Market Return* is the return of the S&P 500 Index in %. *Bond Market Return* is the return to the Barclay's US Corporate Investment Grade Index in %. *Stock Return* is the return to the stock of the underlying issuer in %. *CDS Spread* is the mid price of CDS expressed in basis points. *CDS B/A* is the bid-ask spread of the CDS in basis points. *SD(CDS B/A)* is the volatility of the CDS bid-ask spread in a month. All variables are contemporaneous. *t-stats* use standard errors clustered by time.

Table 11: **Excess Bond Volatility and Firm- and Bond-Level Characteristics**

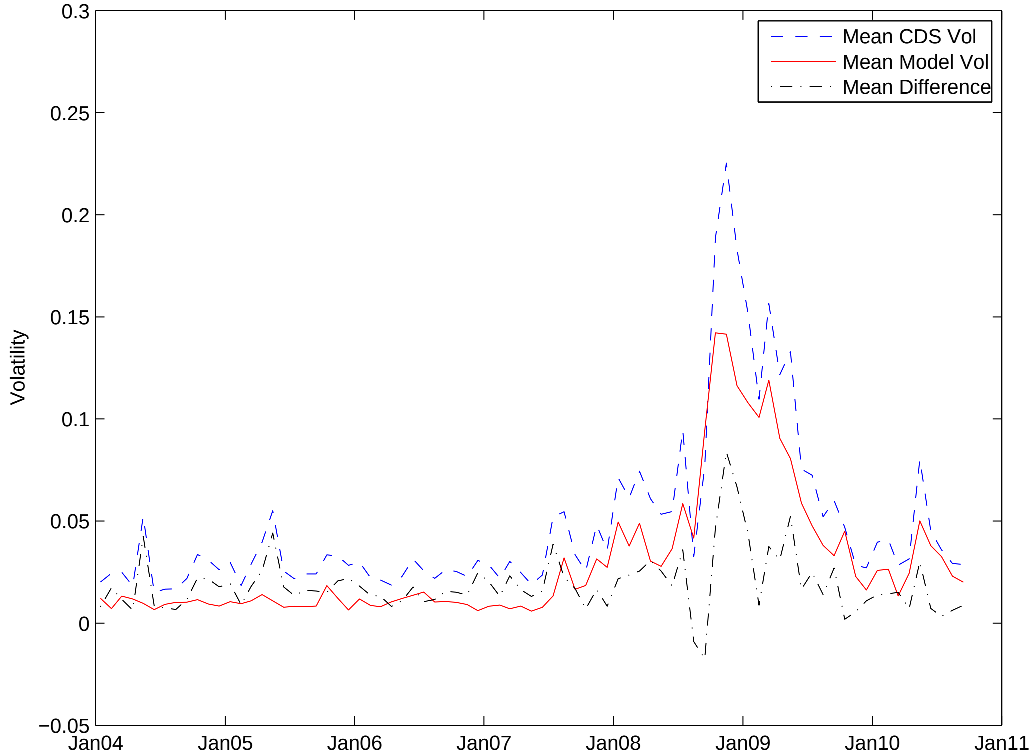
	(1)	(2)	(3)	(4)	(5)	(6)
EBIT/Assets		-14.34		-5.639		-1.963
		(-1.98)		(-1.14)		(-0.41)
Coverage Ratio		0.0318		0.0267		0.0213
		(1.99)		(2.10)		(1.65)
Sales/Assets		0.545		0.185		0.108
		(1.30)		(1.04)		(0.60)
Retained Earnings/Assets		-2.402		0.0363		-0.264
		(-2.59)		(0.07)		(-0.52)
Net Income/Assets		1.878		4.130		2.821
		(0.23)		(0.65)		(0.43)
log(Assets)		-0.570		0.503		0.244
		(-3.66)		(2.06)		(0.94)
Rating	0.363			0.357		0.397
	(4.45)			(3.96)		(3.50)
Maturity	0.172			0.115		0.119
	(8.45)			(4.86)		(5.44)
Age	0.0329		0.0759	0.00962		
	(1.19)		(2.24)	(0.40)		
log(Amt)	-0.246		0.130	-0.111		
	(-2.93)		(1.13)	(-1.37)		
B/A Spread			7.856	3.468		
			(6.66)	(3.41)		
SD(B/A Spread)			11.09	8.286		
			(2.48)	(2.02)		
Bond Zero			0.00908	0.00804		
			(0.94)	(1.69)		
Amihud					-0.300	-0.145
					(-1.76)	(-1.44)
IRC					5.324	2.816
					(4.10)	(2.41)
SD(Amihud)					0.305	0.209
					(3.60)	(2.57)
SD(IRC)					1.667	0.758
					(1.21)	(0.83)
Observations	2,833	2,868	2,600	2,537	2,656	2,594
R-squared	0.360	0.206	0.352	0.419	0.291	0.401

All regressions include time fixed-effects. The dependent variable is $\hat{\sigma}_D - \sigma_D^{Merton}$, where $\hat{\sigma}_D$ is the realized volatility of a corporate bond using monthly returns in a calendar year and σ_D^{Merton} is the volatility implied by the Merton model and realized equity volatility. Both are expressed in annualized %. *EBIT/Assets* is defined using Compustat data as OIADP/AT. *Coverage Ratio* is defined as (OIADP + XINT)/XINT. *Sales/Assets* is defined as SALE/AT. *Retained Earnings/Assets* is defined as RE/AT. *Net Income/Assets* is defined as NI/AT. *Assets* is total book assets in $mm. *Rating* is a bond's Moody rating where Aaa = 1 and C = 21. *Maturity* is a bond's time to maturity in years. *Age* is a bond's time since issuance in years. *Amt* is a bond's amount outstanding in $mm face value. *B/A Spread* is a bond's bid-ask spread divided by its mid price, scaled by 100. *SD(B/A Spread)* is the standard deviation of a bond's bid-ask spread divided by its mid price, scaled by 100. *Bond Zero* is the percentage of days that a bond does not have at least one trade of $100k, following Dick-Nielsen, Feldhutter, and Lando (2012). *Amihud* is the Amihud measure and *IRC* is an implied round-trip cost measure. *SD(Amihud)* and *SD(IRC)* are the standard deviations of these measures. *Amihud*, *IRC*, *SD(Amihud)*, and *SD(IRC)* are defined as in Dick-Nielsen, Feldhutter, and Lando (2012), but scaled by 100 here. *t-stats* are in parentheses and use standard errors clustered by firm.

Table 12: **Excess CDS Volatility and Firm- and Bond-Level Characteristics**

	(1) Base	(2) Base	(3) Base	(4) Short-run	(5) Short-run	(6) Short-run	(7) Base	(8) Base	(9) Base
CDS Spread	0.0117	0.000656	0.00155	0.00513	-0.00214	-0.000711	0.0227	0.0189	0.0207
	(12.91)	(0.78)	(1.73)	(4.69)	(-1.66)	(-0.54)	(14.01)	(5.64)	(6.03)
EBIT/Assets	6.045		4.831	0.853		0.890	5.360		3.404
	(3.26)		(3.17)	(0.36)		(0.40)	(1.40)		(0.98)
Coverage Ratio	-0.00332		-0.00395	-0.00592		-0.00549	-0.00184		-0.00270
	(-0.72)		(-1.03)	(-1.04)		(-1.01)	(-0.24)		(-0.38)
Sales/Assets	-0.147		-0.0687	-0.0193		0.0484	0.217		0.113
	(-1.18)		(-0.73)	(-0.15)		(0.47)	(1.17)		(0.69)
Retained Earnings/Assets	0.525		0.290	0.635		0.479	1.437		0.921
	(1.43)		(0.93)	(1.45)		(1.15)	(2.38)		(1.63)
Net Income/Assets	0.853		-0.425	7.347		5.308	0.236		3.583
	(0.45)		(-0.27)	(2.90)		(2.27)	(0.05)		(0.87)
log(Assets)	0.337		0.265	0.360		0.309	0.563		0.335
	(4.47)		(3.81)	(3.22)		(2.69)	(4.10)		(2.15)
CDS B/A		0.00850	0.0118		-0.0167	-0.0178		-0.187	-0.184
		(0.36)	(0.49)		(-0.56)	(-0.60)		(-2.73)	(-2.50)
SD(CDS B/A)		0.713	0.693		0.489	0.468		0.394	0.380
		(11.42)	(10.68)		(6.20)	(5.83)		(7.04)	(6.59)
Observations	24,294	24,900	24,294	21,667	22,080	21,667	1,794	1,819	1,794
R-squared	0.253	0.381	0.388	0.120	0.195	0.205	0.594	0.634	0.644

All regressions include time fixed-effects. The dependent variable is $\hat{\sigma}_D - \sigma_D^{Merton}$, where $\hat{\sigma}_D$ is the realized CDS volatility and σ_D^{Merton} is the volatility implied by the Merton model and equity volatility. Both are expressed in annualized %. Columns (1)-(3) use CDS volatilities calculated each month from daily returns as described in Section 2.3. Columns (4)-(6) use CDS volatilities calculated each month from daily returns as described in Section 5.1.2. Columns (7) to (9) use CDS volatilities calculated each year using monthly returns. *CDS Spread* is the CDS spread in basis points. *EBIT/Assets* is defined using Compustat data as OIADP/AT. *Coverage Ratio* is defined as (OIADP + XINT)/XINT. *Sales/Assets* is defined as SALE/AT. *Retained Earnings/Assets* is defined as RE/AT. *Net Income/Assets* is defined as NI/AT. Assets is the book value of assets in \$mm. *CDS B/A* is the bid-ask spread of CDS in basis points. *SD(CDS B/A)* is the standard deviation of the CDS bid-ask spread in basis points. *t-stats* are in parentheses and use standard errors clustered by firm.



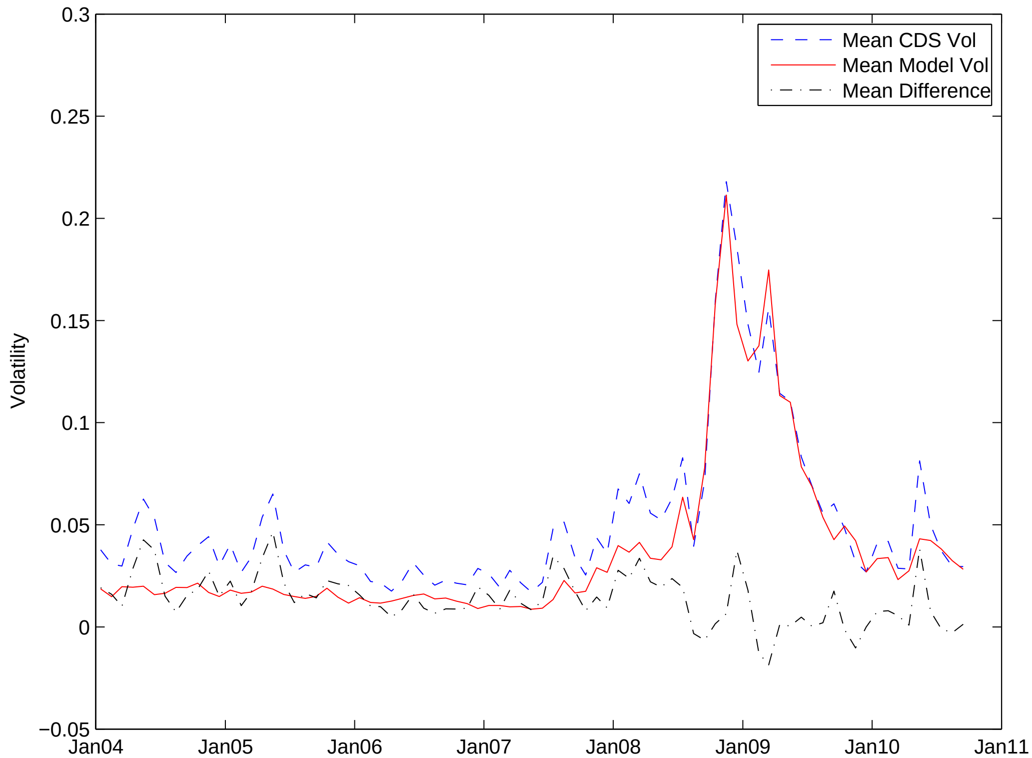


Figure 1: Mean empirical, model, and excess volatility for CDS. Volatilities are calculated each month using daily returns and annualized. The top panel corresponds to our base estimation. The bottom panel corresponds to our conditional volatility estimates.

Appendix

A Interest Rate Calibration

The calibrations for model asset and bond volatility described in sections 2.2 and 2.3 require Vasicek interest rate parameters, κ, θ, and σ_r. Here, we describe the data and methodology used to determine the Vasicek parameters. We use the 3-month and 7-year Constant Maturity Treasury (CMT) series provided by the U.S. Department of Treasury and the Federal Reserve. This yield curve uses close-of-business bid yields for on-the-run securities as inputs and is considered a par curve. Among the inputs are the most recently auctioned 13-week and 7-year Treasury securities.[27]

Using the daily time-series of 3-month Treasury bill rates from 1982 through 2010, we first estimate θ, the long run short-rate, as the mean of the 3-month Treasury time-series, 4.84%. The mean reversion parameter, κ, is estimated to be 0.1526, so that the daily autocorrelation in the Vasicek model matches the sample autocorrelation.

We estimate σ_r, the volatility of the short rate period-by-period by matching the empirically observed volatility of 7-year Treasury bond returns. First, we estimate the return to 7-year Treasury bonds by using the 7-year CMT series. Noting that the CMT curve is a par curve, a 7-year bond at time t with a yield coupon rate equal to the 7-year yield (y_t) trades at par. At $t+1$, the value of this bond can be re-calculated using y_{t+1} to discount the coupon rate of y_t and the par value of the bond. We can then calculate the return to a 7-year Treasury bond. Using daily returns, we calculate volatilities each month and using monthly returns, we calculate volatilities each year.

[27] An alternative to using the CMT series would be to use CRSP Treasury Fixed Term Indices. However, bonds in this series are not necessarily on-the-run and may be less liquid. We do find that the returns to 7-year bonds calculated using our methodology below and the returns to the 7-year bond in the CRSP data have a correlation of 0.98.

Table A.1: **Treasury Volatility**

	Daily Returns Used			Monthly Returns Used		
	mean	med	std	mean	med	std
2003	7.06	7.32	1.39	8.74	N/A	N/A
2004	5.74	5.61	1.12	6.40	N/A	N/A
2005	4.38	4.45	0.61	4.79	N/A	N/A
2006	3.74	3.82	0.63	3.29	N/A	N/A
2007	5.32	5.33	2.01	5.31	N/A	N/A
2008	9.59	9.48	2.42	7.92	N/A	N/A
2009	8.67	8.17	2.94	7.36	N/A	N/A
2010	6.55	6.16	1.68	6.37	N/A	N/A
Full	6.38	5.67	2.56	6.27	6.39	1.78

Annualized volatility of 7-year Treasury bond returns expressed in %. In the first set of columns, daily Treasury returns are used to calculate volatility each month. In the second set of columns, monthly Treasury returns are used to calculate volatilities each year.

Under the Vasicek model, the price of a 7-year Treasury bond is:

$$P_{Treas} = \sum_{t=1}^{14} exp\left(a\left(\frac{t}{2}\right) + b\left(\frac{t}{2}\right) r_0\right)\frac{c}{2} + exp\left(a\left(7\right) + b\left(7\right) r_0\right)$$

and the relation between Treasury volatility and σ_r is given by:

$$\sigma_{Treas}^2 = \left(\frac{\partial \ln P_{Treas}}{\partial r}\right)^2 \sigma_r^2$$

Using the empirical volatility of the 7-year Treasury returns along with the previously estimated κ and θ, we can estimate σ_r each period. Though our estimates of σ_r vary each period with empirical Treasury volatility, the mean estimate is 1.59% when monthly Treasury returns are used and 1.58% when daily Treasury returns are used.

B Model Bond Prices

To calculate corporate bond prices in our setting, it is important to calculate:

$$E^Q\left[\exp\left(-\int_0^{T_2} r_s ds\right)\mathbf{1}_{\{V_{T_1}>K\}}\right] \tag{18}$$

where $T_2 \geq T_1$

$$E^Q\left[\exp\left(-\int_0^{T_2} r_s ds\right) 1_{\{V_{T_1}>K\}}\right] = \exp\left(a(T_2) + b(T_2)r_0\right) N\left(d_3\right) \tag{19}$$

where

$$d_3 = \frac{\ln\left(\frac{V}{K}\right) - a(T_1) - b(T_1)r_0 - \delta T_1 - \frac{1}{2}\Sigma + \sigma_r^2 \frac{b(T_2-T_1)}{\kappa}\left(-b(T_1) + \frac{\exp(-2\kappa T_1)-1}{2\kappa}\right)}{\sqrt{\Sigma}}$$
$$\Sigma = T_1\left(\sigma_v^2 + \frac{\sigma_r^2}{\kappa^2}\right) + \frac{2\sigma_r^2}{\kappa^3}\left(e^{-\kappa T_1} - 1\right) - \frac{\sigma_r^2}{2\kappa^3}\left(e^{-2\kappa T_1} - 1\right)$$

Proof. It can be shown that the above equation satisfies the PDE for arbitrage-free prices:

$$g \cdot r = g_t + g_v(r-\delta)V + \frac{1}{2}g_{vv}V^2\sigma_v^2 + g_r\kappa(\theta - r) + \frac{1}{2}g_{rr}\sigma_r^2 \tag{20}$$

$$\begin{aligned}
g &= \exp(a(T_2) + b(T_2)r_0)N(d_3) \\
g_v &= \frac{Dn(d_3)}{V\sqrt{\Sigma}}, \text{where } D = \exp\left(a(T_2) + b(T_2)r_0\right) \\
g_{vv} &= -\frac{Dd_3n(d_3)}{V^2\Sigma} - \frac{Dn(d_3)}{V^2\sqrt{\Sigma}} \\
g_r &= Db(T_2)N(d_3) - \frac{Dn(d_3)b(T_1)}{\sqrt{\Sigma}} \\
g_{rr} &= D(b(T_2))^2N(d_3) - 2Db(T_2)n(d_3)\frac{b(T_1)}{\sqrt{\Sigma}} - Dn(d_3)d_3\frac{(b(T_1))^2}{\Sigma} \\
g_t &= -DN(d_3)(\theta\kappa b(T_2) + \frac{\sigma_r^2}{2}(b(T_2))^2 - e^{-\kappa T_2}r) \\
&- \frac{Dn(d_3)}{\sqrt{\Sigma}}(-\delta - \theta\kappa b(T_1) - \sigma_r^2(b(T_1))^2 + e^{-\kappa T_1}r - \frac{1}{2}\sigma_v^2 \\
&+ \sigma_r^2\frac{b(T_2-T_1)}{\kappa}(e^{-\kappa T_1} - e^{-2\kappa T_1})) + Dn(d_3)\frac{d_3}{2\Sigma}(\sigma_v^2 + \sigma_r^2(b(T_1))^2)
\end{aligned}$$

After some algebra, we can verify that g satisfies the PDE.

Boundary condition:

$T_1 \to 0$

$$E^Q\left[\exp\left(-\int_0^{T_2} r_s ds\right)\mathbf{1}_{\{V_{T_1}>K\}}\right] \rightarrow \begin{cases} 0 & \text{if V < K} \\ \exp\left(a(T_2-T_1)+b(T_2-T_1)r_{T_1}\right) & \text{if V > K} \end{cases} \quad \blacksquare$$

Special cases include:

1. $T_1 = T_2$

   $$E^Q\left[\exp\left(-\int_0^{T} r_s ds\right)\mathbf{1}_{\{V_T>K\}}\right] = \exp\left(a(T)+b(T)r_0\right)N(d_2)$$

   where d_2 is as defined in the text of the paper

2. $K = 0$ (no default)

   $$E^Q\left[\exp\left(-\int_0^{T} r_s ds\right)\mathbf{1}_{\{V_T>K\}}\right] = \exp\left(a(T)+b(T)r_0\right)$$

Finally, our bond pricing formula (at t = 0) is:

$$\begin{aligned} B = &\sum_{i=1}^{2T}\frac{c}{2}\exp\left(a\left(\frac{i}{2}\right)+b\left(\frac{i}{2}\right)r\right)N\left(d_2\left(\frac{i}{2}\right)\right)+\exp\left(a(T)+b(T)r\right)N\left(d_2(T)\right) \\ &+\sum_{i=1}^{2T}\exp\left(a\left(\frac{i}{2}\right)+b\left(\frac{i}{2}\right)r\right)\left[N\left(d_3\left(\frac{i-1}{2}\right)\right)-N\left(d_2\left(\frac{i}{2}\right)\right)\right]\mathcal{R} \end{aligned} \tag{21}$$

For a zero-coupon bond where the payment contingent on default is paid at maturity, the bond price at t = 0 is:

$$B = \exp\left(a(T)+b(T)r\right)N\left(d_2(T)\right)+exp\left(a(T)+b(T)r\right)\left(1-N\left(d_2(T)\right)\right)\mathcal{R}$$

C Synthetic Floating Rate Bond

C.1 Empirical Volatility

We follow Duffie and Singleton (2003) in constructing a synthetic floating rate corporate bond as a risk-free floating rate bond plus writing a CDS contract. This bond pays quarterly coupon payments equal to the prevailing 3-month interest rate at the previous coupon date (divided by 4) plus $\frac{s}{4}$, where s is the annual CDS premium. Specifically, the synthetic floating rate bond consists of three positions:

1. Risk-free floating rate bond paying quarterly

2. Inflow of $\frac{s}{4}$ each quarter if the underlying remains solvent

3. Outflow of (1 - $\mathcal{R}$) if the underlying defaults

The initial price of this synthetic bond is its face value as a risk-free floater is worth its face value at all ex-coupon dates by arbitrage arguments and the initial CDS spread is set so that the values of (2) and (3) cancel.

To calculate returns, allow one day to elapse. Suppose now that the prevailing CDS spread is $\hat{s}$ and that the prevailing CDS spread for a (5 year - 1 day) CDS that has payments aligned with the above 5 year CDS is the same as the prevailing 5 year CDS spread. We are left to determine the changes in the value of our positions:

(1) To calculate the value of the risk-free floater, note that at the next coupon date, a coupon of $\frac{r_0}{4}$ will be paid and the ex-coupon price of the floater will be its face value. Thus, discount $1 + \frac{r_0}{4}$ at the prevailing interest rate.

(2) & (3) The value of the $\frac{s}{4}$ inflow versus the (1 - $\mathcal{R}$) outflow is the value of a stream of $\frac{s-\hat{s}}{4} = -\frac{\Delta s}{4}$. This is equal to $-\frac{\Delta s}{4}$ times the value of a risky (5 year - 1 day) annuity paying quarterly. The discount rate is the Treasury rate plus the prevailing CDS spread. Finally, add in the accrued CDS premium, properly discounted.

C.2 Model Volatility

The model bond volatility is calculated by noting the three positions that comprise the synthetic floater and applying the formulas derived in Appendix B. In particular, the value of the stream of CDS premia is:

$$\sum_{i=1}^{4T} exp\left(a\left(\frac{i}{4}\right)+b\left(\frac{i}{4}\right)r\right)N\left(d_2\left(\frac{i}{4}\right)\right)\frac{s}{4} \tag{22}$$

The value of the outflow contingent on default is:

$$\sum_{i=1}^{4T} exp\left(a\left(\frac{i}{4}\right)+b\left(\frac{i}{4}\right)r\right)\left[N\left(d_3\left(\frac{i-1}{4}\right)\right)-N\left(d_2\left(\frac{i}{4}\right)\right)\right](1-\mathcal{R}) \tag{23}$$

The value of a default-free floating rate bond can be calculated by noting that the value of a floating rate Treasury at ex-coupon dates is equal to its par value. Thus, we can take the value of a floating rate Treasury at t = 0.25, add the coupon payment, and discount back to t = 0. The value of a floating rate bond is then:

$$\exp\left(a\left(\frac{1}{4}\right)+b\left(\frac{1}{4}\right)r\right)\left(\frac{r_0}{4}+1\right) \tag{24}$$

The value of the synthetic floating rate bond can then be calculated as B = (22) - (23) + (24) and the sensitivities to asset value and interest rates can be calculated as $\frac{\partial \ln B}{\partial \ln V}$ and $\frac{\partial \ln B}{\partial r}$, respectively.

D Standard Errors

In determining the statistical significance of excess volatility, our moment condition for estimating excess volatility is $\hat{\mu} - E(\hat{\sigma}_D - \sigma_D^{Merton}) = 0$ where bond and time subscripts are suppressed. Estimates of σ_D^{Merton} that are noisy, but have errors independent of the errors of the other σ_D^{Merton} estimates will increase the standard error of $\hat{\mu}$. However, given the dependence of σ_D^{Merton} on firm-level parameters, there is likely correlation within firm. To the extent that equity volatility is correlated across firms within a period time, there may also be within period error correlation. Thus, we allow for two-way clustering as in Petersen (2008) and Cameron, Gelbach, and Miller (2011). In the notation of Cameron, Gelbach, and

Miller (2011), our variance matrix is:

$$\hat{V}[\hat{\beta}] = \hat{V}^{G}[\hat{\beta}] + \hat{V}^{H}[\hat{\beta}] - \hat{V}^{G\cap H}[\hat{\beta}] \tag{25}$$

where $\hat{V}^{G}[\hat{\beta}]$ is the variance matrix clustered by firm, $\hat{V}^{H}[\hat{\beta}]$ is the variance matrix clustered by time, and $\hat{V}^{G\cap H}[\hat{\beta}]$ is the variance matrix clustered by firm-time. Cameron, Gelbach, and Miller (2011) show that this variance estimate is consistent under the very mild assumption that errors that do not share a group are uncorrelated. Since cluster-robust standard errors do not require a function form for the cluster error variance matrices, they allow for quite general error correlation, including autocorrelated errors.

The main concern when using two-way clustering to account for cross-sectional and time-series error correlation (other than specifying the correct clusters) is whether there are a sufficient number of clusters. The theory is asymptotic and requires $min(G, H) \to \infty$. This is particularly a concern when we use monthly returns to calculate volatilities each year as there are a small number of cross-sections. To address this, we adopt a wild cluster bootstrap-t procedure, following Cameron, Gelbach, and Miller (2008) that allows for one-way clustering. Empirically, it is the clustering by time that has the greatest effect on the standard errors when monthly returns are used. For CDS volatility, clustering only by time produces a 3.87 t-statistic as compared to a 3.77 t-statistic when clustering by both time and firm. As Cameron, Gelbach, and Miller (2008) show, the wild clustering procedure works well when there are as few as five clusters. We cluster on time and find that our results hold when critical values are calculated using the wild clustering procedure.

When daily CDS returns are used to calculate volatilities each month, serial correlation is much more of a concern. However, in this data, we are less concerned about the number of clusters as we have 81 months of data and over 400 firms. Though two-way clustering allows for consistent standard errors in very general settings including autocorrelation[28], we also

[28]Cameron, Gelbach, and Miller (2011) run Monte Carlo simulations on a case with cross-sectional correlation and also autocorrelation. The sample in their simulation has 21 time periods and 50 observations per cross-section. They find the performance of two-way clustered standard errors to be reasonable as rejection

Table A.2: **Bootstrapped t Statistics**

	$\hat{\sigma}_D - \sigma_D^{Merton}$, s.e. cluster by year				
Sample	obs	mean	t-stat	2.5% Boot t	97.5% Boot t
Straight bonds (month-end prices)	2,883	3.91	4.40	-2.34	2.34
Straight bonds (weighted prices)	2,883	2.19	2.92	-2.27	2.27
CDS (monthly returns)	1,819	2.84	3.87	-2.22	2.22
CDS (daily returns)	24,900	1.92	11.75	-1.98	2.02

Mean excess volatility reported in %. The samples correspond to Tables 5 and 6. *t-stats* clustered by time are reported Bootstrapped t-stat critical values are calculated following the wild cluster bootstrap t-procedure in Cameron, Gelbach, and Miller (2008).

consider taking the average of $\hat{\sigma}_D - \sigma_D^{Merton}$ period-by-period. We then calculate the time-series average of the 81 cross-sectional averages and use Newey-west t-statistics to account for autocorrelation. We find an estimate of 1.89% with a t-statistic of 8.22.

One further concern is that the finite sample properties of cluster-robust standard errors in the presence of skewed data is unclear. While Petersen (2008) and Cameron, Gelbach, and Miller (2011) both run a series of simulations to examine the extent to which different covariance matrix estimates over-reject the null, both papers use fixed effects and errors that are normally distributed. To get a better sense of the effect of skewed data, we run simulations similar to those in Petersen (2008) and Cameron, Gelbach, and Miller (2011), but use our data to impose skewed errors. In particular, we define y_{jt} as the de-meaned excess volatility of bond j in year t and specify

$$y_{jt} = \gamma_i + \alpha_t + e_{jt}$$

where γ_i is a firm fixed-effect, α_t is a time fixed-effect, and e_{jt} is an error term.

We specify the distribution of values of γ_i, α_t, and e_{jt} by regressing y_{jt} on firm and time dummies and obtaining empirical distributions. In our simulations, we draw (with replacement) γ_i for each firm, α_t for each time, and e_{jt} for each observation. For each simulation, we can then calculate a t-stat using two-way clustering. This allows us to generate a distri-

at the 5% level occurs 6.9% of the time. Our sample when daily CDS returns are used is larger in both the time-series and cross-section.

bution of t-statistics for the null of $y_{jt} = 0$ (which is true under our data generating process). In 5,000 simulations using our corporate bond sample, we find that using conventional 5% t-statistics of -1.96 and 1.96, the null hypothesis is rejected 15.64% of the time. However, this over-rejection is asymmetric as the t-stat is less than -1.96 in 14.58% of the simulations and greater than 1.96 in only 1.06% of simulations. That is, the likelihood of making a Type I error with positive excess volatility is low. The asymmetry in our simulations is largely due to the fact that the time fixed-effects, α_t are also positively skewed. Thus, we also consider simulations where we assume that α_t is normally distributed, but continue to sample γ_i and e_{jt} from the empirical distribution. In 5,000 simulations, we find that conventional 5% t-statistics reject the null hypothesis 10.24% of the time. The t-stat is less than -1.96 in 6.46% of cases and greater than 1.96 in 3.78% of cases. Thus, there is some over-rejection, though it is not as severe for positive excess volatility.

E Level of Volatility without Weighting Prices

In constructing monthly bond returns in Section 4.1, we adopt the convention of using transaction size-weighted prices prior to calculating bond returns. As shown in Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011), corporate bonds can have very significant effective bid-ask spreads and this is particularly true for smaller volume trades. Large effective bid-ask spreads imply that there will be a large negative autocovariance when transaction prices are used and this will in turn generate large volatilities. By using transaction size-weighted prices in the main analysis, we weight transactions with lower effective bid-ask spreads more and average across a number of trades (some of which are at bid and others at ask), mitigating these effects. Here, we present the results that use the last trade of a month rather than transaction size-weighted prices and show that using last prices will lead to a greater excess volatility.

As compared to the average excess volatility of 2.19% using volume-weighted prices, the average excess volatility here is higher at 3.91%.

Table A.3: **Data Estimated vs. Model Implied Bond Volatility**

	$\hat{\sigma}_D - \sigma_D^{Merton}$					
	#obs	mean	t-stat	25th	Median	75th
Straight	2,883	3.91	4.11	0.68	2.03	4.82
Callable	9,322	4.21	3.38	0.52	2.19	5.37
By Year						
2003	94	3.53	2.97	0.37	1.65	5.10
2004	362	1.73	10.60	0.52	1.15	2.37
2005	525	3.06	10.50	0.70	1.92	4.24
2006	473	3.17	15.00	0.99	2.16	4.38
2007	358	2.05	8.63	0.48	1.33	2.97
2008	279	8.48	11.46	3.01	6.34	12.54
2009	347	8.41	2.67	-0.13	3.94	11.60
2010	445	2.69	7.14	0.42	1.85	3.73
By Rating						
Aaa & Aa	723	2.03	4.33	0.25	1.31	3.30
A	1,038	2.56	4.80	0.55	1.55	3.49
Baa	621	4.53	3.83	1.00	2.63	5.68
Junk	451	9.01	2.74	2.49	4.82	9.64
By Time to Maturity						
0 - 2	882	1.98	6.99	0.54	1.37	2.80
2 - 4	627	2.89	3.47	0.22	1.27	3.14
4 - 6	311	2.75	3.18	0.38	1.47	3.60
6 - 8	264	3.33	4.67	0.80	1.98	4.72
> 8	799	7.49	4.12	2.52	5.13	9.39

The difference between empirically estimated bond volatility and model-implied bond volatility. Volatilities are expressed in annualized % and are calculated each year using monthly returns. The returns used to calculate empirical bond volatilities are based on the last transaction of the month for a bond. *t-stats* are calculated using standard errors clustered by time and by firm, with the exception of the by-year results which use standard errors clustered by firm.

F Stochastic Volatility

In this section, we describe our implementation of a stochastic volatility model. We specify the firm value and variance processes as:

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sqrt{H_t}\left(\rho dZ_t^{(1)Q} + \sqrt{1-\rho^2}dZ_t^{(2)Q}\right) \tag{26}$$

$$dH_t = \kappa_H(\theta_H - H_t)dt + \sigma_H\sqrt{H_t}dZ_t^{(1)Q} \tag{27}$$

Thus,

$$\sigma_E^2 = \left(\frac{\partial \ln E}{\partial \ln V}\right)^2 H_t + \left(\frac{\partial \ln E}{\partial H}\right)^2 \sigma_H^2 H_t + 2\left(\frac{\partial \ln E}{\partial \ln V}\right)\left(\frac{\partial \ln E}{\partial H}\right)\rho\sigma_H H_t \tag{28}$$

The value of equity and its partials with respect to V and H can be calculated by noting that equity is a call option on firm value and using the methodology in Duffie, Pan, and Singleton (2000). To infer the set of parameters $[\kappa_H, \theta_H, \sigma_H, \rho]$, we start with an empirical time-series of equity variances, estimated each month using daily returns. We then follow the following procedure:

1. Use the time-series of equity variance to estimate initial values for $[\kappa_H, \theta_H, \sigma_H]$. As an initial starting point for ρ, use the correlation between the change in equity variance and log equity returns.

2. Calculate a time-series of H_t using the $[\kappa_H, \theta_H, \sigma_H, \rho]$ and the time-series of equity variances using equation (28).

3. Uisng the time-series of H_t from (2), solve for $[\kappa_H, \theta_H, \sigma_H]$

4. Approximate ρ using H_t, V_t, the parameters $[\kappa_H, \theta_H, \sigma_H]$, and discretized versions of the firm value and variance processes in (26). If $[\kappa_H, \theta_H, \sigma_H, \rho]$ are different from the previous iteration, return to step (2).

After calibrating $[\kappa_H, \theta_H, \sigma_H, \rho]$ and a time-series of H_t above, we can then calculate model bond volatilities using:

$$\sigma_D^2 = \left(\frac{\partial \ln D}{\partial \ln V}\right)^2 H_t + \left(\frac{\partial \ln D}{\partial H}\right)^2 \sigma_H^2 H_t + 2\left(\frac{\partial \ln D}{\partial \ln V}\right)\left(\frac{\partial \ln D}{\partial H}\right)\rho \sigma_H H_t \tag{29}$$

The model value of the CDS part of the synthetic floating rate bond is:

$$CDS = \sum_{t=1}^{20} \frac{s}{4} G_{0,-1}\left(\frac{t}{4}\right) - (1-\mathcal{R})\sum_{t=1}^{20} e^{-r\frac{t}{4}}\left[e^{r\frac{t-1}{4}} G_{0,-1}\left(\frac{t-1}{4}\right) - e^{r\frac{t}{4}} G_{0,-1}\left(\frac{t}{4}\right)\right] \tag{30}$$

where

$$G_{0,-1} = \frac{\psi(0, H_0, t)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im\left[\psi(-iu, H_0, t)e^{iu(\ln k)}\right]}{u} du \tag{31}$$

as in Duffie, Pan, and Singleton (2000).

G Mean-Reverting Leverage Ratio

One potential complication that is not addressed in our main analysis is the possibility that a firm's leverage is mean-reverting. Here, we consider the sensitivity of our volatility estimates to using a mean-reverting leverage model similar to Collin-Dufresne and Goldstein (2001). Define the log asset value and log leverage processes as:

$$dv_t = \left[r - \delta - \frac{1}{2}\sigma_v^2 + \frac{K_t}{V_t}\lambda\left(v_t - \bar{k} - k_t\right)\right] + \sigma_v dZ_t^Q \tag{32}$$

$$dk_t = \lambda\left(v_t - \bar{k} - k_t\right) dt \tag{33}$$

Equations (32) and (33) correspond to equations (11) and (12) in Collin-Dufresne and Goldstein (2001) with one main difference. Collin-Dufresne and Goldstein (2001) assume that the proceeds of new debt issuance are used to repurchase equity, leaving firm value unchanged. Instead, we assume that proceeds from new debt issuance increase firm value by exactly the value of the proceeds and do not change the amount of equity outstanding. This seems more in line with what is typically seen. The important parameters to note above are $\bar{k}$ which is the long-run target value of $\ln(V/K)$ and λ, which governs the speed of mean reversion in leverage.

Defining $x_t \equiv v_t - k_t$, we have:

$$dx_t = \left[r - \delta - \frac{1}{2}\sigma_v^2 + \lambda\left(exp(-x_t) - 1\right)\left(x_t - \bar{k}\right)\right]dt + \sigma_v dZ_t$$

Pricing a zero-coupon bond in this set-up amounts to solving:

$$f \equiv e^{-r(T-t)}E_t^Q\left[1_{x_T>0}\right]$$

This can be solved by numerically solving a PDE through finite difference methods. We then calculate the value of a coupon bond by using a term structure of f. On the equity side, we calculate the sensitivity of equity to asset value, V, by simulating dZ_t^Q.

To check the sensitivity of our calibrations to a mean-reverting leverage ratio model, we first assume that a mean-reverting leverage ratio model is the true data generating process. With a set of assumed parameters in the model, we can then calculate the true bond and equity volatility under the mean-reverting leverage ratio model. Equipped with the true equity volatility, we can then calculate Merton-model implied asset and bond volatilities if we use the observed equity volatility from the mean-reverting leverage ratio model. Effectively, what we examine is a case where the true data generating process is the mean-reverting leverage ratio model, but an Econometrician estimates a Merton model on this data instead. In our analysis, we examine a 5-year bond with a coupon rate of 5%, and a recovery rate of 50%. We also assume that $r = 5\%$ and $\delta = 3\%$. The long-run target leverage, K/V is assumed to be 0.4, which corresponds to $\bar{k} = 0.9163$. In Table A.4, we display the results of analysis for different values of $\frac{K}{V}$, σ_v, and λ.

Generally, estimating a Merton model when the true data generating process is a mean-reverting leverage ratio model produces a higher model bond volatility than the true bond volatility.

Table A.4: **Comparison of Mean-Reverting Leverage Ratio and Merton Model**

Parameters			Mean-Reverting Leverage		Merton	Parameters			Mean-Reverting Leverage		Merton
$\frac{K}{V}$	σ_v	λ	σ_D	σ_E	σ_D	$\frac{K}{V}$	σ_v	λ	σ_D	σ_E	σ_D
0.8	0.3	0.0000	0.1251	0.6112	0.1249	0.4	0.3	0.0000	0.0432	0.4473	0.0431
0.8	0.3	0.1000	0.1418	0.7132	0.1387	0.4	0.3	0.1000	0.0393	0.4579	0.0462
0.8	0.3	0.2000	0.1562	0.7965	0.1430	0.4	0.3	0.2000	0.0355	0.4657	0.0484
0.8	0.3	0.4000	0.1777	0.9249	0.1411	0.4	0.3	0.4000	0.0283	0.4763	0.0515
0.8	0.2	0.0000	0.0985	0.5054	0.0982	0.4	0.2	0.0000	0.0097	0.3109	0.0096
0.8	0.2	0.1000	0.1072	0.5773	0.1177	0.4	0.2	0.1000	0.0075	0.3146	0.0103
0.8	0.2	0.2000	0.1143	0.6346	0.1290	0.4	0.2	0.2000	0.0057	0.3173	0.0108
0.8	0.2	0.4000	0.1224	0.7217	0.1394	0.4	0.2	0.4000	0.0031	0.3213	0.0115
0.8	0.1	0.0000	0.0372	0.3329	0.0368	0.4	0.1	0.0000	0.0000	0.1567	0.0000
0.8	0.1	0.1000	0.0333	0.3627	0.0476	0.4	0.1	0.1000	0.0000	0.1580	0.0000
0.8	0.1	0.2000	0.0288	0.3863	0.0563	0.4	0.1	0.2000	0.0000	0.1591	0.0000
0.8	0.1	0.4000	0.0195	0.4184	0.0683	0.4	0.1	0.4000	0.0000	0.1607	0.0000
0.6	0.3	0.0000	0.0872	0.5313	0.0870	0.2	0.3	0.0000	0.0067	0.3652	0.0067
0.6	0.3	0.1000	0.0888	0.5743	0.0987	0.2	0.3	0.1000	0.0066	0.3660	0.0067
0.6	0.3	0.2000	0.0889	0.6057	0.1064	0.2	0.3	0.2000	0.0063	0.3667	0.0068
0.6	0.3	0.4000	0.0854	0.6487	0.1155	0.2	0.3	0.4000	0.0053	0.3679	0.0070
0.6	0.2	0.0000	0.0465	0.4016	0.0463	0.2	0.2	0.0000	0.0001	0.2442	0.0001
0.6	0.2	0.1000	0.0423	0.4230	0.0533	0.2	0.2	0.1000	0.0001	0.2444	0.0001
0.6	0.2	0.2000	0.0378	0.4379	0.0581	0.2	0.2	0.2000	0.0001	0.2448	0.0001
0.6	0.2	0.4000	0.0285	0.4583	0.0647	0.2	0.2	0.4000	0.0001	0.2456	0.0001
0.6	0.1	0.0000	0.0025	0.2183	0.0025	0.2	0.1	0.0000	0.0000	0.1221	0.0000
0.6	0.1	0.1000	0.0013	0.2236	0.0030	0.2	0.1	0.1000	0.0000	0.1222	0.0000
0.6	0.1	0.2000	0.0006	0.2274	0.0034	0.2	0.1	0.2000	0.0000	0.1224	0.0000
0.6	0.1	0.4000	0.0001	0.2331	0.0040	0.2	0.1	0.4000	0.0000	0.1227	0.0000

The columns under "Mean-Reverting Leverage" represent the volatilities under the true data generating process, the Mean-Reverting Leverage Ratio Model. Assumed parameters are T = 5, coupon rate = 5%, recovery rate = 50%, $\delta = 3\%$, and $\bar{k} = 0.9163$. Additional assumed parameters for each row are in the columns labeled "Parameters". The columns under "Merton" represent the Merton model implied bond volatilities when the equity volatility from the true data generating process is used.

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