Class 2: Estimating Market Volatility
Financial Markets, Spring 2021, SAIF

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The importance of measuring market volatility:
▶ Portfolio managers performing optimal asset allocation.
▶ Risk managers assessing portfolio risk (e.g., Value-at-Risk).
▶ Derivatives investors trading non-linear contracts.

Volatility $\sigma$ can be better measured than expected returns $\mu$:
▶ Backward looking: estimate volatility using historical data.
▶ Forward looking: from derivatives prices.

Estimating volatility using financial time series:
▶ SMA: simple moving average model (traditional approach).
▶ EWMA: exponentially weighted moving average model (RiskMetrics).
▶ ARCH and GARCH models (Nobel Prize).

EWMA for covariances and correlations.
Daily Returns on the S&P 500 Index

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The Simple Moving Average Model

- Unlike expected returns, volatility can be measured with better precision using higher frequency data. So let’s use daily data.
- Some have gone into higher frequency by using intra-day data. But micro-structure noises such as bid/ask bounce start to dominate in the intra-day domain. So let’s not go there in this class.
- Suppose in month $t$, there are $N$ trading days, with $R_n$ denoting $n$-th day return. The simple moving average (SMA) model:

$$\sigma = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (R_n)^2}$$

- To get an annualized number: $\sigma \times \sqrt{252}$. (252 trading days per year).
The Monthly SMA Volatility Estimates

Annualized Volatility (%)
The Precision of the SMA Estimates

![SMA estimates of $\sigma$ and their 95% confidence intervals](image1)

The left graph shows the SMA estimates of annualized volatility ($\sigma$) and their 95% confidence intervals over time from 1970 to 2010. The volatility peaks in the late 1980s and early 1990s.

![SMA estimates of $\mu$ and their 95% confidence intervals](image2)

The right graph depicts the SMA estimates of the monthly average of daily returns ($\mu$) and their 95% confidence intervals for the same period. The returns fluctuate significantly, with sharp increases and decreases, especially in the late 1980s and early 1990s.
Time-Varying Volatility and Business Cycles
SMA Volatility and Option-Implied Volatility

![Graph showing SMA Volatility and Option-Implied Volatility over time from 1990 to 2010. The graph compares SMA Vol Estimator and CBOE VXX.](image-url)
Exponentially Weighted Moving Average Model

- The simple moving average (SMA) model fixes a time window and applies equal weight to all observations within the window.
- In the exponentially weighted moving average (EWMA) model, the more recent observation carries a higher weight in the volatility estimate.
- The relative weight is controlled by a decay factor $\lambda$.
- Suppose $R_t$ is today’s realized return, $R_{t-1}$ is yesterday’s, and $R_{t-n}$ is the daily return realized $n$ days ago. Volatility estimate $\sigma$:

\[
\text{Equally Weighted: } \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (R_{t-n})^2} \\
\text{Exponentially Weighted: } \sqrt{(1 - \lambda) \sum_{n=0}^{N-1} \lambda^n (R_{t-n})^2}
\]
EWMA Weighting Scheme

The chart illustrates the weight on past observations over time for different values of \( \lambda \):

- Red line: \( \lambda = 0.8 \)
- Blue line: \( \lambda = 0.94 \)
- Green line: \( \lambda = 0.97 \)

The y-axis represents the weight on past observations, while the x-axis represents the number of days in the past.
SMA and EWMA Estimates after a Crash

Chart 5.2
Log price changes in GBP/DEM and VaR estimates (1.65σ)

Computing EWMA recursively

- One attractive feature of the exponentially weighted estimator is that it can be computed recursively.
- Let $\sigma_{t-1}$ be the EWMA volatility estimator using all the information available on day $t - 1$ for the purpose of forecasting the volatility on day $t$.
- Moving one day forward, it’s now day $t$. After the day is over, we observe the realized return $R_t$.
- We now need to update our EWMA volatility estimator $\sigma_t$ using the newly arrived information (i.e. $R_t$):

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) R_t^2$$
Decay factor, Strong or Weak?

- A strong decay factor (that is, small $\lambda$) underweights the far away events more strongly, making the effective sample size smaller.
- A strong decay factor improves on the timeliness of the volatility estimate, but that estimate could be noisy and suffers in precision.
- On the other hand, a weak decay factor improves on the smoothness and precision, but that estimate could be sluggish and slow in response to changing market conditions.
- So there is a tradeoff.
Fast, Medium, and Slow Decay

Annualized EWMA Volatility Estimate using Daily S&P 500 Index Returns

- $\lambda=0.8$
- $\lambda=0.94$

Annualized Volatility (%) vs. Year (2007-2011)
ARCH and GARCH models

- The ARCH model, autoregressive conditional heteroskedasticity, was proposed by Professor Robert Engle in 1982. The GARCH model is a generalized version of ARCH.
- ARCH and GARCH are statistical models that capture the time-varying volatility:
  \[
  \sigma_t^2 = a_0 + a_1 R_t^2 + a_2 \sigma_{t-1}^2
  \]
- As you can see, it is very similar to the EWMA model. In fact, if we set \( a_0 = 0 \), \( a_2 = \lambda \), and \( a_1 = 1 - \lambda \), we are doing the EWMA model.
- This model has three parameters while the EWMA has only one. So it offers more flexibility (e.g., allows for mean reversion and better captures volatility clustering).

8 October 2003

The Royal Swedish Academy of Sciences has decided that the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, 2003, is to be shared between

Robert F. Engle
New York University, USA

“for methods of analyzing economic time series with time-varying volatility (ARCH)”

and

Clive W. J. Granger
University of California at San Diego, USA

“for methods of analyzing economic time series with common trends (cointegration)”.

Robert F. Engle III
1/2 of the prize
USA
New York University
New York, NY, USA
b. 1942
Our goal is to create the variance-covariance matrix for the key risk factors influencing our portfolio.

For the moment, let’s suppose that there are only two risk factors affecting our portfolio.

Let $R_t^A$ and $R_t^B$ be the day-$t$ realized returns of these two risk factors. The covariance between A and B:

$$
cov_t = \lambda \, cov_{t-1} + (1 - \lambda) \, R_{t-1}^A \times R_{t-1}^B
$$

And their correlation:

$$
corr_t = \frac{cov_t}{\sigma_t^A \sigma_t^B},
$$

where $\sigma_t^A$ and $\sigma_t^B$ are the EWMA volatility estimates.
Negative Correlation between $R^M$ and $\Delta VIX$
Correlation between SPX and SSE
From Volatility Estimates to VaR

from Goldman Sachs 2010 10-K form

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<tr>
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<tbody>
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<td>Interest rates</td>
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<tr>
<td>Equity prices</td>
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<td>66</td>
<td>72</td>
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<tr>
<td>Currency rates</td>
<td>32</td>
<td>36</td>
<td>30</td>
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<tr>
<td>Commodity prices</td>
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<td>36</td>
<td>44</td>
</tr>
<tr>
<td>Diversification effect</td>
<td>(92)</td>
<td>(96)</td>
<td>(108)</td>
</tr>
<tr>
<td>Total</td>
<td>$134</td>
<td>$218</td>
<td>$180</td>
</tr>
</tbody>
</table>

1. Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.