Class 2: Estimating Market Volatility
Financial Markets, Spring 2021, SAIF

Jun Pan

Shanghai Advanced Institute of Finance (SAIF)
Shanghai Jiao Tong University

May 29-30, 2021
The importance of measuring market volatility:
- Portfolio managers performing optimal asset allocation.
- Risk managers assessing portfolio risk (e.g., Value-at-Risk).
- Derivatives investors trading non-linear contracts.

Volatility $\sigma$ can be better measured than expected returns $\mu$:
- Backward looking: estimate volatility using historical data.
- Forward looking: from derivatives prices.

Estimating volatility using financial time series:
- SMA: simple moving average model (traditional approach).
- EWMA: exponentially weighted moving average model (RiskMetrics).
- ARCH and GARCH models (Nobel Prize).

EWMA for covariances and correlations.
The Simple Moving Average Model

- Unlike expected returns, volatility can be measured with better precision using higher frequency data. So let’s use daily data.
- Some have gone into higher frequency by using intra-day data. But micro-structure noises such as bid/ask bounce start to dominate in the intra-day domain. So let’s not go there in this class.
- Suppose in month \( t \), there are \( N \) trading days, with \( R_n \) denoting \( n \)-th day return. The simple moving average (SMA) model:

\[
\sigma = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (R_n)^2}
\]

- To get an annualized number: \( \sigma \times \sqrt{252} \). (252 trading days per year).
The Monthly SMA Volatility Estimates
The Precision of the SMA Estimates

![Graphs showing SMA estimates of σ and μ with 95% confidence intervals over time.](image)

**SMA estimates of σ and their 95% confidence intervals**

- **Annualized Volatility (%):** 0 to 140
- **X-axis:** 1970 to 2010 (Yearly increments)

**SMA estimates of μ and their 95% confidence intervals**

- **Monthly Average of Daily Returns (%):** -4 to 2
- **X-axis:** 1970 to 2010 (Yearly increments)
Time-Varying Volatility and Business Cycles
SMA Volatility and Option-Implied Volatility

The graph shows the annualized volatility (%) over time from 1990 to 2010. The blue line represents the SMA Vol Estimator, and the red line represents the CBOE VIX. The volatility fluctuates significantly, with peaks and troughs indicating periods of higher and lower market volatility.
The simple moving average (SMA) model fixes a time window and applies equal weight to all observations within the window.

In the exponentially weighted moving average (EWMA) model, the more recent observation carries a higher weight in the volatility estimate.

The relative weight is controlled by a decay factor \( \lambda \).

Suppose \( R_t \) is today’s realized return, \( R_{t-1} \) is yesterday’s, and \( R_{t-n} \) is the daily return realized \( n \) days ago. Volatility estimate \( \sigma \):

\[
\text{Equally Weighted} \quad \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (R_{t-n})^2}
\]

\[
\text{Exponentially Weighted} \quad \sqrt{(1 - \lambda) \sum_{n=0}^{N-1} \lambda^n (R_{t-n})^2}
\]
EWMA Weighting Scheme

![Graph showing weight on past observations over days in the past for different values of λ (λ=0.8, λ=0.94, λ=0.97)]

- **λ=0.8**
- **λ=0.94**
- **λ=0.97**

**weight on past observations**

**days in the past**

Financial Markets, Spring 2021, SAIF

Class 2: Estimating Market Volatility

Jun Pan 10 / 20
SMA and EWMA Estimates after a Crash

Computing EWMA recursively

- One attractive feature of the exponentially weighted estimator is that it can be computed recursively.
- Let $\sigma_{t-1}$ be the EWMA volatility estimator using all the information available on day $t - 1$ for the purpose of forecasting the volatility on day $t$.
- Moving one day forward, it’s now day $t$. After the day is over, we observe the realized return $R_t$.
- We now need to update our EWMA volatility estimator $\sigma_t$ using the newly arrived information (i.e. $R_t$):

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) R_t^2$$
A strong decay factor (that is, small $\lambda$) underweights the far away events more strongly, making the effective sample size smaller.

A strong decay factor improves on the timeliness of the volatility estimate, but that estimate could be noisy and suffers in precision.

On the other hand, a weak decay factor improves on the smoothness and precision, but that estimate could be sluggish and slow in response to changing market conditions.

So there is a tradeoff.
Fast, Medium, and Slow Decay

Annualized EWMA Volatility Estimate using Daily S&P 500 Index Returns

- $\lambda=0.8$
- $\lambda=0.94$

Annualized Volatility (%)


Class 2: Estimating Market Volatility

Jun Pan 14 / 20
The ARCH model, autoregressive conditional heteroskedasticity, was proposed by Professor Robert Engle in 1982. The GARCH model is a generalized version of ARCH.

ARCH and GARCH are statistical models that capture the time-varying volatility:

\[ \sigma_t^2 = a_0 + a_1 R_t^2 + a_2 \sigma_{t-1}^2 \]

As you can see, it is very similar to the EWMA model. In fact, if we set \( a_0 = 0 \), \( a_2 = \lambda \), and \( a_1 = 1 - \lambda \), we are doing the EWMA model.

This model has three parameters while the EWMA has only one. So it offers more flexibility (e.g., allows for mean reversion and better captures volatility clustering).

8 October 2003

The Royal Swedish Academy of Sciences has decided that the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, 2003, is to be shared between

Robert F. Engle
New York University, USA

“for methods of analyzing economic time series with time-varying volatility (ARCH)”

and

Clive W. J. Granger
University of California at San Diego, USA

“for methods of analyzing economic time series with common trends (cointegration)”.
Our goal is to create the variance-covariance matrix for the key risk factors influencing our portfolio.

For the moment, let’s suppose that there are only two risk factors affecting our portfolio.

Let $R_A^t$ and $R_B^t$ be the day-$t$ realized returns of these two risk factors. The covariance between A and B:

$$\text{cov}_{t+1} = \lambda \text{cov}_t + (1 - \lambda) R_A^t \times R_B^t$$

And their correlation:

$$\text{corr}_{t+1} = \frac{\text{cov}_{t+1}}{\sigma_A^{t+1} \sigma_B^{t+1}}$$

where $\sigma_A^{t+1}$ and $\sigma_B^{t+1}$ are the EWMA volatility estimates.
Negative Correlation between $R^M$ and $\Delta VIX$

![Graph showing correlation between $R^M$ and $\Delta VIX$ over time.](image)
Correlation between SPX and SSE
From Volatility Estimates to VaR

*from Goldman Sachs 2010 10-K form*

<table>
<thead>
<tr>
<th>Risk Categories</th>
<th>December 2010</th>
<th>December 2009</th>
<th>November 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rates</td>
<td>$93</td>
<td>$176</td>
<td>$142</td>
</tr>
<tr>
<td>Equity prices</td>
<td>68</td>
<td>66</td>
<td>72</td>
</tr>
<tr>
<td>Currency rates</td>
<td>32</td>
<td>36</td>
<td>30</td>
</tr>
<tr>
<td>Commodity prices</td>
<td>33</td>
<td>36</td>
<td>44</td>
</tr>
<tr>
<td>Diversification effect</td>
<td>(92)</td>
<td>(96)</td>
<td>(108)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$134</strong></td>
<td><strong>$218</strong></td>
<td><strong>$180</strong></td>
</tr>
</tbody>
</table>

1. Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.