Class 3: Optimal Risk-Taking in Theory

Financial Markets, Fall 2020, SAIF

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November 23, 2020
Mean-variance analysis:
- Abstracting from reality, we model investors with utility functions.
- A mean-variance investor likes expected returns $\mu$ and dislikes risk $\sigma$.

Optimal risk and return tradeoff:
- We ask the investor to make the optimal decision in choosing
  - risky asset with random return $R_t^M$,
  - and riskfree asset with constant return $r_f$.
- Key intuition: the optimal risk and return tradeoff.

Diversification and the optimal risky portfolio:
- We introduce multiple risky assets: $R_t^1$ and $R_t^2$.
- Key intuition: the power of diversification.
- The optimal mix of $R_t^1$ and $R_t^2$ becomes the optimal risky portfolio, which has the highest Sharpe ratio.
## Policy Portfolio, Harvard Management Company, 2002

<table>
<thead>
<tr>
<th>Category</th>
<th>Min</th>
<th>Policy</th>
<th>Max</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic equities</td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>80% S&amp;P 500, 8% S&amp;P 400, 12% Russell 2000</td>
</tr>
<tr>
<td>Foreign equities</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>93% EAFE, 7% MSCI Small Cap ex US ex EAFE</td>
</tr>
<tr>
<td>Emerging markets</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>80% MSCI EM Investable, 20% MSCI EM Inv + 5%</td>
</tr>
<tr>
<td>Private equities</td>
<td>8</td>
<td>13</td>
<td>18</td>
<td>Cambridge Associates Weighted Composite</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>30</td>
<td>43</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Absolute return</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20% equity composite, 20% LIBOR+5%, 60% funds of funds</td>
</tr>
<tr>
<td>High-yield</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>60% Sal. High-Yield/Bankrupt Weighted Composite, 40% EMBI+</td>
</tr>
<tr>
<td>Commodities</td>
<td>8</td>
<td>13</td>
<td>18</td>
<td>23% GSCI and 77% NCREIF Timberland Index</td>
</tr>
<tr>
<td>Real estate</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>50% CPI+6, 25% NCREIF, 25% REIT. Leverage adjusted</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>25</td>
<td>40</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Domestic bonds</td>
<td>6</td>
<td>11</td>
<td>21</td>
<td>Lehman 5+ year Treasury Index</td>
</tr>
<tr>
<td>Foreign bonds</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>J.P. Morgan Non U.S.</td>
</tr>
<tr>
<td>Inflation-indexed</td>
<td>0</td>
<td>6</td>
<td>15</td>
<td>Salomon 5+ year TIPS</td>
</tr>
<tr>
<td>Cash</td>
<td>-10</td>
<td>-5</td>
<td>10</td>
<td>One-month LIBOR</td>
</tr>
</tbody>
</table>
One Risky and One Riskfree

Mean-variance investor:

\[ \text{Utility} = \text{mean} - \frac{1}{2} \times \text{risk aversion} \times \text{variance}. \]

Portfolio weights:

- Invest \( y \) in the risky portfolio \( R^M_t \)
- Leave \( 1 - y \) in riskfree \( r_f \)

Portfolio return:

\[ R^y_t = y R^M_t + (1 - y) r_f. \]

The optimal portfolio weight:

\[ y^* = \frac{\text{risk premium}}{\text{variance} \times \text{risk aversion}} = \frac{E (R^M_t) - r_f}{\text{var} (R^M_t) \times \text{risk aversion}}. \]
Two Risky and One Riskfree

- Portfolio weights: invest $w_1$ in risky asset one $R^1_t$ and $w_2$ in risky asset two $R^2_t$

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

- Portfolio return:

$$R^w_t = w_1 R^1_t + w_2 R^2_t + (1 - w_1 - w_2) r_f$$

- Risk premium:

$$\text{risk premium} = \begin{pmatrix} E(R^1_t) - r_f \\ E(R^2_t) - r_f \end{pmatrix} = \begin{pmatrix} \mu_1 - r_f \\ \mu_2 - r_f \end{pmatrix}$$

- Variance-Covariance:

$$\Sigma = \begin{pmatrix} \text{variance 1} & \text{covariance} \\ \text{covariance} & \text{variance 2} \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix}$$
The Optimal Risky Portfolio

- The Optimal Portfolio Weights:
  \[ w^* = \frac{1}{\text{risk aversion}} \times \Sigma^{-1} \times \text{risk premium} \]

- The Optimal Risky Portfolio Weights:
  \[
  \frac{1}{\sum_{i=1}^{N}(w_i^*)} \begin{pmatrix}
  w_1^* \\
  w_2^* \\
  \cdot \\
  \cdot \\
  w_i^* \\
  \cdot \\
  w_N^*
  \end{pmatrix}
  \]

- Investors with different risk aversion hold the same optimal risky portfolio, differing only on their relative weight on the risky portfolio.
- It is also the tangent portfolio, the portfolio with the highest Sharpe ratio.
Matrix Operations

Some useful tips for matrix operation in *Excel*:
- the command for summation is still “+”
- the command for multiplication is “mmult”
- the command for inverse, say $\Sigma^{-1}$, is “minverse”

Some useful tips for matrix operation in *Matlab*:
- the command for summation is still “+”
- the command for multiplication is still “*”
- the command for inverse, say $\Sigma^{-1}$, is “inv($\Sigma$)”
Group Assignment and Presentation

Download the data and report the estimates:

- Obtain monthly returns of China’s stock market from my website under Chinese Data and report $\mu$, $\sigma$, and standard error of your estimate of $\mu$.

- Repeat the same for the US. Use the monthly returns data I downloaded from Prof Ken French’s website: US Data. The monthly return will be the column of Mkt-RF plus the column of RF. The two other factors will be useful for us in later classes.

- The above data also contains the monthly US riskfree return $r_f$, which is the column RF. Since in our setting, RF is a constant. So pick your sample period and use the average of the RF. We will do this exercise from the US investor’s perspective. As such, we will use the US riskfree rate.

- For simplicity, let’s ignore the currency risk in this exercise.

- Estimate the correlation $\rho$ between the monthly returns of US and China. Since the correlation varies over time, please indicate the correlation you would like to use.
Group Assignment and Presentation

- Risk and return tradeoff:
  - Let’s assume that the global investor’s risk aversion coefficient is 4.
  - Calculate his/her optimal risky portfolio weight $y^*$ for China and US separately.
  - Add any comments and observations you find interesting.

- The optimal risky portfolio:
  - Construct the optimal risky portfolio using US and China as the two risky assets.
  - Report the Sharpe ratio of your optimal risky portfolio.
  - Report the optimal risky portfolio weights.
  - Suppose that the global investor’s risk aversion coefficient is 4. Report his/her optimal portfolio weights on US, China, and the riskfree asset.
  - Add any comments and observations you find interesting.
Use the above calculations as the core content of your presentation.

Add motivations on why this problem is interesting and timely.

Add discussions and analyses on how sensitive your results are with respect to the parameters, \( \mu, \sigma, \rho \), and risk aversion.

As a financial adviser to this global investor, write a one-page recommendations advising his/her of the key costs and benefits of investing in China.

Add any further discussions that you find interesting.