Class 9: Time-Varying Volatility

Financial Markets, Spring 2020, SAIF

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The importance of measuring market volatility:
- Portfolio managers performing optimal asset allocation.
- Risk managers assessing portfolio risk (e.g., Value-at-Risk).
- Derivatives investors trading non-linear contracts.

Volatility $\sigma$ can be better measured than expected returns $\mu$:
- Backward looking: estimate volatility using historical data.
- Forward looking: from derivatives prices.

Estimating volatility using financial time series:
- SMA: simple moving average model (traditional approach).
- EWMA: exponentially weighted moving average model (RiskMetrics).
- ARCH and GARCH models (Nobel Prize).

EWMA for covariances and correlations.
The Evolution of an Investment Bank

1930: Sidney Weinberg takes over (builds Investment Banking Business)

1929: Crash

Time of very little business and virtually no profits

1929-32: Gov’t antitrust case against 17 banks

1924-29: Proliferation of Investment Trusts; Cheap & easy credit fuels leverage

1913: Federal Reserve Act

1919-20: Financial Panic

1969-70: Goldman Sachs Trading Corporation

1984: Goldman+ Sachs

1990: Firm Capital: $100K

1990: Marcus Goldman
Buy/Sell Commercial Papers

1990: Whitehead and Weinberg
14 business principles “Our clients’ interests always come first”

1984: Lead Underwriter of Ford IPO

1975: Lead Underwriter with Lehman

1982: Sears IPO

1986: Hires from Salomon (fixed income)

1987: Huge trading losses in FICC

1990-91: Record year in trading profits

1991-95: Corzine and Paulson

1996-2000: Firm-wide risk management

2000: IPO; CEO Paulson

2001: Acquired SLK (NYSE specialist)

2006: CEO Blankfein

2008: LTCM crisis

2009: IPO withdrawn

2008-10: Firm-wide risk management

1984: Acquired J. Aron (commodity)

1990-91: Hires from Salomon (fixed income)

1990-91: Huge trading losses in FICC

1994: Record year in trading profits

1994: Rubin and Friedman

1997: Freeman, head of risk arbitrage arrested for insider trading

1969-94: Paulson
Derivatives Losses by Non-Financial Corporations in 1990s

- Orange County: $1.7 billion, leverage (reverse repos) and structured notes
- Showa Shell Sekiyu: $1.6 billion, currency derivatives
- Metallgesellschaft: $1.3 billion, oil futures
- Barings: $1 billion, equity and interest rate futures
- Codelco: $200 million, metal derivatives
- Proctor & Gamble: $157 million, leveraged currency swaps
- Air Products & Chemicals: $113 million, leveraged interest rate and currency swaps
- Dell Computer: $35 million, leveraged interest rate swaps
- Louisiana State Retirees: $25 million, IOs/POs
- Arco Employees Savings: $22 million, money market derivatives
- Gibson Greetings: $20 million, leveraged interest rate swaps
- Mead: $12 million, leveraged interest rate swaps
By early 1990s, the increasing activity in securitization and the increasing complexity in the financial instruments made the trading books of many investment banks too complex and diverse for the chief executives to understand the overall risk of their firms.

Market risk management tools such as Value-at-Risk are ways to aggregate the firm-wide risk to a set of numbers that can be easily communicated to the chief executives. By the mid-1990s, most Wall Street firms have developed risk measurement into a firm-wide system.

Daily estimates of market volatility, along with correlations across financial assets, constitute the key inputs to Value-at-Risk. JP Morgan’s RiskMetrics uses exponentially weighted moving average (EWMA) model to estimate the volatilities and correlations of over 480 financial time series in order to construct a variance-covariance matrix of 480x480.
Daily Returns on the S&P 500 Index

Daily SPX Returns (%)
The Simple Moving Average Model

- Unlike expected returns, volatility can be measured with better precision using higher frequency data. So let’s use daily data.
- Some have gone into higher frequency by using intra-day data. But micro-structure noises such as bid/ask bounce start to dominate in the intra-day domain. So let’s not go there in this class.
- Suppose in month $t$, there are $N$ trading days, with $R_n$ denoting $n$-th day return. The simple moving average (SMA) model:

$$\sigma = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (R_n)^2}$$

- To get an annualized number: $\sigma \times \sqrt{252}$. (252 trading days per year).
The Monthly SMA Volatility Estimates

Annualized Volatility (%)
The Precision of the SMA Estimates

**SMA estimates of \( \sigma \) and their 95% confidence intervals**

**SMA estimates of \( \mu \) and their 95% confidence intervals**
Time-Varying Volatility and Business Cycles
SMA Volatility and Option-Implied Volatility

![Graph showing annualized volatility over time]

- **SMA Vol Estimator**
- **CBOE VXXO**

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The simple moving average (SMA) model fixes a time window and applies equal weight to all observations within the window.

In the exponentially weighted moving average (EWMA) model, the more recent observation carries a higher weight in the volatility estimate.

The relative weight is controlled by a decay factor $\lambda$.

Suppose $R_t$ is today’s realized return, $R_{t-1}$ is yesterday’s, and $R_{t-n}$ is the daily return realized $n$ days ago. Volatility estimate $\sigma$:

\[
\text{Equally Weighted} \quad \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (R_{t-n})^2}
\]

\[
\text{Exponentially Weighted} \quad \sqrt{(1 - \lambda) \sum_{n=0}^{N-1} \lambda^n (R_{t-n})^2}
\]
EWMA Weighting Scheme
SMA and EWMA Estimates after a Crash

One attractive feature of the exponentially weighted estimator is that it can be computed recursively.

Let $\sigma_t$ be the EWMA volatility estimator using all the information available on day $t - 1$ for the purpose of forecasting the volatility on day $t$.

Moving one day forward, it's now day $t$. After the day is over, we observe the realized return $R_t$.

We now need to update our EWMA volatility estimator $\sigma_{t+1}$ using the newly arrived information (i.e. $R_t$). It turns out that we can do so by

$$\sigma_{t+1}^2 = \lambda \sigma_t^2 + (1 - \lambda) R_t^2$$
A strong decay factor (that is, small $\lambda$) underweights the far away events more strongly, making the effective sample size smaller.

A strong decay factor improves on the timeliness of the volatility estimate, but that estimate could be noisy and suffers in precision.

On the other hand, a weak decay factor improves on the smoothness and precision, but that estimate could be sluggish and slow in response to changing market conditions.

So there is a tradeoff.
Fast, Medium, and Slow Decay

Annualized EWMA Volatility Estimate using Daily S&P 500 Index Returns

Annualized Volatility (%)

2007 2008 2009 2010 2011

\( \lambda = 0.8 \)
\( \lambda = 0.94 \)

Annualized EWMA Volatility Estimate using Daily S&P 500 Index Returns

Annualized Volatility (%)

2007 2008 2009 2010 2011

\( \lambda = 0.97 \)
\( \lambda = 0.94 \)
The ARCH model, autoregressive conditional heteroskedasticity, was proposed by Professor Robert Engle in 1982. The GARCH model is a generalized version of ARCH.

ARCH and GARCH are statistical models that capture the time-varying volatility:

\[ \sigma_{t+1}^2 = a_0 + a_1 R_t^2 + a_2 \sigma_t^2 \]

As you can see, it is very similar to the EWMA model. In fact, if we set \( a_0 = 0 \), \( a_2 = \lambda \), and \( a_1 = 1 - \lambda \), we are doing the EWMA model.

This model has three parameters while the EWMA has only one. So it offers more flexibility (e.g., allows for mean reversion and better captures volatility clustering).
Press Release: The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel
2003

8 October 2003

The Royal Swedish Academy of Sciences has decided that the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, 2003, is to be shared between

**Robert F. Engle**
New York University, USA

“for methods of analyzing economic time series with time-varying volatility (ARCH)”

and

**Clive W. J. Granger**
University of California at San Diego, USA

“for methods of analyzing economic time series with common trends (cointegration)”.

---

**Robert F. Engle III**

1/2 of the prize
USA

New York University
New York, NY, USA

b. 1942
Review: EWMA Volatility Estimates
Our goal is to create the variance-covariance matrix for the key risk factors influencing our portfolio.

For the moment, let’s suppose that there are only two risk factors affecting our portfolio.

Let $R_t^A$ and $R_t^B$ be the day-$t$ realized returns of these two risk factors. The covariance between A and B:

$$\text{cov}_{t+1} = \lambda \text{cov}_t + (1 - \lambda) R_t^A \times R_t^B$$

And their correlation:

$$\text{corr}_{t+1} = \frac{\text{cov}_{t+1}}{\sigma_{t+1}^A \sigma_{t+1}^B} ,$$

where $\sigma_{t+1}^A$ and $\sigma_{t+1}^B$ are the EWMA volatility estimates.
Negative Correlation between $R^M$ and $\Delta VIX$
Correlation between SPX and SSE

EWMA Correlation of SPX and SSE using 5-Day Returns

Correlation (%)

-20 -10 0 10 20 30 40


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From Volatility Estimates to VaR

*from Goldman Sachs 2010 10-K form*

<table>
<thead>
<tr>
<th>Risk Categories</th>
<th>December 2010</th>
<th>December 2009</th>
<th>November 2008</th>
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<tbody>
<tr>
<td>Interest rates</td>
<td>$ 93</td>
<td>$176</td>
<td>$142</td>
</tr>
<tr>
<td>Equity prices</td>
<td>68</td>
<td>66</td>
<td>72</td>
</tr>
<tr>
<td>Currency rates</td>
<td>32</td>
<td>36</td>
<td>30</td>
</tr>
<tr>
<td>Commodity prices</td>
<td>33</td>
<td>36</td>
<td>44</td>
</tr>
<tr>
<td>Diversification effect</td>
<td>(92)</td>
<td>(96)</td>
<td>(108)</td>
</tr>
<tr>
<td>Total</td>
<td><strong>$134</strong></td>
<td><strong>$218</strong></td>
<td><strong>$180</strong></td>
</tr>
</tbody>
</table>

1. Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.
The Main Takeaways